Neutrino-Electron Vertex

Only left-handed neutrinos are observed in beta decays: $$u_\nu \rightarrow \left(\frac{1-\gamma^5}{2}\right)u_\nu$$

This leads to the following electron-neutrino vertex
(assuming vector coupling between LH neutrino and e): $$\bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu$$

If one further exploits that $P_l = 1/2(1 - \gamma^5)$ is a projection operator one finds:

$$\bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu = \bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)^2 u_\nu = u_e^+\left(\frac{1-\gamma^5}{2}\right)\gamma^0\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu = \overline{(u_e)}_L\gamma^\mu(u_\nu)_L$$

The left-handed neutrino thus couples only to left-handed electrons (vector current).

V-A structure: $$\bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu = \frac{1}{2}\bar{u}_e\left(\gamma^\mu - \gamma^\mu\gamma^5\right)u_\nu$$

V - A (vector – axial-vector)

3.3 V – A Theory

Careful analysis of experimental data (parity violation, neutrino helicity spin change in nuclear β-decays, muon decay properties together w/ universality) finally led to the V-A theory of (nuclear) weak decays:

V − A (nucleon), V−A (lepton / fund. fermion)

$$M = \frac{G_F}{\sqrt{2}} \left(\bar{u}_p \gamma^\mu (c_V - c_A \gamma^5) u_n \right) \cdot \left(\bar{u}_e \gamma^\mu (1 - \gamma^5) v_\nu \right)$$

nucleon — lepton / fund. fermion

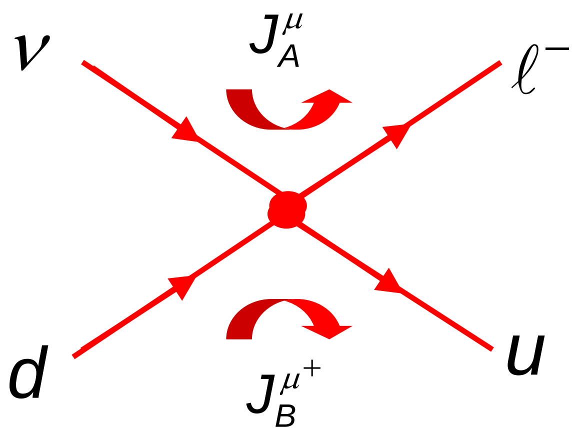
Composed objects

c_V, c_A vector and axial-vector couplings of nucleons:

$$c_A / c_V = 1.2695 \pm 0.0029$$ PDG 2004

V-A ansatz for fundamental fermions

Four-Fermion Theory



J_A and J_B are lepton and quark currents

$$J_\ell^\mu = \bar{u}_\ell \gamma^\mu (1-\gamma^5) u_\nu$$

$$J_q^\mu = \bar{u}_u \gamma^\mu (1-\gamma^5) u_d$$

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{A,\mu} \cdot J_B^{\mu +}$$

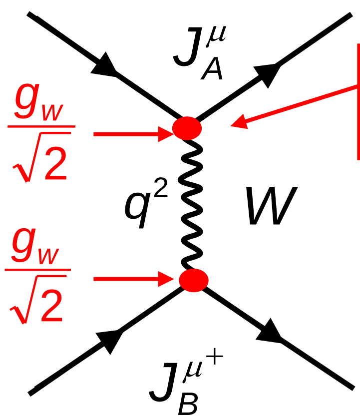
Reminder

$$u_L = \frac{1}{2}(1-\gamma^5) u$$

$$u_R = \frac{1}{2}(1+\gamma^5) u$$

Electroweak Theory

According today's understanding the 4-fermion coupling is the $q^2 \to 0$ limit of W propagator:



g_w = coupling for weak interaction

$$M = \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_{A,\mu} \frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2})}{q^2 - M_W^2} \cdot \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_B^{\nu +}$$

for $q^2 \to 0$: $= \frac{1}{M_W^2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8 M_W^2}$$

With $G_F \approx 1.16\times10^{-5}\,GeV^{-2}$ follows w $M_W \approx 80\,GeV$: $g_w \approx 0.65$

3.4 V-A coupling of leptons and quarks

Reminder

$$\bar{u}_\ell \gamma^\mu (1-\gamma^5) u_\nu = \bar{u}_\ell^L \gamma^\mu u_\nu^L$$

In V-A theory the weak interaction couples left-handed lepton/quark currents (right-handed anti-lepton/quark currents) with an **universal coupling strength**:

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

Charged weak transition appear only inside weak-isospin doublets:

Problem: Not equal to the mass eigenstate

Lepton currents:

1. $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L$ $\quad j^\mu_{e\nu} = \bar{u}_e \gamma^\mu (1-\gamma^5) u_\nu$

2. $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L$ $\quad j^\mu_{\mu\nu} = \bar{u}_\mu \gamma^\mu (1-\gamma^5) u_\nu$

3. $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L$ $\quad j^\mu_{\tau\nu} = \bar{u}_\tau \gamma^\mu (1-\gamma^5) u_\nu$

Quark currents:

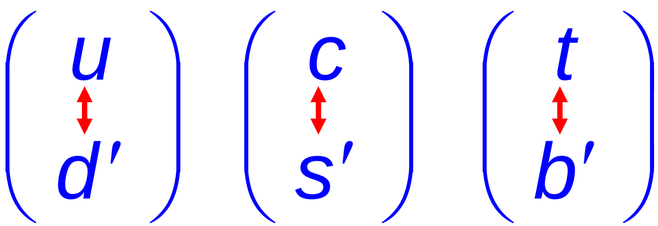
1. $\begin{pmatrix} u \\ d' \end{pmatrix}_L$ $\quad j^\mu_{d'u} = \bar{u}_{d'} \gamma^\mu (1-\gamma^5) u_u$

2. $\begin{pmatrix} c \\ s' \end{pmatrix}_L$ $\quad j^\mu_{s'c} = \bar{u}_{s'} \gamma^\mu (1-\gamma^5) u_c$

3. $\begin{pmatrix} t \\ b' \end{pmatrix}_L$ $\quad j^\mu_{b't} = \bar{u}_{b'} \gamma^\mu (1-\gamma^5) u_t$

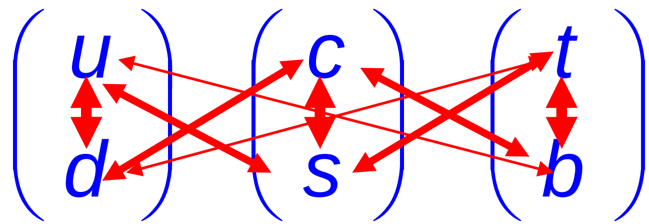
CKM matrix to describe the quark mixing

Weak eigenstates:



Weak transitions

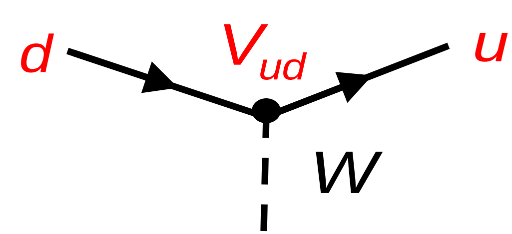
Mass/flavor eigenstates:



One finds that the weak eigenstates of the down type quarks entering the weak isospin doublets are not equal to the their mass/flavor eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

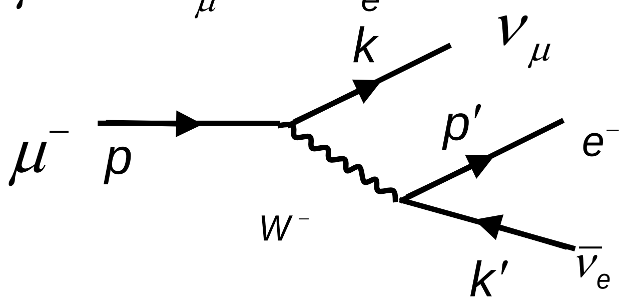
Cabibbo-**K**obayashi-**M**askawa mixing matrix



3.6 Test of V-A structure in particle decays

a) Muon decay

$$\mu^- \to \overline{\nu}_\mu + e^- \overline{\nu}_e$$



Applying the Feynman rules:

4-fermion interaction – ignore propagator

$$M = \frac{G_F}{\sqrt{2}} \left[\overline{u}_\nu(k)\gamma_\alpha(1-\gamma^5)u_\mu(p)\right]\left[\overline{u}_e(p')\gamma^\alpha(1-\gamma^5)v_\nu(k')\right]$$

Analogous to the QED calculations of ee→μμ one finds after a lengthy calculation:

$$M = \frac{1}{2}\sum_{Spins}|M|^2 = 64G_F^2(k\cdot p')(k'\cdot p)$$

Using $d\Gamma = \frac{1}{2E}|M|^2 d\Phi$ one obtains the electron spectrum in the muon rest frame:

"incoming flux"

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$

with E' = electron energy

$$\frac{1}{\tau} = \Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE'} dE' = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Measurement of the muon lifetime thus provides a determination of the fundamental coupling $\mathbf{G_F}$

$$\tau_\mu = (2.19703 \pm 0.00004)\cdot 10^{-6} s$$

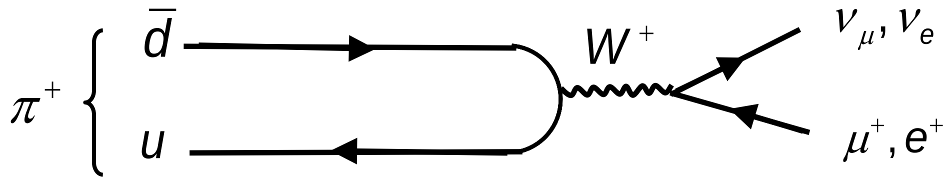
$$G_F = (1.16639 \pm 0.00001)\cdot 10^{-5} \mathrm{GeV}^{-2}$$

Fermi constant measured in muon decays is often called $\mathbf{G_\mu}$

b) Pion decay

$$\pi^+ \rightarrow \mu^+ \nu_\mu$$

$$\pi^+ \rightarrow e^+ \nu_e$$



<u>Naïve expectation:</u>

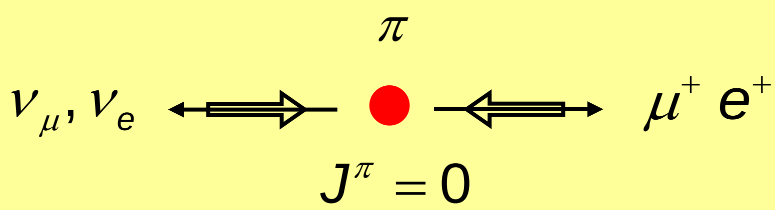
Assuming the same decay dynamics the decay rate to e$^+$ should be much larger than to μ^+ as the phase space is much bigger.

<u>Measurement:</u> (PDG)

$$\frac{\Gamma(\pi^+ \rightarrow e^+ \nu_e)}{\Gamma(\pi^+ \rightarrow \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \cdot 10^{-4}$$

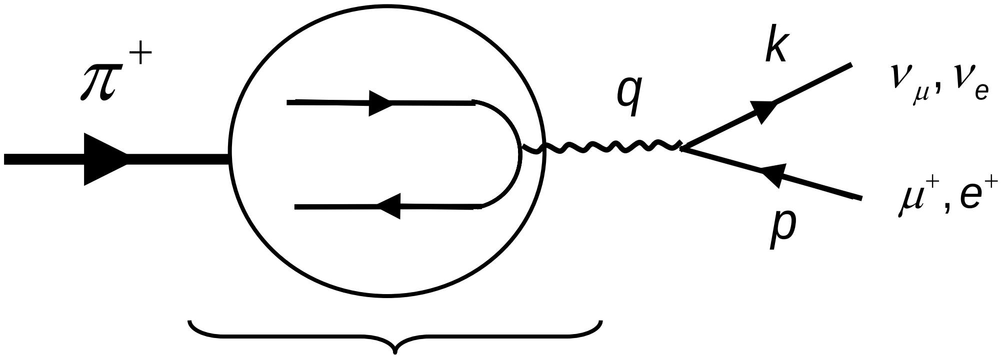
Large suppression due to a dynamic effect.

<u>Qualitative explanation within V-A theory:</u>



Angular momentum conservation forces the lepton into the “wrong” helicity state: suppressed ~ (1-v/c) i.e. for vanishing lepton masses the pion could not decay into leptons.

Determination of decay rates:



Quarks in pion are bound

Effective interaction – ignore propagator

$$M = \frac{G_F}{\sqrt{2}} \cdot (\pi)_\mu \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

scalar particles

$$M = \frac{G_F}{\sqrt{2}} \cdot \left(p_\mu + k_\mu\right) \cdot f_\pi \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

As the pion spin $s_\pi=0$, q is the only relevant 4-vector:

$$q^\mu = p^\mu + k^\mu$$

$$(\pi)_\mu = q_\mu \cdot f_\pi(q^2)$$

Pion form factor:

$$q^2 = m_\pi^2: \quad f_\pi(q^2) = f_\pi(m_\pi^2) = f_\pi$$

Must be measured !

$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_\mu^2 (1 - \frac{m_\mu^2}{m_\pi^2})$$

$$\Gamma(\pi^+ \to e^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_e^2 (1 - \frac{m_e^2}{m_\pi^2})$$

$$\frac{\Gamma(\pi^+ \to e^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) = 1.275 \cdot 10^{-4}$$

$$(1.230 \pm 0.004) \cdot 10^{-4}{}_{PDG}$$

Matrix element:

$$|M|^2(\ell) \sim m_\ell^2 (m_\pi^2 - m_\ell^2)$$

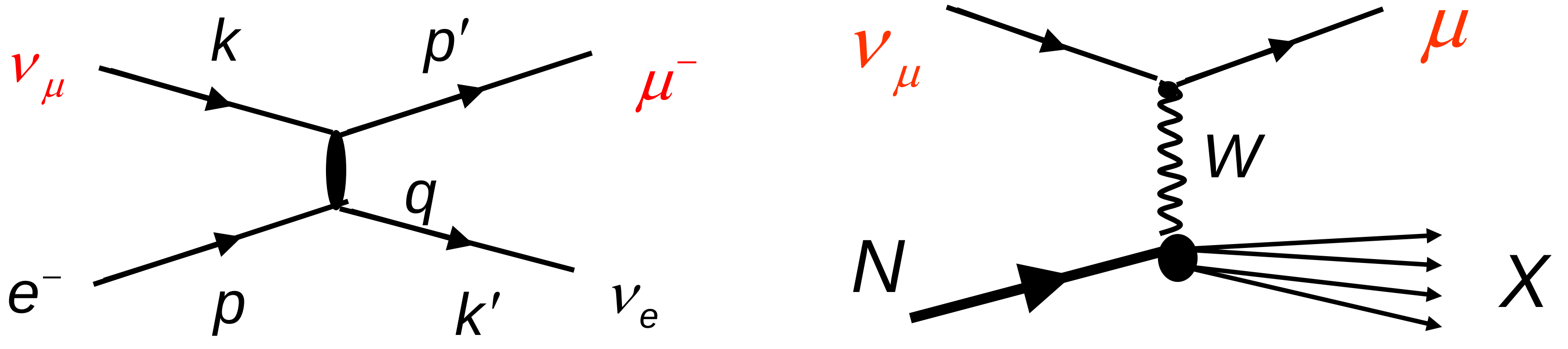
$$\frac{|M|^2(e)}{|M|^2(\mu)} = \frac{m_e^2 (m_\pi^2 - m_e^2)}{m_\mu^2 (m_\pi^2 - m_\mu^2)} = 5.5 \cdot 10^{-5}$$

Phase phase:

$$\sim p_\ell = \frac{1}{2m_\pi}(m_\pi^2 - m_\ell^2)$$ ➡ e/μ ~ 2.4

The prediction of the V-A theory is confirmed by the experimental observation.

3.7 Neutrino scattering in V-A theory



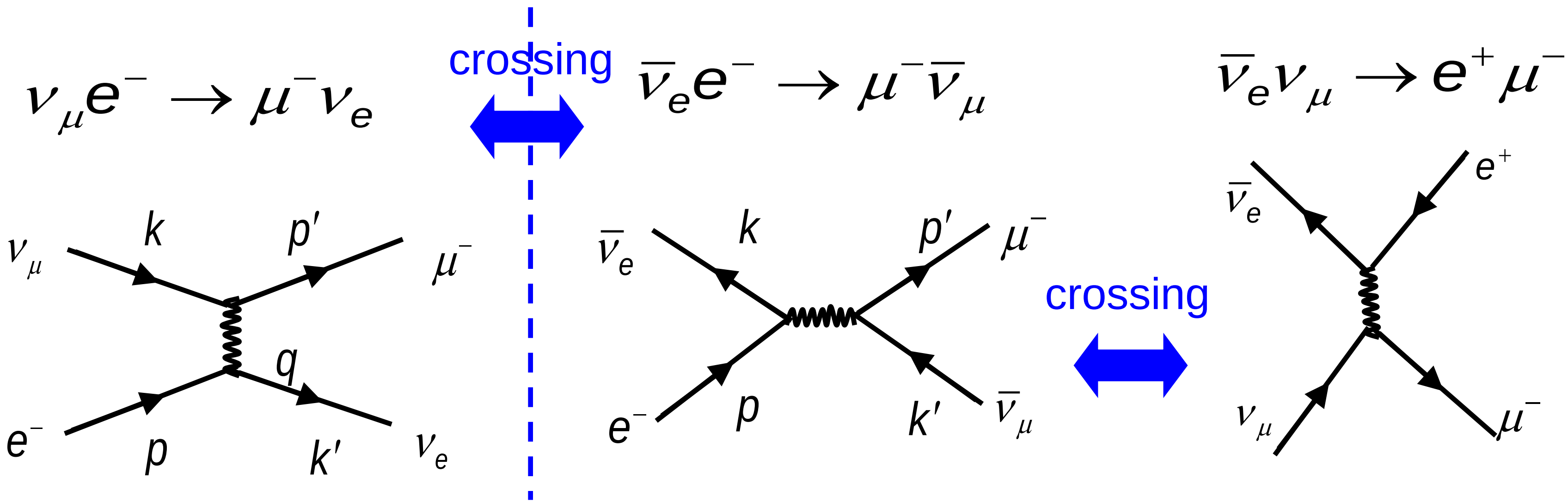
Very small cross section for νN scattering: $\sigma(\nu N) \approx E_\nu[\text{GeV}]\text{x}10^{-38}\,\text{cm}^2$

$= E_\nu[\text{GeV}]\text{x}10\,\text{fb}$

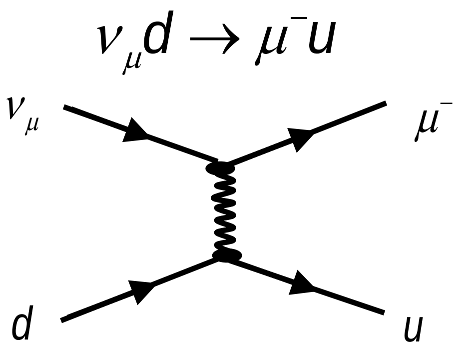
Calculation: see below.

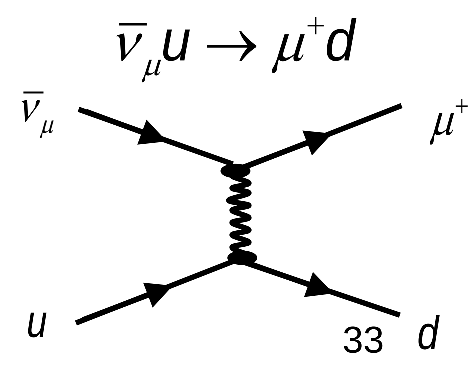
- intense neutrino beams
- large instrumented targets

Neutrino-lepton and neutrino-quark reactions

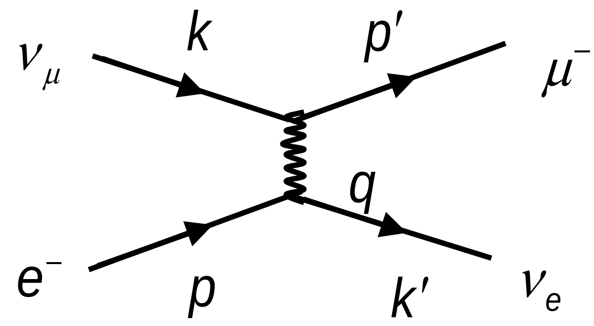


(Anti)neutrino-lepton interaction similar to (Anti)neutrino-quark interaction: neutrino-lepton results can be applied to deep-inelastic νN scattering.





a) Neutrino-electron scattering



$$\nu_\mu e^- \to \mu^- \nu_e$$

ν_e $\qquad$ e^-

$$M = \frac{G_F}{\sqrt{2}}\left[\bar{u}_\nu(k')\gamma_\alpha(1-\gamma^5)u_e(p)\right]\left[\bar{u}_\mu(p')\gamma^\alpha(1-\gamma^5)u_\nu(k)\right]$$

$$\overline{|M|^2} = \frac{1}{2}\sum_{Spins}|M|^2 = \ldots = 64G_F^2(k\cdot p)(k'\cdot p') = 16G_F^2\cdot s^2$$

Limit $m_e \approx m_\mu \approx 0$ $\qquad$ $s = (k+p)^2 = 2kp = 2k'p'$

Using the phase space factor of chapter II:

Although effective 4-fermion theory works well for low q^2 it violates unitarity bound for high q^2!

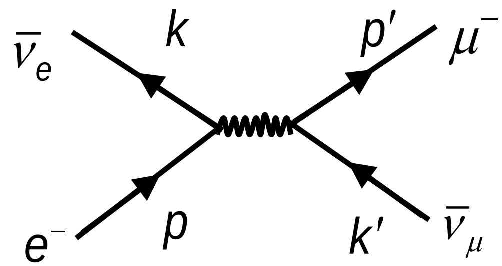
$$\frac{d\sigma}{d\Omega}(\nu_\mu e^-) = \frac{1}{64\pi^2 s}\overline{|M|^2} = \frac{G_F^2 s}{4\pi^2}$$

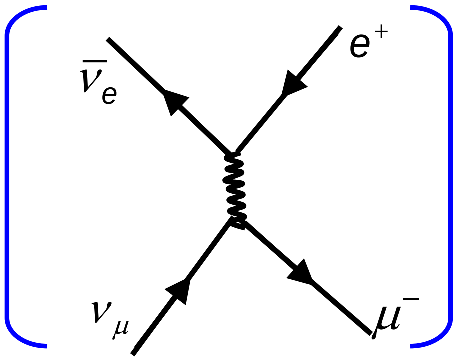
$$\sigma(\nu_\mu e^-) = \frac{G_F^2 s}{\pi} \longleftarrow = 2m_e E_\nu$$

This is a clear indication that the 4-fermion interaction is only an effective low energy approximation – not valid at high energies !!

b) Anti-Neutrino-electron scattering (V-A)

$$\bar{\nu}_e e^- \to \bar{\nu}_\mu \mu^-$$





Crossing: s ⇔ t (u)

$$\overline{|M|^2} = \frac{1}{2}\sum_{Spins}|M|^2 = 16 G_F^2 \cdot t^2 = 4 G_F^2 \cdot s^2 (1 - \cos\theta)^2$$

$$\frac{d\sigma}{d\Omega}(\bar{\nu}e^-) = \frac{G_F^2 s}{16\pi^2}(1 - \cos\theta)^2$$

$$\sigma(\bar{\nu}e^-) = \frac{G_F^2 s}{3\pi}$$

Result of V-A structure

For the charged current (CC) contribution to the (anti) neutrino electron scattering one finds

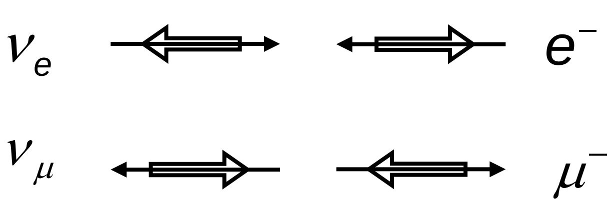
$$\frac{\sigma_{\nu e}^{cc}}{\sigma_{\bar{\nu} e}^{cc}} = 3$$

Different angular distribution of (anti) neutrino scattering can be understood from a helicity discussion

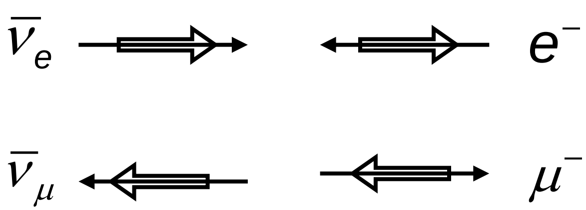
$$\frac{d\sigma}{d\Omega}(\nu_e e^- \to \nu_\mu \mu^-) = \frac{G_F^2 s}{4\pi^2}$$

$$\frac{d\sigma}{d\Omega}(\bar{\nu}_e e^- \to \bar{\nu}_\mu \mu^-) = \frac{G_F^2 s}{16\pi^2}(1-\cos\theta)^2$$

$\nu_e e^- \to \nu_\mu \mu^-$



$\bar{\nu}_e e^- \to \bar{\nu}_\mu \mu^-$



initial

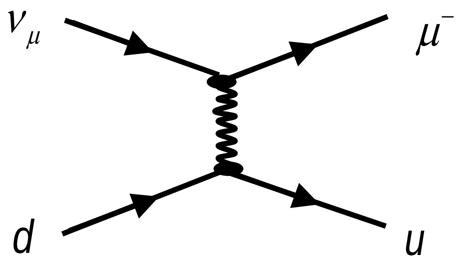
final state

Allowed $\to$ isotropic

Forbidden (angular mom.) $\to$ (1-cosθ) suppression

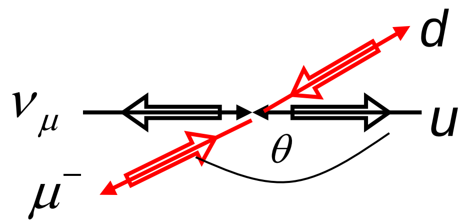
c) <u>(Anti) neutrino-quark scattering</u>

$$\nu_\mu d \to \mu^- u$$



$$\frac{d\sigma}{d\Omega}(\nu_\mu d) = \frac{G_F^2 s}{4\pi^2}$$

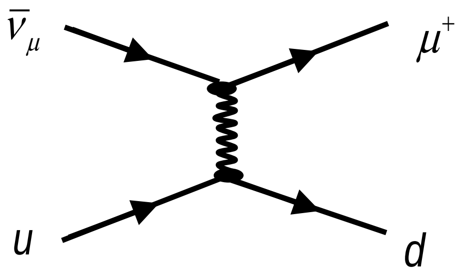
$$\sigma(\nu_\mu d) = \frac{G_F^2 s}{\pi}$$



$$\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}$$

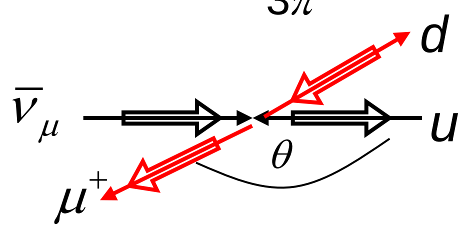
$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu \bar{d}) = \frac{d\sigma}{d\Omega}(\nu_\mu d)$$

$$\bar{\nu}_\mu u \to \mu^+ d$$



$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u) = \frac{G_F^2 s}{16\pi^2}(1+\cos\theta)^2$$

$$\sigma(\bar{\nu}_\mu u) = \frac{G_F^2 s}{3\pi}$$



$$\nu_\mu \bar{u} \to \mu^- \bar{d}$$

$$\frac{d\sigma}{d\Omega}(\nu_\mu \bar{u}) = \frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u)$$

Neutrinos only interact w/ d and anti-u quarks
Anti-neutrinos only interact w/ u and anti-d quarks

d) Neutrino-nucleon N scattering

$$\sigma(\nu N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[Q_l + \frac{1}{3}\overline{Q}_l \right]$$

$$\sigma(\overline{\nu} N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[\overline{Q}_l + \frac{1}{3} Q_l \right]$$

with $Q_l = \int xQ(x)dx$

(integral of quark / anti-quark distribution)

$$R = \frac{\sigma_{\overline{\nu}N}}{\sigma_{\nu N}} = \frac{1 + 3\overline{Q}_l / Q_l}{3 + \overline{Q}_l / Q_l}$$

If nucleon consists only of valence quarks ($\overline{Q}$=0): R=1/3 , because of V-A structure

Measurement: $R = \frac{0.34}{0.67} \quad \Rightarrow \quad \overline{Q}_l / Q_l \approx 0.15$

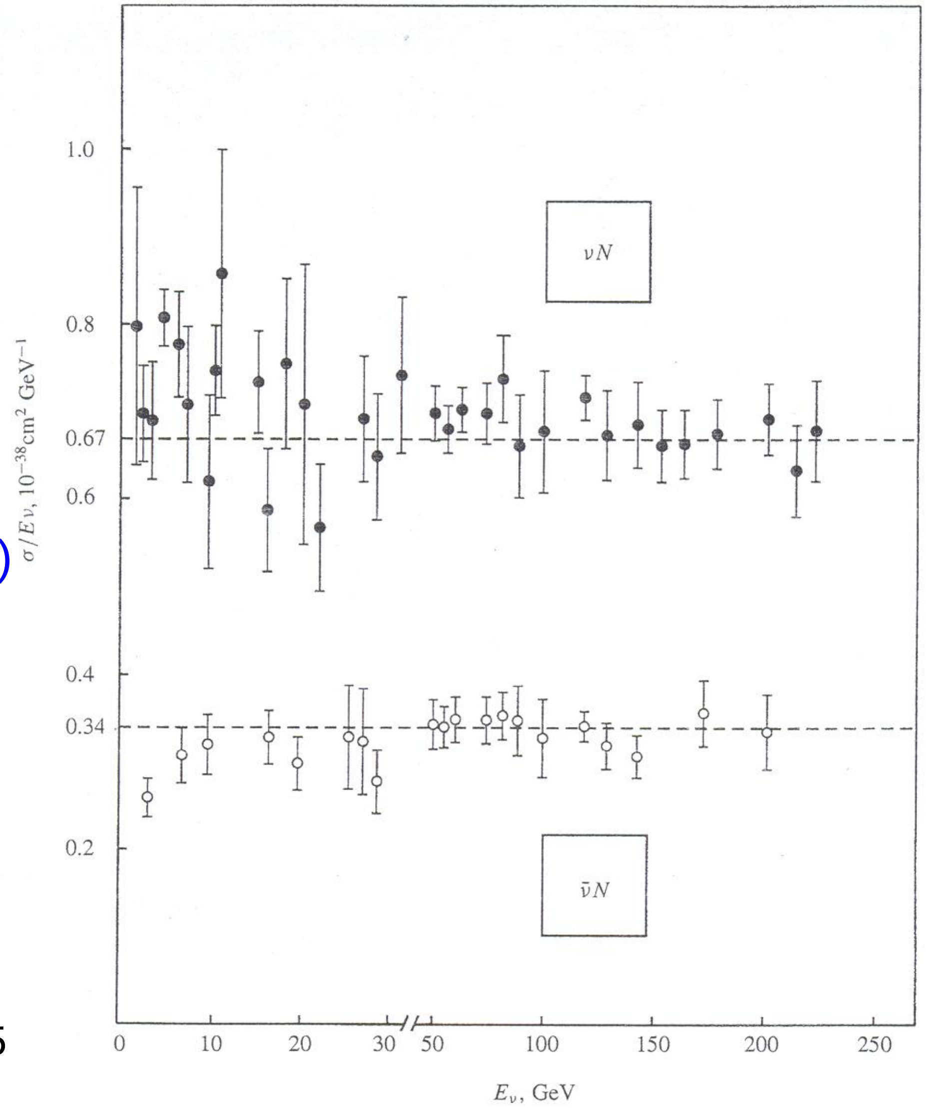


Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio σ / E_ν is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).

⇒V-A theory confirmed, there are sea quarks

3.8 Problems with pure V-A theory

- Cross section for $\nu e^- \to e^- \nu_e$ in 4-fermion ansatz: i.e. cross section goes to infinity if s→∞: violates unitarity

$$\sigma(\nu e^-) = \frac{G_F^2 s}{\pi}$$

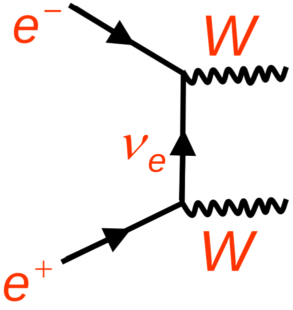
- Lee and Wu (1965) introduced a massive exchange boson. Effect of propagator:

$$\frac{G_F}{\sqrt{2}} \mapsto \frac{G_F}{\sqrt{2}} \frac{1}{1 - q^2 / M_W^2} \qquad \sigma(\nu e^-) \mapsto const.$$

Not trivial, see e.g.:
C.Quigg, Gauge Theory of Strong and Weak interaction

This fix leads to a new problem, namely the violation of unitarity of the predicted W pair production !

W pair production:



Violates unitarity

➡ **We need a new theory.**

Standard Model including a Z boson:

$$e^+ e^- \to Z \to WW$$

4. Neutral currents (CERN, 1973)

Gargamelle Bubble Chamber

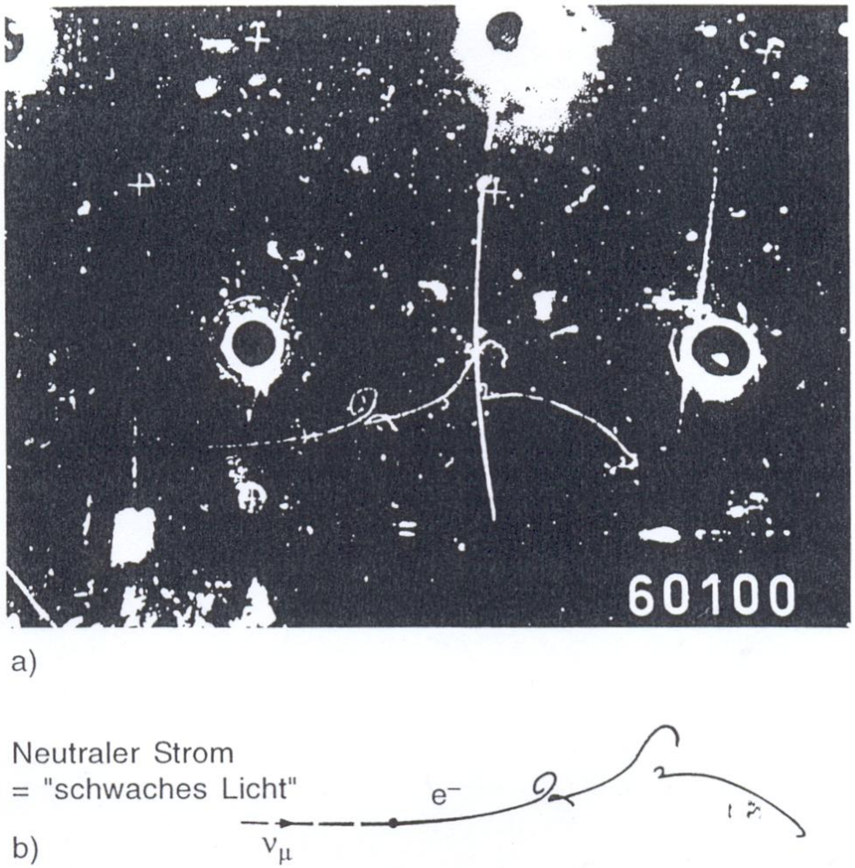
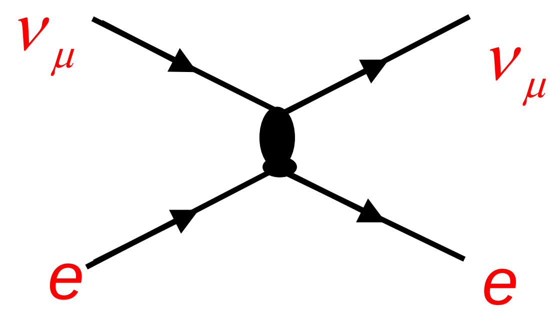


Abb. 9. Dieses erste Ereignis mit einem neutralen schwachen Strom wurde in Aachen entdeckt. Ein Neutrino dringt von links in die Blasenkammer ein (auf dem Bild nicht sichtbar) und wird elastisch an einem Elektron gestreut. Das Elektron ist als rechte Spurkaskade (Bremsstrahlung) zu erkennen. Dieses Bild ist in die Geschichte des CERN eingegangen

One out of three $\nu e \rightarrow \nu e$ events

$$\nu_\mu + e^- \rightarrow \nu_\mu + e^-$$



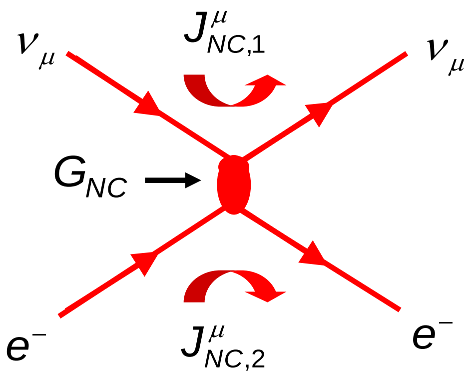
Neutral current νN events appear with a significant rate:

$$R_\nu = \frac{\sigma_{NC}(\nu N \rightarrow \nu X)}{\sigma_{CC}(\nu N \rightarrow \mu X)} = 0.307 \pm 0.008$$

i.e. approx.1/3 of the νN interactions are neutral current interactions.

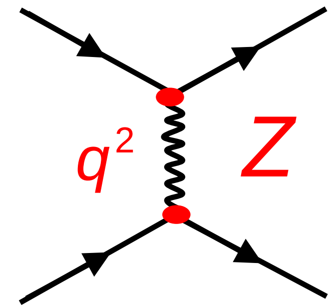
Structure of Neutral currents

Ansatz: four-fermion interaction



$$M = \frac{8G_{NC}}{\sqrt{2}} \cdot J_{NC,1,\mu} \cdot J^{\mu}_{NC,2}$$

as $q^2 \to 0$ approximation of:



Experimental determination of the structure of the weak neutral currents:

$$J^\mu_{NC} = \bar{u}\,\gamma^\mu \frac{1}{2}(g_V - g_A\gamma^5)u$$

Neutral weak interaction couples to left- and right-handed chiral fermion currents differently:

$$g_L = \frac{1}{2}(g_V + g_A) \qquad g_R = \frac{1}{2}(g_V - g_A)$$

$$J^\mu_{NC} = \bar{u}\gamma^\mu(g_R \frac{1+\gamma^5}{2} + g_L \frac{1-\gamma^5}{2})u$$