Hadronic Effects in B-Decays

(from the b-quark to the B-meson)

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Outline

1. $b \to c\bar{u}$ decays
   - $b \to c\bar{u}$ decays at Born level
   - Quantum-loop contributions to $b \to c\bar{u}$ decay
   - From $b \to c\bar{u}$ to $B \to D\pi$
2. $b \to s(d)\, q\bar{q}$ decays
   - Penguin operators
   - Charmless non-leptonic decays: $B \to \pi\pi$ and $B \to \pi K$
3. $b \to s(d)\gamma$ decays
   - Inclusive $B \to X_s\gamma$ decays
   - Sensitivity to the B-meson shape function
   - Exclusive $B \to K^*\gamma$ decay
   - $B \to K^*\ell^+\ell^-$

...Some introductory remarks ...

- Physical processes involve different typical energy/length scales
- Different physical phenomena are described in terms of different degrees of freedom and different input parameters.
- In particle physics, the description of low-energy phenomena can be formulated as an effective theory, where the physics at small scales (high energies) is irrelevant, or can be absorbed into appropriate effective quantities.

In our case, the relevant scales are:

The scale of possible new physics	:	$M_X > 100$ GeV
The electroweak scale	:	$M_W \simeq 80$ GeV
The heavy quark masses	:	$m_b \simeq 4.6$ GeV
		$m_c \simeq 1.4$ GeV
The strong interaction scale	:	$\Lambda_{\rm QCD} \simeq 0.3$ GeV

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Central Notions to be explained

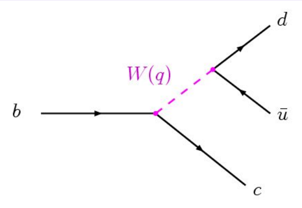
The dynamics of strong interactions in B-decays is very complex and has many faces. I will not be able to cover everything, but I hope that some theoretical and phenomenological concepts become clearer ...

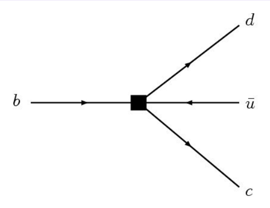
- Factorization
  - separation of scales in perturbation theory
  - simplification of exclusive hadronic matrix elements
- Operators in the weak effective Hamiltonian (current-current, strong penguins, electroweak penguins)
- Naive factorization and its improvement (BBNS)
- Form factors, light-cone distribution amplitudes, ...
- Isospin and $SU(3)_F$
- Inclusive decays and shape functions

First Example: $b \to c d \bar{u}$ decays

$b \longrightarrow c d \bar{u}$ decay at Born level

Full theory (SM) $\longrightarrow$ Fermi model





$$\left(\frac{g}{2\sqrt{2}}\right)^2 J_\alpha^{(b\to c)} \, \frac{-g^{\alpha\beta}+\frac{q^\alpha q^\beta}{M_W^2}}{q^2-M_W^2} \, \bar{J}_\beta^{(d\to u)} \quad \xrightarrow{|q|\ll M_W} \quad \frac{G_F}{\sqrt{2}} \, J_\alpha^{(b\to c)} \, g^{\alpha\beta} \, \bar{J}_\beta^{(d\to u)}$$

- Energy/Momentum transfer limited by mass of decaying b-quark.
- b-quark mass much smaller than W-boson mass.

$$\boxed{|q| \leq m_b \ll m_W}$$

Effective Theory:

- Analogously to muon decay, transition described in terms of current-current interaction, with left-handed charged currents

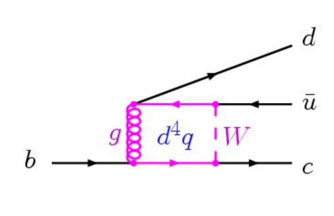
$$J_\alpha^{(b\to c)} = V_{cb}\,[\bar c\,\gamma_\alpha(1-\gamma_5)\,b]\ , \qquad \bar J_\beta^{(d\to u)} = V_{ud}^*\,[\bar d\,\gamma_\beta(1-\gamma_5)\,u]$$

- Effective operators only contains light fields ("light" quarks, electron, neutrinos, gluons, photons). $\surd$

- Effect of large scale M_W in effective Fermi coupling constant:

$$\boxed{\frac{g^2}{8M_W^2} \longrightarrow \frac{G_F}{\sqrt{2}} \simeq 1.16639\cdot 10^{-5}\,\text{GeV}^{-2}}$$

Quantum-loop contributions to $b \to cd\bar{u}$ decay



- Momentum q of the W-boson is an internal loop momentum that is integrated over and can take values between $-\infty$ and $+\infty$.

$\Rightarrow$ We cannot simply expand in $|q|/M_W$!

$\Rightarrow$ Need a method to separate the cases $|q| \gg M_W$ and $|q| \ll M_W$.

$\longrightarrow$ Factorization

For illustration – 1-dimensional toy integral

"Full theory" integral with two distinct "scales": $m \ll M$

$$I \equiv \int_0^\infty dk \, \frac{M}{(k+M)(k+m)} = -\frac{M \ln[m/M]}{M-m} \approx -\ln\left[\frac{m}{M}\right]$$

In this simple example, the calculation in the "full theory" is easy.

Why to switch to effective theories anyway?

- Integrals with different mass scales become more difficult to calculate in realistic cases, in particular at higher-loop order.
- The product of coupling constants and large logarithms may be too large for fixed-order perturbation theory to work,

$$g^2 \ln \frac{m}{M} \sim \mathcal{O}(1)$$

- The physics at the small scale m might involve non-perturbative phenomena, for instance, if m represents the light quark masses in QCD.

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Hard and soft regions of an integral via cut-off procedure

Intuitive procedure: introduce a cut-off $m < \mu < M$:

$$I_s^{\rm cut}(\mu) = \int_0^{\mu} dk \, \frac{M}{(k+M)(k+m)} \overset{k \leq \mu \ll M}{\simeq} \int_0^{\mu} dk \, \frac{1}{k+m} \simeq \ln\left[\frac{\mu}{m}\right]$$

$$I_h^{\rm cut}(\mu) = \int_{\mu}^{\infty} dk \, \frac{M}{(k+M)(k+m)} \overset{k \geq \mu \gg m}{\simeq} \int_{\mu}^{\infty} dk \, \frac{M}{(k+M)(k)} \simeq -\ln\left[\frac{\mu}{M}\right]$$

Interpretation:

- The soft (low-energy) part does not depend on $\ln M$.
  Can be calculated within the low-energy effective theory. $\surd$
- The hard (short-distance) does not depend on $\ln m$.
  Take into account by re-adjusting the (Born-level) effective coupling constants. $\surd$
- The cut-off dependence cancels in the combination of soft and hard pieces. $\surd$
- Sub-leading terms correspond to power-suppressed operators.

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Comment:

- For technical reasons, one often uses dimensional regularization instead of momentum cut-offs, to separate hard and soft momentum regions . . .

Back to the real case:

- Hard gluon corrections to the current-current interaction modify the effective coupling constants.
- Colour matrices attached to the quark-gluon vertex,
  $\Rightarrow$ second current-current operator with another colour structure.

$$H_{\rm eff} = -V_{cb} V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_{i=1,2} C_i(\mu)\, \mathcal{O}_i \qquad (b \to c d \bar{u})$$

- The so-called Wilson coefficients $C_i(\mu)$ contain all the information about short-distance physics above the scale μ (SM and NP)

[see Buchalla/Buras/Lautenbacher 1996]

What did we gain?

- At 1-loop, Wilson coefficients have generic form:

$$C_i(\mu) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + \frac{\alpha_s(\mu)}{4\pi} \left(a_i^{(1)} \ln \frac{\mu}{M_W} + \delta_i^{(1)} \right) + \mathcal{O}(\alpha_s^2)$$

- Wilson coefficients depend on the scale μ

"Matching"

For $\mu \sim M_W$ the logarithmic term is small, and $C_i(M_W)$ can be calculated in fixed-order perturbation theory, since $\alpha_s(M_W)/\pi \ll 1$.

$$C_i(M_W) = \begin{Bmatrix} 1 \\ 0 \end{Bmatrix} + \delta_i^{(1)} \frac{\alpha_s(M_W)}{4\pi} + \ldots$$

Here M_W is called the matching scale.

Evolution of effective couplings to small scales

- Remember: soft loop integrals in effective theory involve $\ln \mu/m$.
- In order not to obtain large logarithmic coefficients, we would like to perform calculations in the low-energy effective theory in terms of $C_i(\mu \sim m_b)$ rather than $C_i(M_W)$.
- In perturbation theory, we can calculate the scale dependence:

$$\frac{\partial}{\partial \ln \mu} C_i(\mu) \equiv \gamma_{ji}(\mu)\, C_j(\mu) = \left(\frac{\alpha_s(\mu)}{4\pi}\, \gamma_{ji}^{(1)} + \ldots \right) C_j(\mu)$$

- $\gamma_{ij}(\mu)$ is called anomalous dimension matrix

Comment: RG evolution at leading-log accuracy

- Formal solution of differential equation: (separation of variables)

$$C(\mu) \;=\; C(M)\cdot \exp\left[\int_{\ln M}^{\ln \mu} d\ln\mu'\,\gamma(\mu')\right]$$

- Perturbative expansion of anomalous dimension and β-function

$$\gamma(\mu) \;=\; \frac{\alpha_s}{4\pi}\,\gamma^{(1)} + \ldots$$

$$2\beta(\mu) \;\equiv\; \frac{d}{d\ln\mu}\,\alpha_s(\mu) = -\frac{2\beta_0}{4\pi}\,\alpha_s^2(\mu) + \ldots$$

- change variables, $d\ln\mu = d\alpha_s/2\beta$, to obtain

$$C(\mu) \simeq C(M_W)\cdot\left(\frac{\alpha_s(\mu)}{\alpha_s(M_W)}\right)^{-\gamma^{(1)}/2\beta_0} \qquad \text{(LeadingLogApprox)}$$

Numerical values for $C_{1,2}$

operator:	$\mathcal{O}_1 = (\bar{s}_L \gamma_\mu u_L)(\bar{c}_L \gamma^\mu b_L)$	$\mathcal{O}_2 = (\bar{s}_L \gamma_\mu T^a u_L)(\bar{c}_L \gamma^\mu T^a b_L)$
$C_i(m_b)$:	1.026 (LL) 1.008 (NLL)	-0.514 (LL) -0.303 (NLL)

- depends on $M_W, M_Z, \sin\theta_W, m_t, m_b, \alpha_s$ (SM)
- to be modified in NP scenarios

Summary: Effective Theory for *b*-quark decays

"Full theory" $\leftrightarrow$ all modes propagate Parameters: $M_W, m_q, g, \alpha_s \ldots$	$\uparrow \mu > M_W$
$C_i(M_W) = C_i\big|_{\rm tree} \left(1 + \delta_i^{(1)} \, \frac{\alpha_s(M_W)}{4\pi} + \ldots\right)$	matching: $\mu \sim M_W$
"Eff. theory" $\leftrightarrow$ low-energy modes propagate. High-energy modes are "integrated out". Parameters: $m_b, \alpha_s, C_i(\mu) \ldots$	$\downarrow \mu < M_W$
$\frac{\partial}{\partial \ln \mu} \, C_i(\mu) = \gamma_{ji}(\mu) \, C_j(\mu)$	anomalous dimensions
Expectation values of operators $\langle O_i \rangle$ at $\mu = m_b$. All dependence on M_W absorbed into $C_i(m_b)$	resummation of logs

From $b \to cd\bar{u}$ to $\bar{B}^0 \to D^+\pi^-$

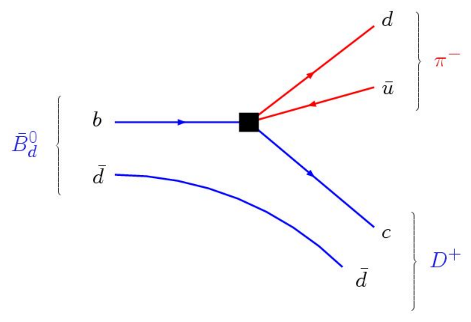
- In experiment, we cannot see the quark transition directly.
- Rather, we observe exclusive hadronic transitions, described by hadronic matrix elements, like e.g.

$$\langle D^+\pi^-|\mathcal{H}^{\rm SM}|\bar{B}_d^0\rangle = V_{cb}V_{ud}^* \frac{G_F}{\sqrt{2}} \sum_i C_i(\mu)\, r_i(\mu)$$

$$r_i(\mu) = \langle D^+\pi^-|\mathcal{O}_i|\bar{B}_d^0\rangle \Big|_{\mu}$$

- The hadronic matrix elements r_i contain QCD (and also QED) dynamics below the scale $\mu \sim m_b$.

"Naive" Factorization of hadronic matrix elements



$$r_i = \underbrace{\langle D^+ | J_i^{(b \to c)} | \bar{B}_d^0 \rangle}\ \underbrace{\langle \pi^- | J_i^{(d \to u)} | 0 \rangle}$$

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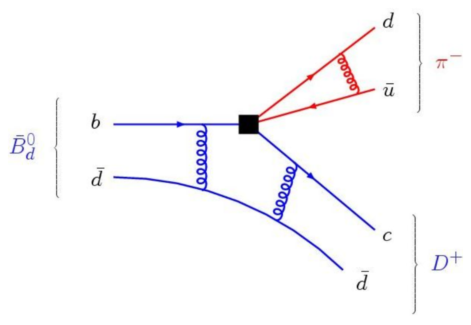

$$r_i = \underbrace{\langle D^+ | J_i^{(b\to c)} | \bar{B}_d^0 \rangle}_{\text{form factor}} \; \underbrace{\langle \pi^- | J_i^{(d\to u)} | 0 \rangle}_{\text{decay constant}}$$

- Part of the gluon effects encoded in simple/universal had. quantities

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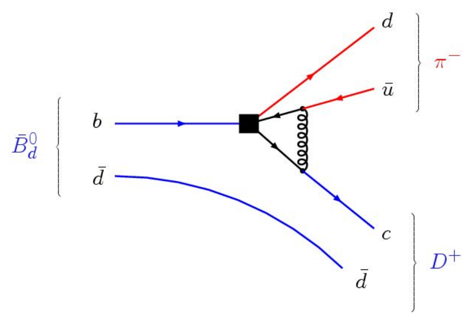

$$r_i(\mu) \quad = \quad \underbrace{\langle D^+|J_i^{(b\to c)}|\bar{B}_d^0\rangle}_{\text{form factor}} \; \underbrace{\langle \pi^-|J_i^{(d\to u)}|0\rangle}_{\text{decay constant}} + \text{corrections}(\mu)$$

- Gluon cross-talk between π^- and $B \to D$ $\Rightarrow$ QCD corrections

QCD factorization [Beneke/Buchalla/Neubert/Sachrajda 2000]

- light quarks in π^- have large energy (in B rest frame)
- gluons from the $B \to D$ transition see "small colour-dipole"

$\Rightarrow$ corrections to naive factorization dominated by gluon exchange at short distances $\sim 1/m_b$

New feature: Light-cone distribution amplitudes $\phi_\pi(u)$

- momenta/energies of light quarks in π^- cannot be neglected compared to reference scale $\mu \sim m_b$
- Short-distance corrections to naive factorization given as convolution

$$r_i(\mu) \simeq \sum_j F_j^{(B\to D)}(m_\pi^2) \int_0^1 du \left(1 + \frac{\alpha_s C_F}{4\pi} t_{ij}(u,\mu) + \ldots\right) f_\pi \, \phi_\pi(u,\mu)$$

- $\phi_\pi(u)$: distribution of momentum fraction u of a quark in the pion.
- $t_{ij}(u,\mu)$: perturbative coefficient function (depends on u)

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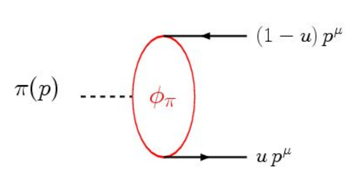
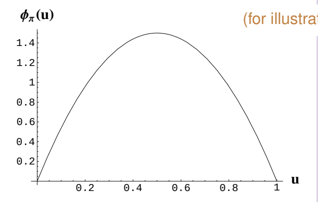
Light-cone distribution amplitude for the pion





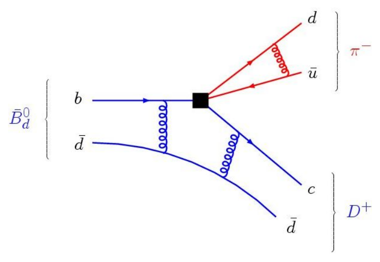
(for illustration only)

- Exclusive analogue of parton distribution function:
  - PDF: probability density (all Fock states)
  - LCDA: probability amplitude (one Fock state, e.g. $q\bar{q}$)
- Phenomenologically relevant $\langle u^{-1}\rangle_\pi \simeq 3.3 \pm 0.3$

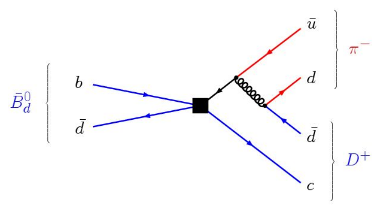
[from sum rules, lattice, exp.]

Complication: Annihilation in $\bar{B}_d \to D^+\pi^-$

Second topology for hadronic matrix element possible:



"Tree" (class-I)

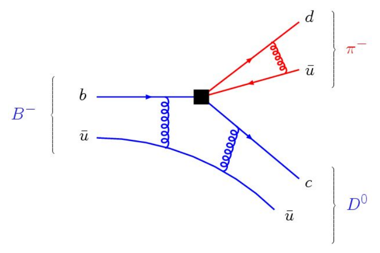


"Annihilation" (class-III)

- annihilation is power-suppressed by Λ/m_b
- numerically difficult to estimate

Still more complicated: $B^- \to D^0\pi^-$

Second topology with spectator quark going into light meson:



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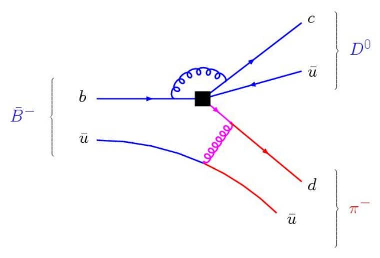
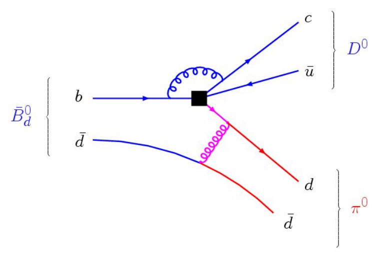


"Tree" (class-II)

- class-II amplitude does not factorize into simpler objects
  (colour-transparency argument does not apply)
- again, it is power-suppressed compared to class-I topology

Non-factorizable: $\bar{B}^0 \to D^0 \pi^0$

In this channel, class-I topology is absent:



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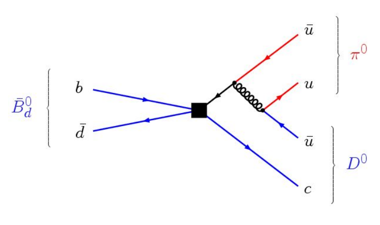

"Annihilation" (class-III)

- The whole decay amplitude is power-suppressed!
- Naive factorization is not even an approximation!

QCDF - Generic statements:

- QCD corrections to hadronic matrix elements match μ dependence of Wilson coefficients $\surd$
- Strong phases are suppressed by α_s or $\Lambda_{\rm QCD}/m_b$

$\rightarrow$ to be tested in experiment

Discussion of hadronic input

- $B \to D$ form factors rather well known (heavy-quark limit, exp. data on $B \to D\ell\nu$) $\surd$
- pion decay constant $\surd$
- pion distribution amplitude: $\surd$
  - expanded into set of polynomials
  - first few coefficients known from lattice/sum rules
- power corrections of order $\Lambda_{\rm QCD}/m_b$ contain genuinely non-perturbative ("non-factorizable") correlations between B, D and π. (?)

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Alternative: Isospin analysis for $B \to D\pi$

- Employ isospin symmetry between (u, d) of strong interactions.
- Final-state with pion ($I = 1$) and D-meson ($I = 1/2$).
- The three possible decay modes (+ CP conjugates) described by only two isospin amplitudes:

$$
\begin{aligned}
\mathcal{A}(\bar{B}_d \to D^+\pi^-) &= \sqrt{\frac{1}{3}}\,\mathcal{A}_{3/2} + \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\
\sqrt{2}\,\mathcal{A}(\bar{B}_d \to D^0\pi^0) &= \sqrt{\frac{4}{3}}\,\mathcal{A}_{3/2} - \sqrt{\frac{2}{3}}\,\mathcal{A}_{1/2}\,, \\
\mathcal{A}(B^- \to D^0\pi^-) &= \sqrt{3}\,\mathcal{A}_{3/2}\,,
\end{aligned}
$$

- QCDF: $\mathcal{A}_{1/2}/\mathcal{A}_{3/2} = \sqrt{2} + \text{corrections}$

Isospin amplitudes from experimental data [BaBar hep-ph/0610027]

$$\left|\frac{\mathcal{A}_{1/2}}{\sqrt{2}\,\mathcal{A}_{3/2}}\right| = 0.655^{+0.015+0.042}_{-0.014-0.042}\,, \qquad \cos\Delta\theta = 0.872^{+0.008+0.031}_{-0.007-0.029}$$

- corrections to naive factorization of order 35%
- relative strong phases from FSI of order 30°

Next Example: $b \to s(d)\, q\bar{q}$ decays

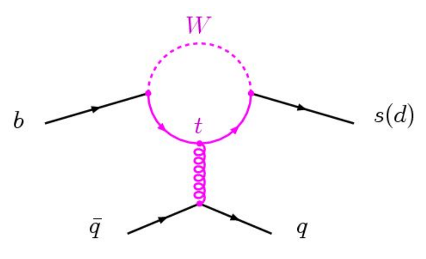
- Now, there are two possible flavour structures:

$$
\begin{aligned}
V_{ub} V^*_{us(d)}\, (\bar{u}_L \gamma_\mu b_L)(\bar{d}(s)_L \gamma^\mu u_L) &\equiv V_{ub} V^*_{us(d)}\, \mathcal{O}_1^u\,, \\
V_{cb} V^*_{cs(d)}\, (\bar{c}_L \gamma_\mu b_L)(\bar{d}(s)_L \gamma^\mu c_L) &\equiv V_{cb} V^*_{cs(d)}\, \mathcal{O}_1^c\,,
\end{aligned}
$$

- Again, α_s corrections induce independent colour structure $\mathcal{O}_2^{u,c}$.

$b \to s(d)\, q\bar{q}$ decays – strong penguin operators

- New feature:
  Penguin diagrams induce additional operator structures



$\longrightarrow V_{tb} V^*_{ts(d)}\, \mathcal{O}_{3-6}$

- Wilson coeff. C_{3-6} numerically suppressed by loop factor.
- Potential sensitivity to new-physics contribution.

Comments on operator basis for $b \to s(d)\, q\bar{q}$

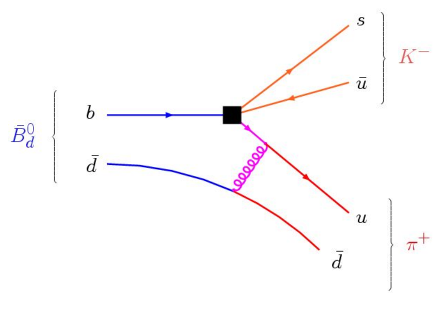
- Also electroweak penguin diagrams with γ/Z instead of gluon.
  $\to$ electroweak penguin operators $\mathcal{O}_{7-10}$
- Also electromagnetic and chromomagnetic operators $\mathcal{O}_7^\gamma$ and $\mathcal{O}_8^g$.
- Mixing between operators under renormalization.
- Use unitarity of CKM matrix,

$$V_{tb} V_{ts(d)}^* = -V_{ub} V_{us(d)}^* - V_{cb} V_{cs(d)}^*$$

yields two sets of operators with different weak phase.

Charmless non-leptonic decays: $B \to \pi\pi$ and $B \to \pi K$

Naive factorization:



- Both final-state mesons are light and energetic.
- Colour-transparency argument applies for class-I and class-II topologies.
- $B \to \pi(K)$ form factors fairly well known (QCD sum rules)

- 4 kinds of partonic momentum configurations (in B rest frame)
  - heavy b quark: $p_b \simeq m_b\,(1, 0_\perp, 0)$
  - soft spectator in B-meson: $p_s \simeq (0, 0_\perp, 0)$
  - collinear quarks in meson$_1$: $p_{c1} \simeq m_b/2\,(1, 0_\perp, +1)$
  - collinear quarks in meson$_2$: $p_{c2} \simeq m_b/2\,(1, 0_\perp, -1)$

- Internal interactions:

	heavy	soft	coll$_1$	coll$_2$
heavy	–	heavy	hard	**hard**
soft	heavy	soft	hard-coll$_1$	**hard-coll$_2$**
coll$_1$	hard	hard-coll$_1$	coll$_1$	**hard**
coll$_2$	**hard**	**hard-coll$_2$**	**hard**	coll$_2$

where

- hard modes have virtualities of order m_b
- hard-collinear modes have invariant mass $\sim \sqrt{\Lambda m_b}$

Corrections to naive factorization

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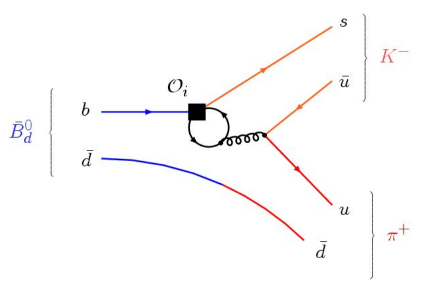
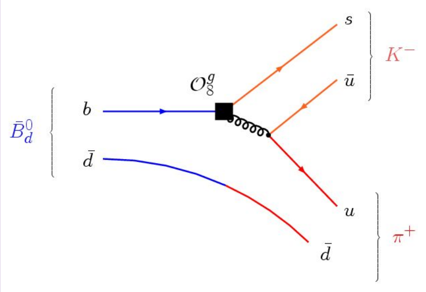
QCDF for $B \to \pi\pi$ and $B \to \pi K$ decays (BBNS 1999)

Factorization formula has to be extended:

- "Hard" corrections are treated as in $B \to D\pi$
  - Take into account new penguin diagrams! ($\to$ Fig.)
- "Hard-collinear" corrections involve spectator quark in B-meson ($\to$ Fig.)
  - Sensitive to the distribution of the spectator momentum ω $\longrightarrow$ light-cone distribution amplitude $\phi_B(\omega)$

Additional diagrams for hard corrections in QCDF (example)





$\longrightarrow$ additional contributions to the hard coefficient functions $t_{ij}(u, \mu)$

$$r_i(\mu)\Big|_{\rm hard} \simeq \sum_j F_j^{(B\to\pi)}(m_K^2) \int_0^1 du \left(1 + \frac{\alpha_s}{4\pi} t_{ij}(u,\mu) + \ldots\right) f_K\, \phi_K(u,\mu)$$

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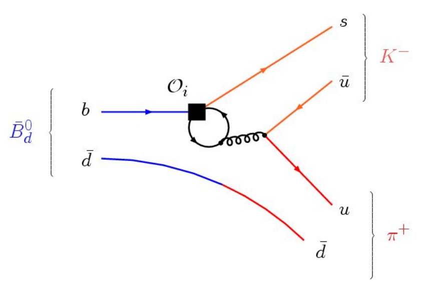

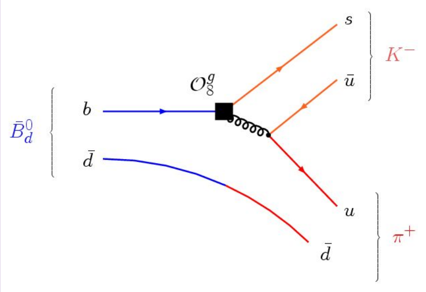

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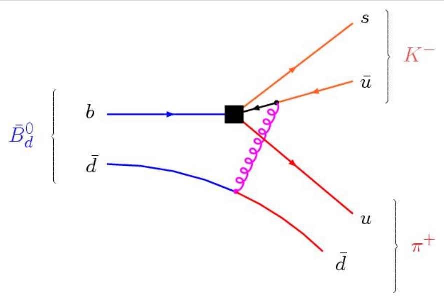
Spectator corrections with hard-collinear gluons in QCDF

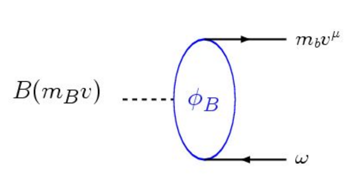


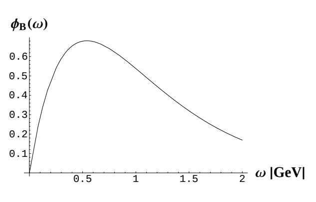
$\longrightarrow$ additive correction to naive factorization

$$\Delta r_i(\mu)\Big|_{\text{spect.}} = \int du\, dv\, d\omega \left(\frac{\alpha_s}{4\pi}\, h_i(u,v,\omega,\mu) + \ldots\right) \times f_K\, \phi_K(u,\mu)\, f_\pi\, \phi_\pi(v,\mu)\, f_B\, \phi_B(\omega,\mu)$$

Distribution amplitudes for all three mesons involved!

New ingredient: LCDA for the B-meson





- Phenomenologically relevant: $\langle \omega^{-1} \rangle_B \simeq (1.9 \pm 0.2)\ \mathrm{GeV}^{-1}$

(at $\mu = \sqrt{m_b \Lambda} \simeq 1.5$ GeV)

(from QCD sum rules [Braun/Ivanov/Korchemsky])

(from HQET parameters [Lee/Neubert])

- Large logarithms with ratio of hard and hard-collinear scale appear!
- Can be resummed using Soft-Collinear Effective Theory $\surd$

Complications for QCDF in $B \to \pi\pi, \pi K$ etc.

- Annihilation topologies are numerically important. BBNS use conservative model estimates.
- Some power-corrections are numerically enhanced by "chiral factor"

$$\frac{\mu_\pi}{f_\pi} = \frac{m_\pi^2}{2 f_\pi\, m_q}$$

- Many decay topologies interfere with each other.
- Many hadronic parameters to vary.

$\to$ Hadronic uncertainties sometimes quite large.

Phenomenology of $B \to \pi\pi$ decays

- Phenomenological parametrization: (using isospin; neglect EW peng.)

$$
\begin{aligned}
\mathcal{A}(\bar{B}_d^0 \to \pi^+\pi^-) &= V_{ub} V_{ud}^* \, T_{\pi\pi} + V_{cb} V_{cd}^* \, P_{\pi\pi} \,, \\
\sqrt{2}\,\mathcal{A}(B^- \to \pi^-\pi^0) &\simeq V_{ub} V_{ud}^* \, (T_{\pi\pi} + C_{\pi\pi}) \,, \\
\sqrt{2}\,\mathcal{A}(\bar{B}_d^0 \to \pi^0\pi^0) &= V_{ub} V_{ud}^* \, C_{\pi\pi} - V_{cb} V_{cd}^* \, P_{\pi\pi}
\end{aligned}
$$

- Recent predictions from QCDF (incl. part of NNLO corrections)

Ratio	Value/Range
$P_{\pi\pi}/T_{\pi\pi}$	$-0.122^{+0.033}_{-0.063} + (-0.024^{+0.047}_{-0.048})\, i$
$C_{\pi\pi}/T_{\pi\pi}$	$+0.363^{+0.277}_{-0.156} + (+0.029^{+0.166}_{-0.103})\, i$

[Beneke/Jäger, hep-ph/0610322]

- Remark: P/T ratio important to extract the CKM angle α from CP asymmetries in $B \to \pi\pi$.

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Phenomenology of $B \to \pi K$ decays

- Important: Tree amplitudes are CKM suppressed.
- Electroweak penguins are non-negligible.
- Phenomenological parametrization: (4th amplitude from isospin)

$$
\begin{aligned}
\mathcal{A}(B^- \to \pi^- \bar{K}^0) &= P\left(1 + \epsilon_a\, e^{i\phi_a}\, e^{-i\gamma}\right), \\
-\sqrt{2}\,\mathcal{A}(B^- \to \pi^0 K^-) &= P\left(1 + \epsilon_a\, e^{i\phi_a}\, e^{-i\gamma} - \epsilon_{3/2}\, e^{i\phi}\left(e^{-i\gamma} - q e^{i\omega}\right)\right), \\
-\mathcal{A}(\bar{B}_d^0 \to \pi^+ K^-) &= P\left(1 + \epsilon_a\, e^{i\phi_a}\, e^{-i\gamma} - \epsilon_T\, e^{i\phi_T}\left(e^{-i\gamma} - q_C e^{i\omega_C}\right)\right)
\end{aligned}
$$

with

$$
\begin{aligned}
\epsilon_{3/2,T,a} &\propto \left|\frac{V_{ub}V_{us}^*}{V_{cb}V_{cs}^*}\right| \equiv \epsilon_{KM} = \mathcal{O}(\lambda^2) \\
q, q_C &= \mathcal{O}(1/\lambda^2)
\end{aligned}
$$

- makes a priori 11 independent hadronic parameters !

Qualitative Results for $B \to \pi K$ parameters:

- $SU(3)_F$ symmetry predicts:

$$q\, e^{i\omega} \simeq \frac{-3}{2\epsilon_{KM}} \frac{C_9 + C_{10}}{C_1 + C_2} \simeq 0.69$$

- $\epsilon_a\, e^{i\phi_a}$ is negligible in QCDF.
  Consequence: Direct CP asymmetry in $B^- \to \pi^- K^0$ is tiny. $\surd$
- $q_C\, e^{i\omega_C}$ is of minor numerical importance.
- ϵ_T and $\epsilon_{3/2}$ are of the order 30%.
  Related strong phases are of the order of 10°.

Exploiting the approximate $SU(3)_F$ symmetry

- Relate e.g. $B \to \pi K$ and $B \to \pi\pi$ amplitudes
- Explicitely realized in BBNS approach
  (small $SU(3)_F$ violation from $f_\pi \neq f_K$, $F^{B\to\pi} \neq F^{B\to K}$ etc.)
- To control $SU(3)_F$ violations in a model-independent way, one needs the whole set of $B_{u,d,s} \to (\pi, K, \eta)^2$ decays

[see, e.g., Buras/Fleischer/Recksiegel/Schwab]

Other 2-body charmless B-decays

- Similar considerations for $B \to PV$ and $B \to VV$ decays.

  [$\to$ Beneke/Neubert 2004]

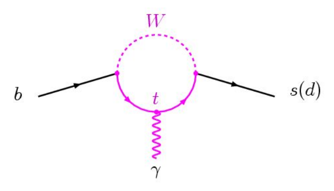
- Different interference pattern between various decay topologies.
- New issues with flavour-singlet mesons $(\eta, \eta', \ldots)$

Last Example: $b \to s(d)\gamma$ decays

Operator basis for $b \to s(d)\gamma$ decays

Same operator basis as for $b \to s(d)\, q\bar{q}$ decays:

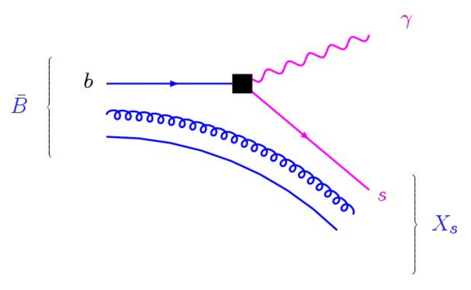
- current-current operators: $\mathcal{O}_{1,2}^{u,c}$
- strong penguins: $\mathcal{O}_{3-6}$
- EW penguins: $\mathcal{O}_{7-10}$
- chromomagnetic: $\mathcal{O}_8^g$
- **electromagnetic:** $\mathcal{O}_7^\gamma$



Inclusive $B \to X_s\gamma$ decays

Decay signature allows for an *almost* inclusive measurement of the $b \to s\gamma$ transition:

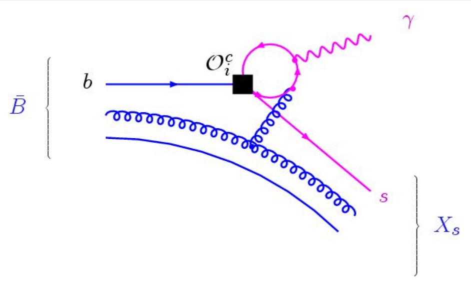
- Sum over all final states with one (energetic) photon, and hadronic system with open strangeness.
- To first approximation, the inclusive decay rate equals the partonic rate for $b \to s\gamma$, since the probability to find a b-quark with (almost) the same momentum as the B-meson is nearly one.
- Radiative corrections to the partonic rate can be calculated in terms of $\alpha_s(m_b)$.
- Corrections to the partonic rate arise from the binding effects of the b-quark in the B-meson, and can be systematically expanded in Λ/m_b.



Leading partonic contribution

$$\Gamma^{\mathcal{O}_7}_{B\to X_s\gamma} = \frac{\alpha G_F^2 m_b^5}{32\pi^4} \, |V_{tb}V_{ts}^*|^2 \, \left[C_7^\gamma(m_b)\right]^2 \left(1 + \frac{\delta}{m_b^2} + \ldots\right)$$

- $\delta = \frac{\lambda_1}{2} - \frac{9\lambda_2}{2}$ arises from the kinetic and chromomagnetic term in HQET.
- related uncertainty can be reduced by normalizing to $B \to X_c \ell\nu$ rate.



Example: Correction term involving O_2^c

$$\Delta\Gamma_{B\to X_s\gamma}^{\mathcal{O}_2^c} = \Gamma_{B\to X_s\gamma}^{\rm LO}\left(-\frac{1}{9}\,\frac{C_2}{C_7^\gamma}\,\frac{\lambda_2}{m_c^2}+\ldots\right)$$

- Only suppressed by $1/m_c^2$.
- Gives an order 3% correction.

Status of the theoretical calculation: [Misiak et al, hep-ph/0609232]

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm th(1)} = (3.15 \pm 0.23) \times 10^{-4}$$

for photon-energy cut $E_\gamma > 1.6$ GeV.

- includes corrections of order α_s^2
- 5% uncertainty from non-perturbative parameters
- 3% uncertainty from perturbative input parameters
- 3% from higher-order effects
- 3% from m_c-dependence of loop integrals (not exactly treated)

Comparison with exp. value (HFAG):

$$\mathcal{B}(\bar{B} \to X_s\gamma)_{\rm exp} = (3.55 \pm 0.24^{+0.09}_{-0.10} \pm 0.03) \times 10^{-4} \qquad (E_\gamma > 1.6)$$

Sensitivity to the B-meson shape function

- Presence of the photon-energy cut induces a second scale

$$\Delta = m_b - 2E_\gamma \leq 1.4\,\text{GeV}\,, \qquad \Delta \ll m_b$$

- Perturbative coefficients enhanced by $\ln[\Delta/m_b] \gg 1$
- What scale to choose for α_s ?

$$\alpha_s(m_b) \simeq 0.2\,, \quad \text{vs.} \quad \alpha_s(\Delta) \simeq 0.3 - 0.4$$

Need method to separate the scales Δ and m_b

(At least, if we aim for precision tests of the SM)

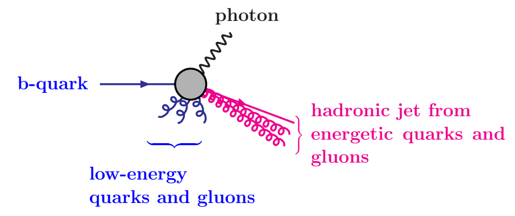
Decay kinematics:

$$
\begin{aligned}
p_b^\mu &:= m_b\,(1,\vec{0}) + k^\mu\,, \qquad |k| \sim \Lambda \\
p_\gamma^\mu &:= E_\gamma\,(1, 0_\perp, 1)\,, \qquad \Delta_\gamma \equiv m_b - 2E_\gamma < \Delta_{\rm cut} \\
\Rightarrow\ p_s^2 &= (p_b - p_\gamma)^2 \\
&= m_b^2 + 2m_b\,k_0 + k^2 - 2E_\gamma(m_b + k_0 - k_z) \\
&= m_b\,(\Delta_\gamma + k_0 + k_z) + \Delta_\gamma(k_0 - k_z) + k^2 \\
&\simeq m_b\,(\Delta_\gamma + k_0 + k_z)
\end{aligned}
$$

- For small Δ_γ invariant mass of hadronic system small $\Rightarrow$ Jet
- Dynamics at the intermediate scale $p_s^2 \ll m_b^2$ sensitive to residual momentum of the b-quark in the B-meson!

Shape function: $S(k_+ = k_0 + k_z)$ $=$ Parton Distribution Function

Momentum configuration



B-Meson rest frame:

$$m_b - 2E_\gamma = \Delta_\gamma \ll m_b$$

long-distance dynamics:

- heavy b-quark: $p_b^\mu = m_b v + k_s^\mu$
- soft quarks and gluons: $p_s^\mu \sim \Lambda \ll m_b$

$\Rightarrow$ soft matrix elements in HQET (shape function)

short-distance dynamics:

- "hard" modes: $p^2 \sim \mathcal{O}(m_b^2)$

$\Rightarrow$ perturbative coefficients from QCD

- ("collinear") jet modes: $p_{\rm jet}^2 \simeq m_b \Delta \ll m_b^2$

$\Rightarrow$ perturbative jet function from SCET

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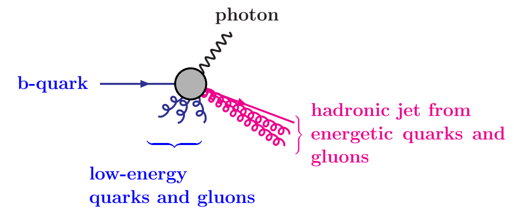

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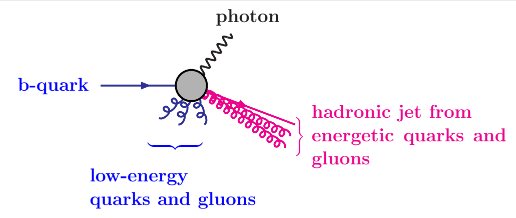

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Ingredients of the factorization theorem

- Hard Function:
  - describes the dynamics on the scale $\mu = m_b$
  - perturbatively calculable in QCD in terms of $\alpha_s(m_b)$
- Jet Function:
  - describes the dynamics on the intermediate scale $\mu_i = \sqrt{m_b \Delta}$
  - perturbatively calculable in SCET in terms of $\alpha_s(\mu_i)$
  - large logarithms resummed via renormalization in SCET
- Shape Function:
  - describes the non-perturbative dynamics of the *b*-quark
  - if Δ is not too small it can be expanded in terms of moments, which are related to the usual HQET parameters.

Numerical estimate: [Neubert/Becher]

$$\mathcal{B}(\bar{B} \to X_s \gamma)_{\mathrm{th}(2)} = (2.98^{+0.49}_{-0.53}) \times 10^{-4}$$

(5% smaller value than fixed order result, more conservative(?) error estimate, 19% smaller than experiment (1.4 σ))

Alternative view: E_γ spectrum "measures" the shape function:

Shape-function independent relations

- Shape functions are universal (depend only on B-meson bound state)
- Only **one** SF at leading power in $1/m_b$ (= "leading twist" in DIS)
- Decay spectra for $B \to X_u \ell \nu$ and $B \to X_s \gamma$ only differ by perturbatively calculable short-distance functions

$\Rightarrow$ Shape-function independent relations

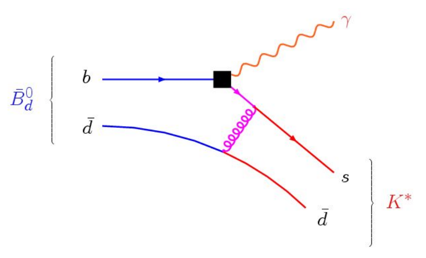
(re-weighting the $B \to X_s \gamma$ spectrum to predict $B \to X_u \ell \nu$)

Rather precise determination of $|V_{ub}|$ (2-loop + estimate of power corr.)

[Lange/Neubert/Paz, hep-ph/0508178]

Exclusive $B \to K^*\gamma$ decay

Naive factorization



- $B \to K^*$ form factor fairly well known (QCD sum rules)
- The photon does not couple to gluons, . . . ,
  . . . should there be any corrections at all ?!
- Actually, the photon can split into $(q\bar{q})$ just like a meson !
- Factorization of QCD effects, similar to $B \to \pi\pi$.

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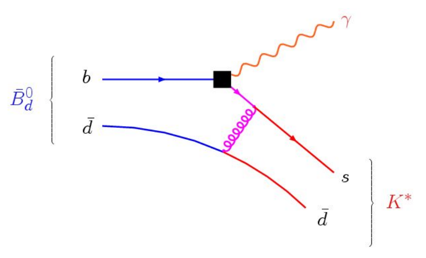

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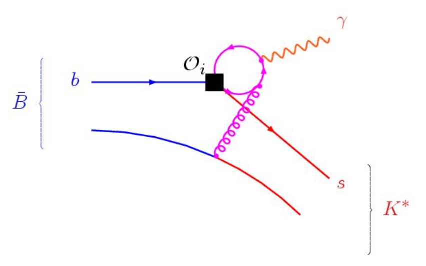
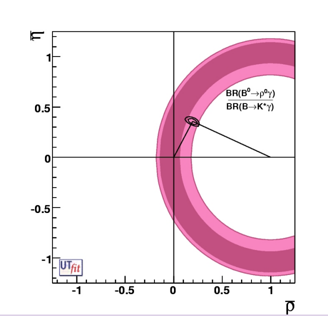
Example: Spectator scattering correction to $B \to K^*\gamma$



- LCDAs for B- and K^*-meson enter
- Photon still behaves point-like, to first approximation
- Power corrections also require LCDA for the photon !

Application: $|V_{td}/V_{ts}|$ from $B \to \rho\gamma$ vs. $B \to K^*\gamma$

Constraint on Wolfenstein parameters $(\bar\rho, \bar\eta)$ compared to global CKM fit
[Plot from UTfit Collaboration]



Ratio of decay widths (B-factories):

$$\frac{\Gamma[B^0 \to \rho^0\gamma]}{\Gamma[B^0 \to K^{*0}\gamma]} \propto \left|\frac{V_{td}}{V_{ts}}\right|^2 \frac{F^{B\to\rho}}{F^{B\to K^*}} (1+\Delta)$$

- form factor ratio $= 1$ in $SU(3)_F$ symmetry limit
- deviations can be estimated from non-perturbative methods
- correction term $|\Delta|$ estimated from QCD factorization

[for more details, see http://utfit.roma1.infn.it/]

Generalization to $B \to K^* \ell^+ \ell^-$

- Two new operators

$$\mathcal{O}_9^{\ell\ell} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \ell)\,, \quad \mathcal{O}_{10}^{\ell\ell} = (\bar{s}_L \gamma_\mu b_L)(\bar{\ell}\gamma^\mu \gamma_5 \ell)$$

- Form factors for $B \to K^* \ell\ell$ fulfill approximate symmetry relations.
- QCD corrections perturbatively calculable for $m_\rho^2 < q^2 < 4m_c^2$.

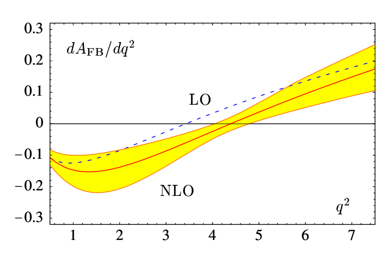
$$\frac{d\Gamma[B \to K^* \ell^+ \ell^-]}{dq^2\, d\cos\theta} \propto \Bigg[(1+\cos^2\theta)\,\frac{2q^2}{m_B^2}\left[f_\perp(q^2)\right]^2 \left(|\mathcal{C}_{9,\perp}(q^2)|^2 + C_{10}^2\right)$$
$$+(1-\cos^2\theta)\left[f_\parallel(q^2)\right]^2 \left(|\mathcal{C}_{9,\parallel}(q^2)|^2 + C_{10}^2\right)$$
$$-\cos\theta\,\frac{8q^2}{m_B^2}\left[f_\perp(q^2)\right]^2 \mathrm{Re}\left[\mathcal{C}_{9,\perp}(q^2)\right] C_{10}\Bigg]$$

- Functions $\mathcal{C}_{9,\perp,\parallel}(q^2)$ contain the non-trivial QCD corrections !

Important application: FB asymmetry zero

Leading hadronic uncertainties from form factors drop out:

$$0 \stackrel{!}{=} \mathrm{Re}\left[\mathcal{C}_{9,\perp}(q_0^2)\right] = \mathrm{Re}\left[C_9^{\ell\ell} + \frac{2m_b m_B}{q_0^2}\, C_7^{\gamma} + Y(q_0^2)\right] + \text{QCDF corr.} + \text{power corr.}$$



Theoretical uncertainties incl.:

- hadronic input parameters
- electro-weak SM parameters
- variation of factorization scale

[from Beneke/TF/Seidel 01]

Asymmetry zero in SM: $q_0^2 = (4.2 \pm 0.6)\ \mathrm{GeV}^2$

(incl. estimated 10% uncertainty from undetermined power corrections)

Summary

" When looking for new physics, . . .
. . . do not forget about the complexity of the old physics ! "