Heidelberg University
Summer term 2024

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Geometric Algebra for Physicists - Problem Set 1

to be handed in by **TBA1**

1 Rotors and matrices

If we want to handle rotations in two dimensions, we normally either use complex numbers or two-by-two rotation matrices. Geometric algebra allows us to link these two concepts together.
Let $\mathbf{v} = v_1 e_1 + v_2 e_2$ be some 2D vector. We want to rotate it by an angle θ. In 2D geometric algebra, we construct the even multivector

$$z = \exp(e_{12}\theta) = \cos(\theta) + e_{12}\sin(\theta) \tag{1.1}$$

We can apply it from the right to get the rotated vector.

$$\mathbf{v}' = \mathbf{v}z \tag{1.2}$$

1. Derive the 2×2 rotation matrix $\mathsf{R}(\theta)$ corresponding to this rotation. Use the expression for $\mathbf{v}z$.
2. We might have also multiplied z from the left, $\mathbf{v} \mapsto z\mathbf{v}$. Is the result any different?
3. We can convert the vector $\mathbf{v}$ to a "complex number" (even 2D multivector) by premultiplying it with e_1, i.e. $v = e_1\mathbf{v}$. What is the expression for v in terms of the components of $\mathbf{v}$? Can we also rotate it by multiplying z? Does it matter from which side?
4. Could we also have chosen another vector than e_1 to convert $\mathbf{v}$ to a complex number?

2 Pythagorean theorem

Given two orthogonal vectors $\mathbf{a}$ and $\mathbf{b}$, expand the geometric product $(\mathbf{a}+\mathbf{b})^2 = (\mathbf{a}+\mathbf{b})(\mathbf{a}+\mathbf{b})$.

3 Multivector expressions

Simplify the following multivector-valued expressions. Use $e_i^2 = 1$ for all i.

1. e_{1234}^2
2. $e_{12}^{-3} \wedge e_{34}$
3. $e_{41} \cdot e_{1234}$
4. $e_{12} \cdot (e_{23} \cdot e_2)$
5. $e_1 \wedge 1$
6. $(e_{12} + e_{23})^2$
7. $e_{123} \cdot (e_1 \wedge e_{12})$
8. $\langle (e_1 \wedge e_4) e_{13} e_2^{-1} \rangle_3$
9. $-\langle (1 + e_{12})^5 \rangle_2$
10. $(\cos\gamma + e_{12}\sin\gamma)^2$