Neutrino Oscillation

For massive neutrinos one could introduce in "analogy" to the quark mixing a mixing matrix describing the relation between mass and flavor states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 3} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

Massive neutrinos propagate differently in time.

$$\left|\nu_i(t)\right\rangle = \left|\nu_i(0)\right\rangle e^{-iE_i t} = \left|\nu_i(0)\right\rangle e^{-i(p_i + \frac{m_i^2}{2p_i})}$$

for masses $m_i << E_i$:

$$E_i = \sqrt{p_i^2 + m_i^{\,2}} = p_i + \frac{m_i^{\,2}}{2p_i}$$

$\rightarrow$ there will be a mixing of the flavor states with time.

$$\left|\nu(t)\right\rangle_\alpha = \sum_i U_{\alpha i} e^{-iE_i t}\left|\nu_i(0)\right\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-iE_i t}\left|\nu_\beta\right\rangle$$

1. Mixing in the 2 neutrino case

$$\begin{pmatrix} \nu_\alpha \\ \nu_\beta \end{pmatrix} = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \end{pmatrix}$$

Definite momentum p; same for all mass eigenstate components

$$E_i = \sqrt{p^2 + m_i^2} = p + \frac{m_i^2}{2p}$$

$$E_2 - E_1 = \frac{m_1^2 - m_2^2}{2p} \approx \frac{\Delta m^2}{2E}$$

(assuming p_i is the same)

$t = L/\beta$ w/ $\beta \approx 1$:

$$\left(E_2 - E_1\right) t = \frac{\Delta m^2}{2E} L$$

Time development for an initially pure $|\nu_\alpha>$ beam:

$$\left|\nu_\alpha(t)\right\rangle = \cos\theta\, e^{-iE_1t}\left|\nu_1\right\rangle + \sin\theta\, e^{-iE_2t}\left|\nu_2\right\rangle$$

$$= \left[\cos^2\theta\, e^{-iE_1t} + \sin^2\theta\, e^{-iE_2t}\right]\cdot\left|\nu_\alpha\right\rangle$$

$$+ \left[\cos\theta \sin\theta (e^{-iE_1t} - e^{-iE_2t})\right]\cdot\left|\nu_\beta\right\rangle$$

Mixing probability:

$$P(\nu_\alpha \to \nu_\beta, t) = \left|\left\langle \nu_\beta \middle| \nu_\alpha(t)\right\rangle\right|^2 = 2(\cos\theta\sin\theta)^2\left[1 - \cos^2\frac{E_2 - E_1}{2}t\right]$$

$$\boxed{P(\nu_\alpha \to \nu_\beta, t) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E}L\right)} = \sin^2 2\theta \sin^2\left(\frac{1.27\cdot\Delta m^2[eV]}{4E[GeV]}L[km]\right)$$

How to search for neutrino oscillation ?

$$P(\nu_\alpha \rightarrow \nu_\beta, t) = \sin^2 2\theta \sin^2\left(\frac{\Delta m^2}{4E} L\right)$$

- Disappearance:
  (I) With known neutrino flux: Measurement of flux at distance L: reactor experiments (sun). ➡ Solar neutrinos, atmospheric neutrinos

  (II) Measure neutrino flux at position 1 and verify flux after distance L. ➡ Reactor neutrinos

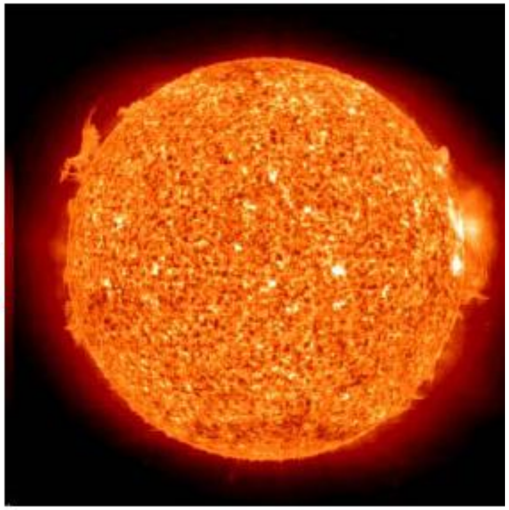
  Accelerator neutrinos

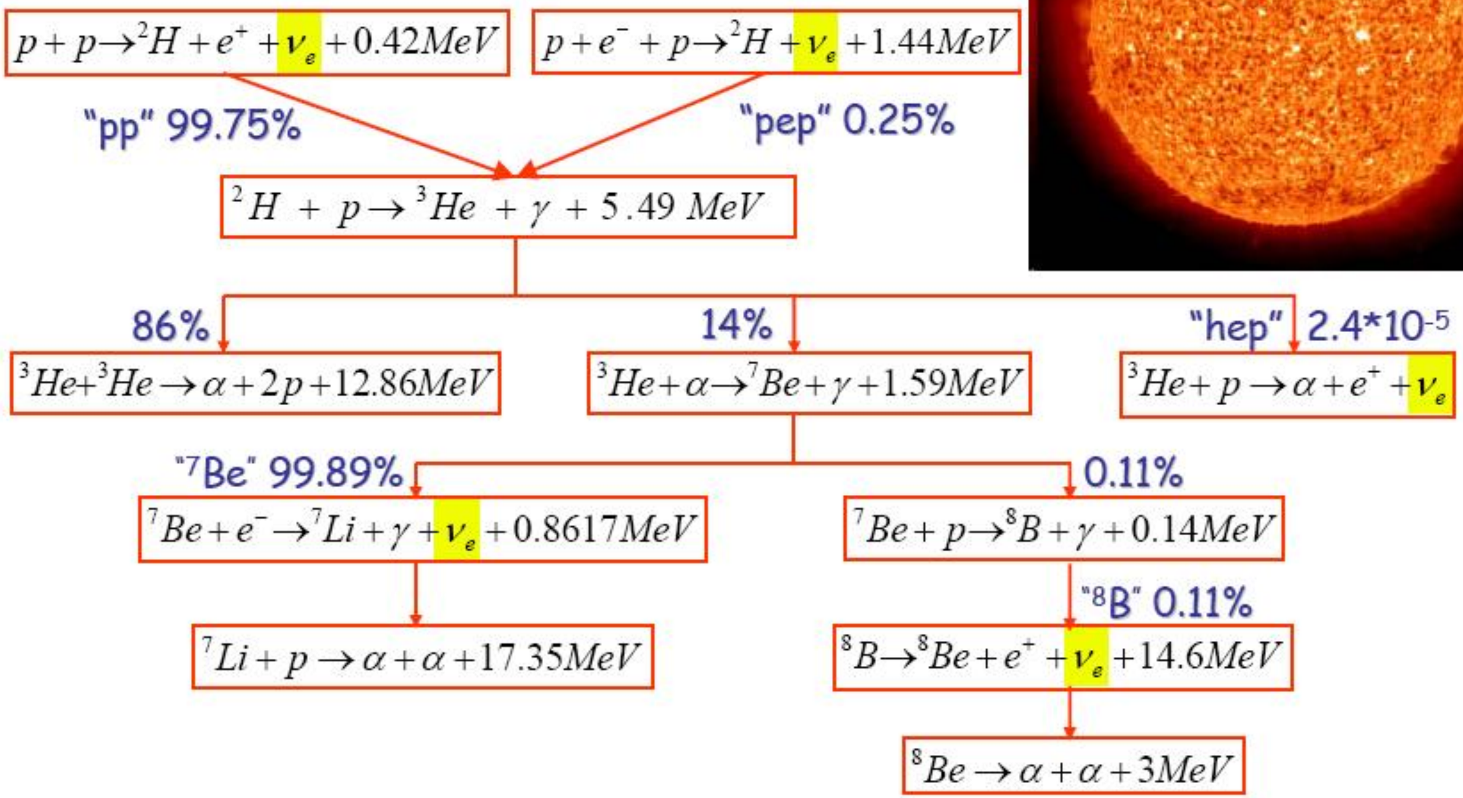
- Appearance:
  Use neutrino beam of type A and search at distance L for neutrinos of type B. ➡ Accelerator neutrinos

2. Solar Electron Neutrino Problem

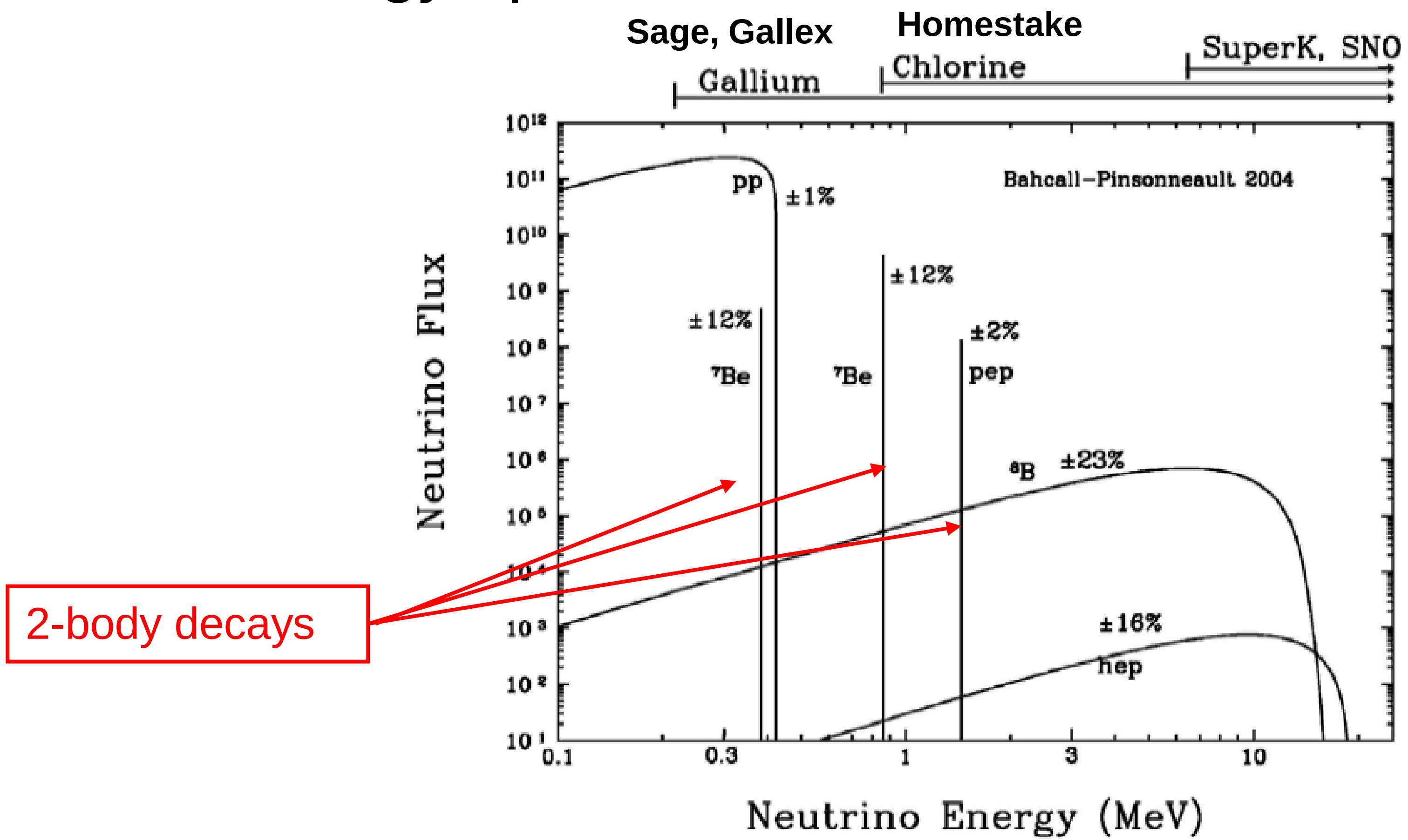
Neutrino production

ν_e are abundant by-products of nuclear fusion in the sun





Neutrino energy spectrum



<u>Neutrino detection:</u>

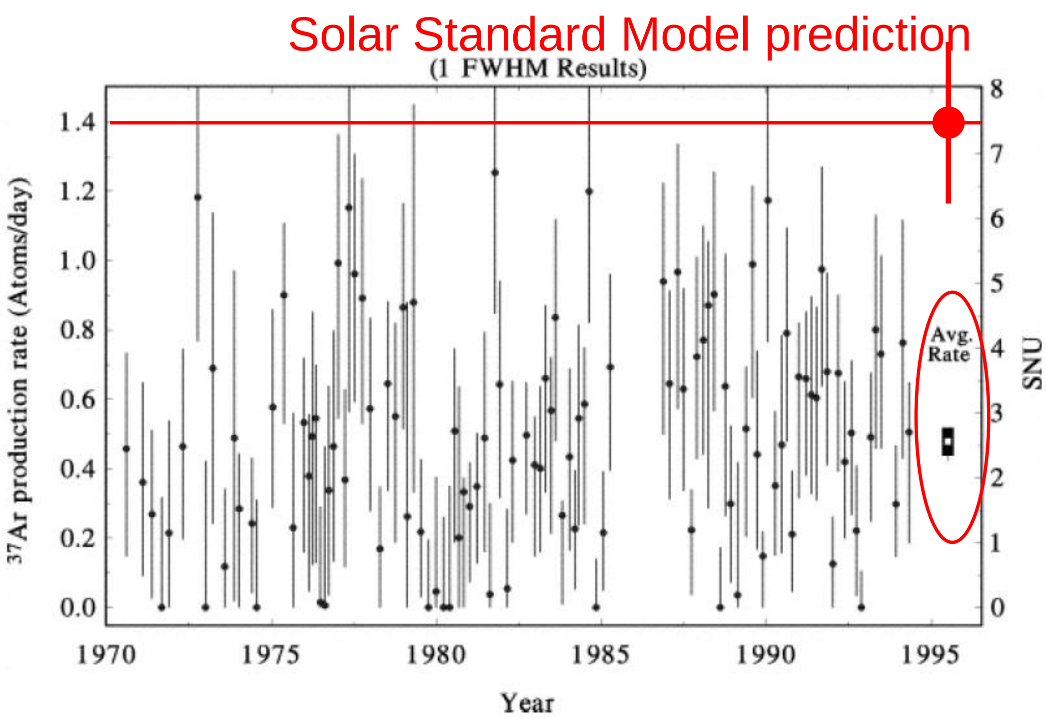
Cl_2 detectors	$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e$, ${}^{37}Ar \rightarrow {}^{37}Cl$ (EC)	E_ν>0.8 MeV
Ga detectors	$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e$	E_ν>0.2 MeV
H_2O detectors	Elastic scattering: $\nu_e + e \rightarrow \nu_e + e$	E_ν>5 MeV (detection)

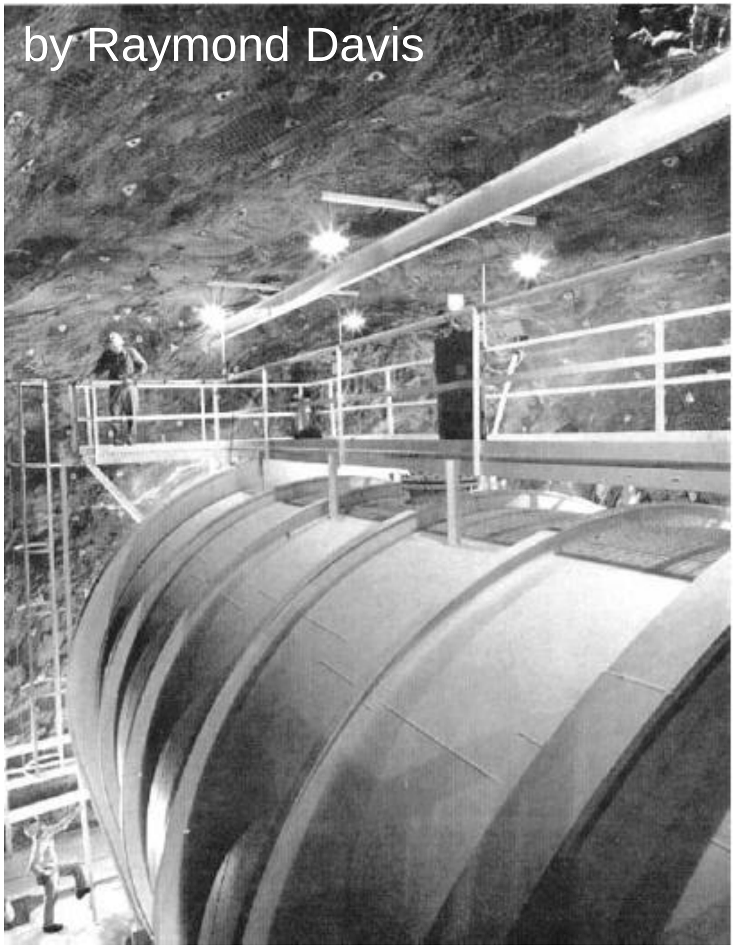
Radio-chemical experiments:

Homestake, SAGE, GALLEX MPI-K

- Homestake mine, 1400 m underground
- 615 t of C_2Cl_4 (perchloroethilene) = 2.2×10^{30} atoms of ^{37}Cl
- Use ^{36}Ar and ^{38}Ar to carry-out the few atoms of ^{37}Ar (~ 1 atom/day)
- Count radioactive ^{37}Ar decays

Homestake Cl_2 experiment

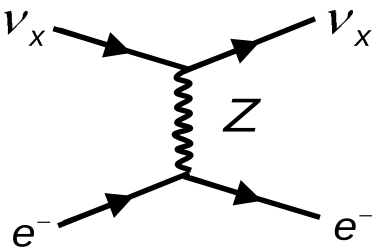


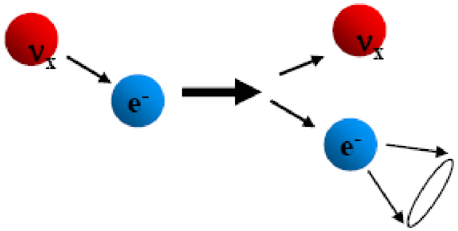


Neutrino detection with water detectors [$E_\nu \sim O(\mathrm{GeV})$]

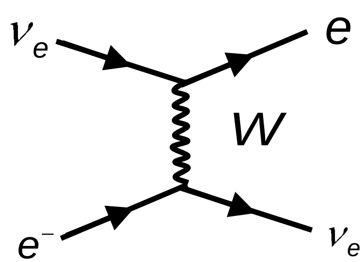
Water = "active target" (Cherenkov effect)

Elastic scattering ES

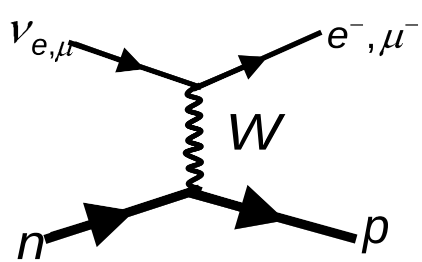




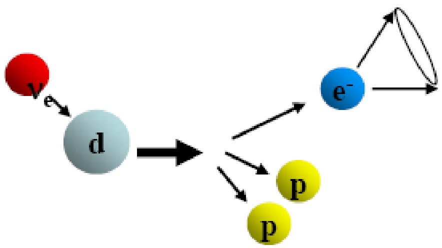
Cherenkov Light



Charged current CC



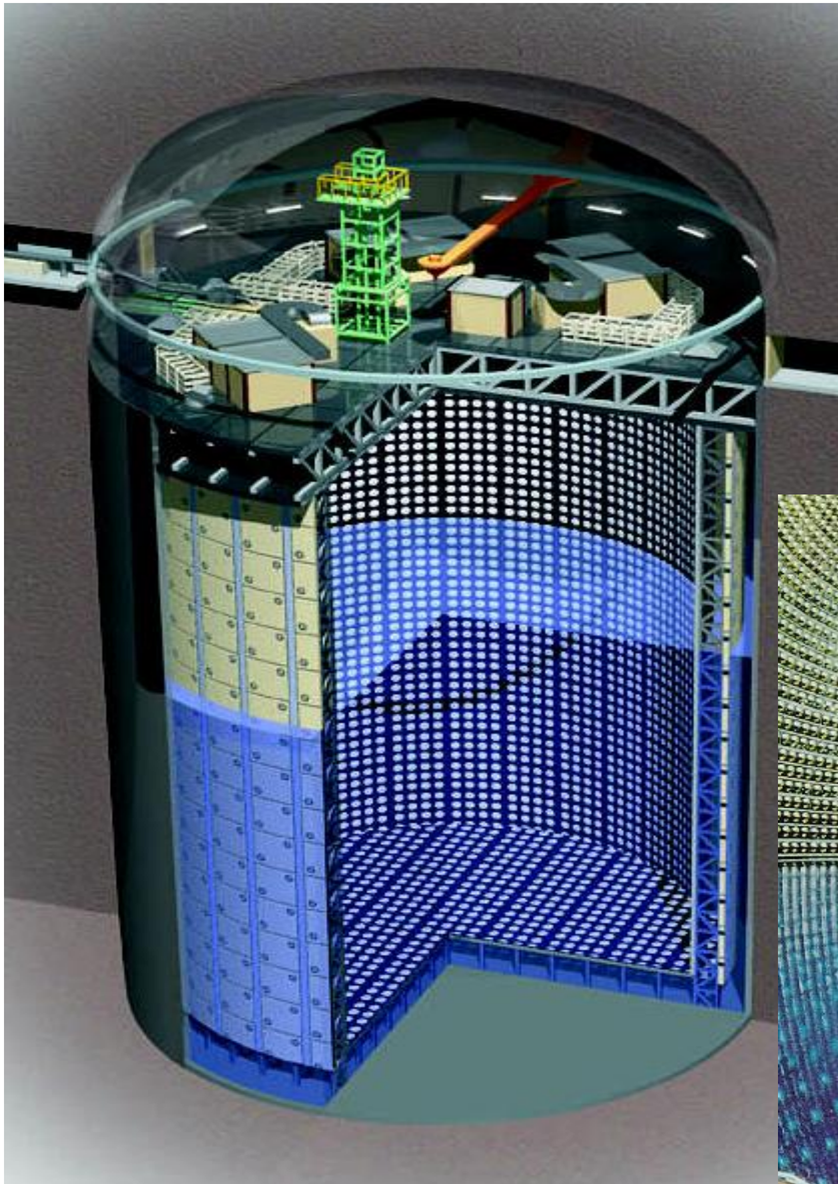
Kinematical limit for ν_μ: $E_\nu > m_\mu$



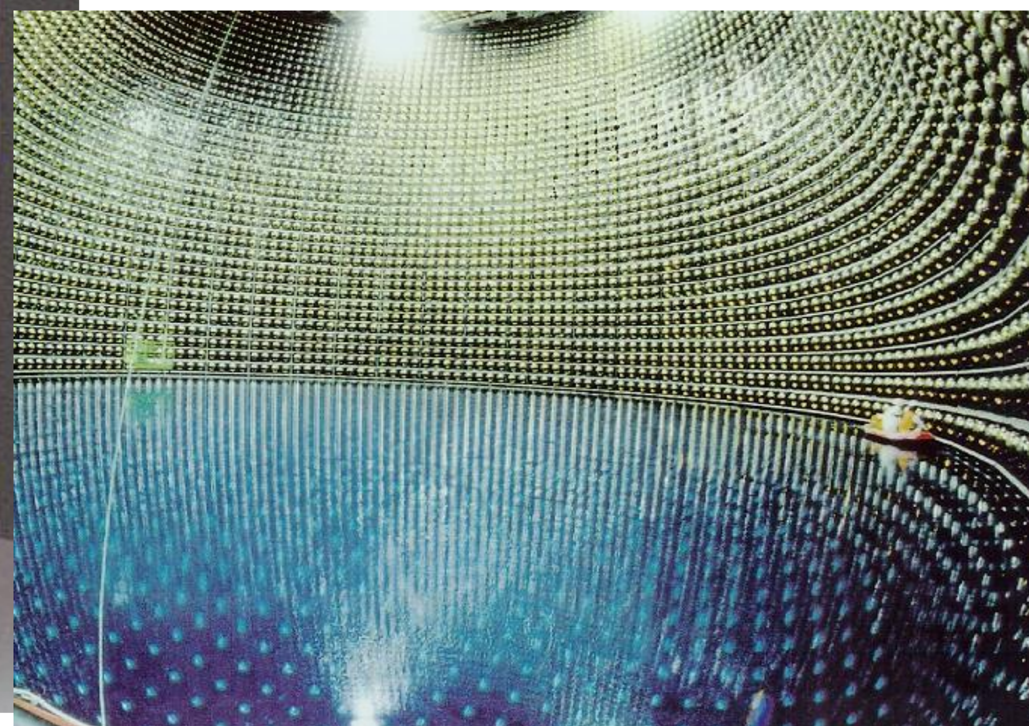
Detection of Cherenkov photons: Photo multiplier

Experiments: (Super)-Kamiokande

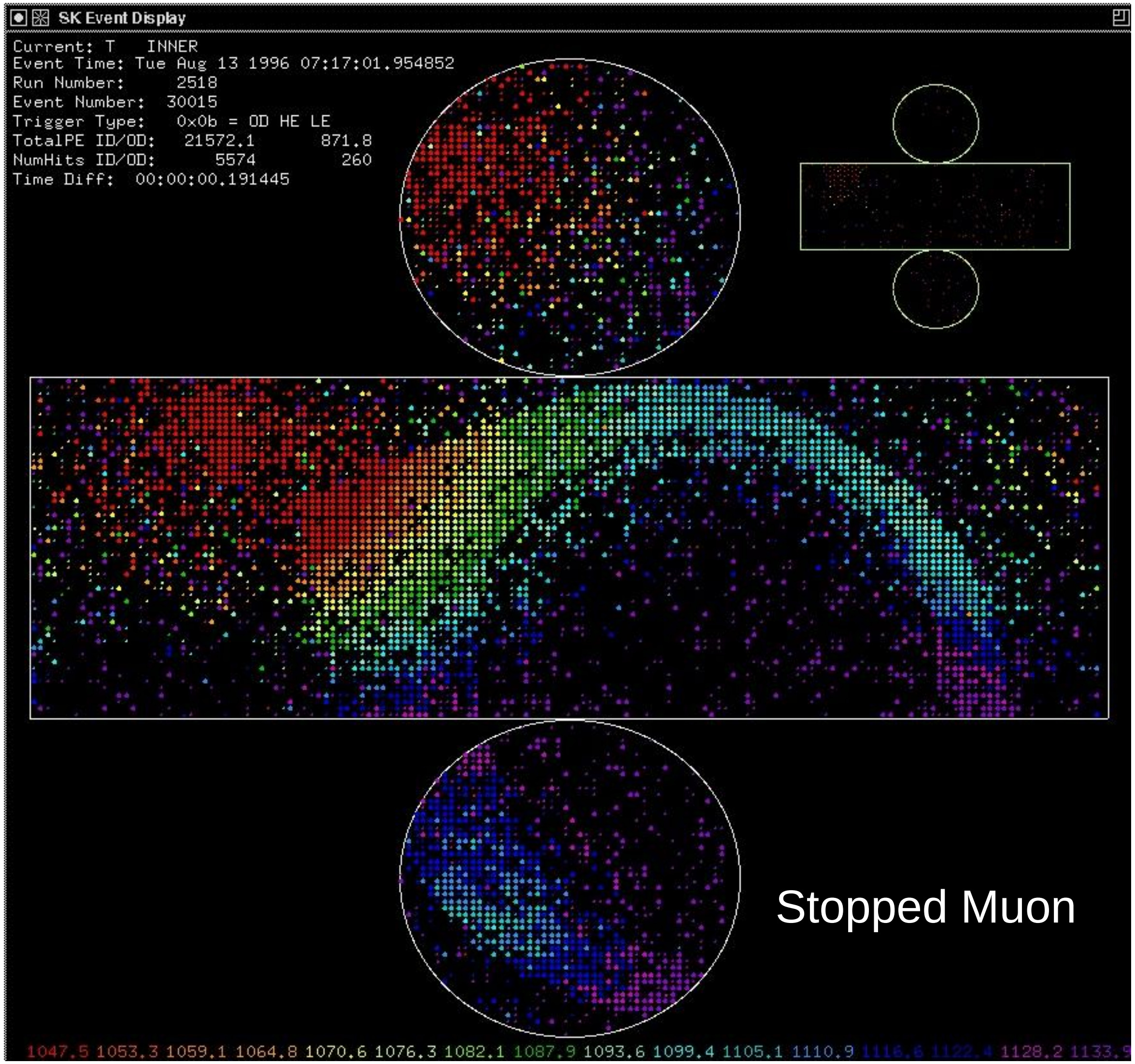
Super-Kamiokande

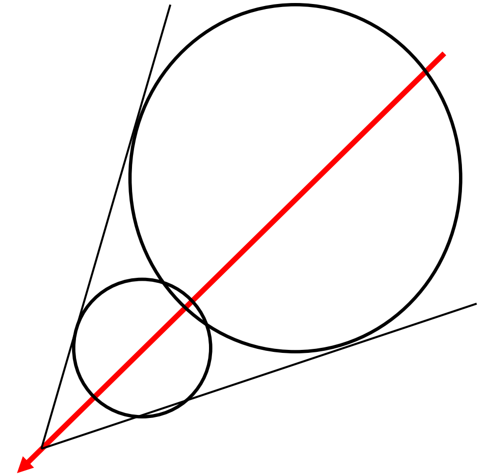


- Largest artificial water detector (50 kt)
- 11000 PMTs (50 cm tubes!): 40% of surface covered with photo-cathode



$\nu_\mu \rightarrow \mu$ stopped





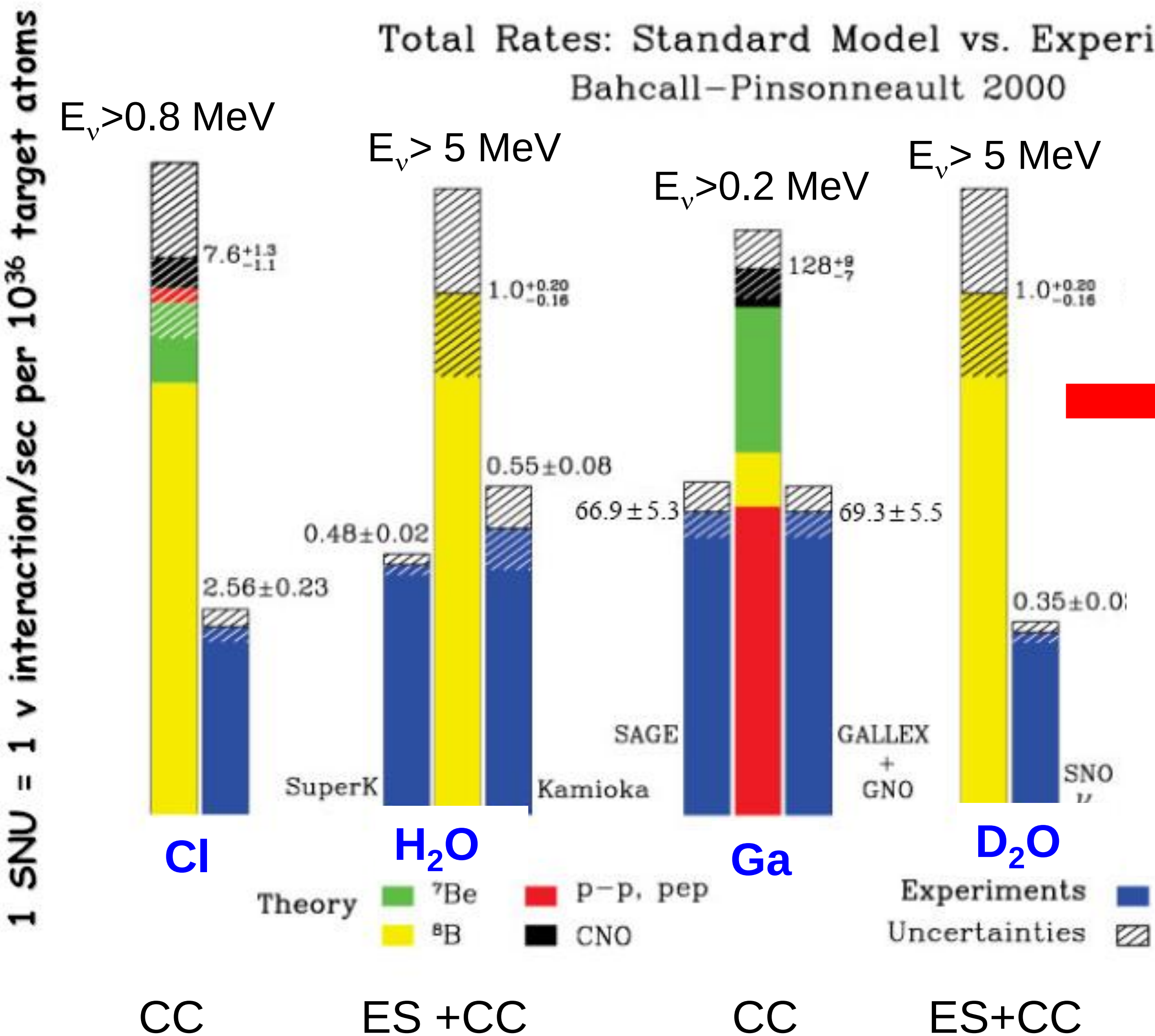
Cherenkov cone:

$$\cos\theta = \frac{1}{\beta n}$$

$$\Leftrightarrow \theta = 42^o \ (\beta = 1)$$

Experiment can distinguish electron and muon events, can measure energy

Solar Neutrino Problem: Experimental summary



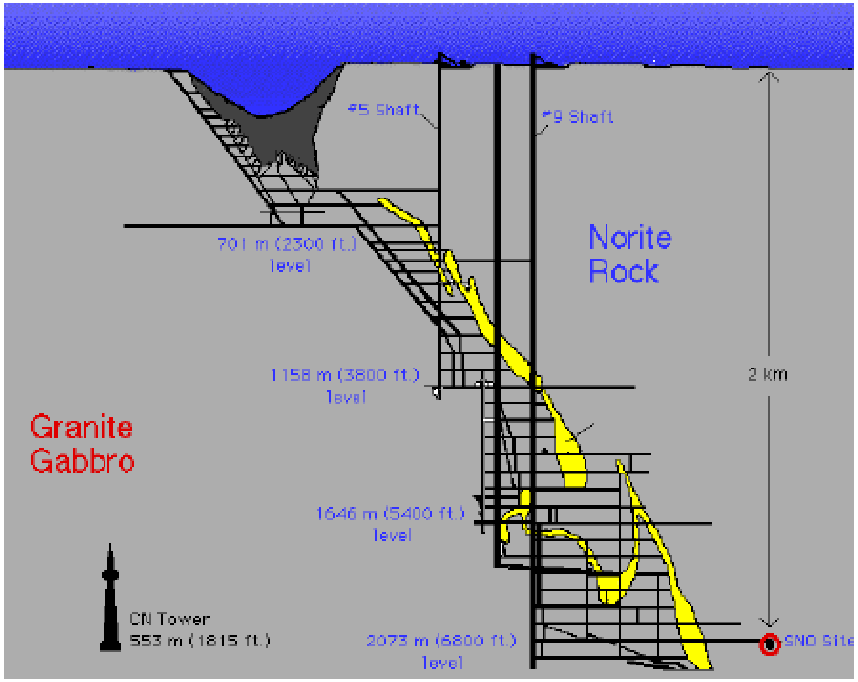
All to low: Neutrino disappearance

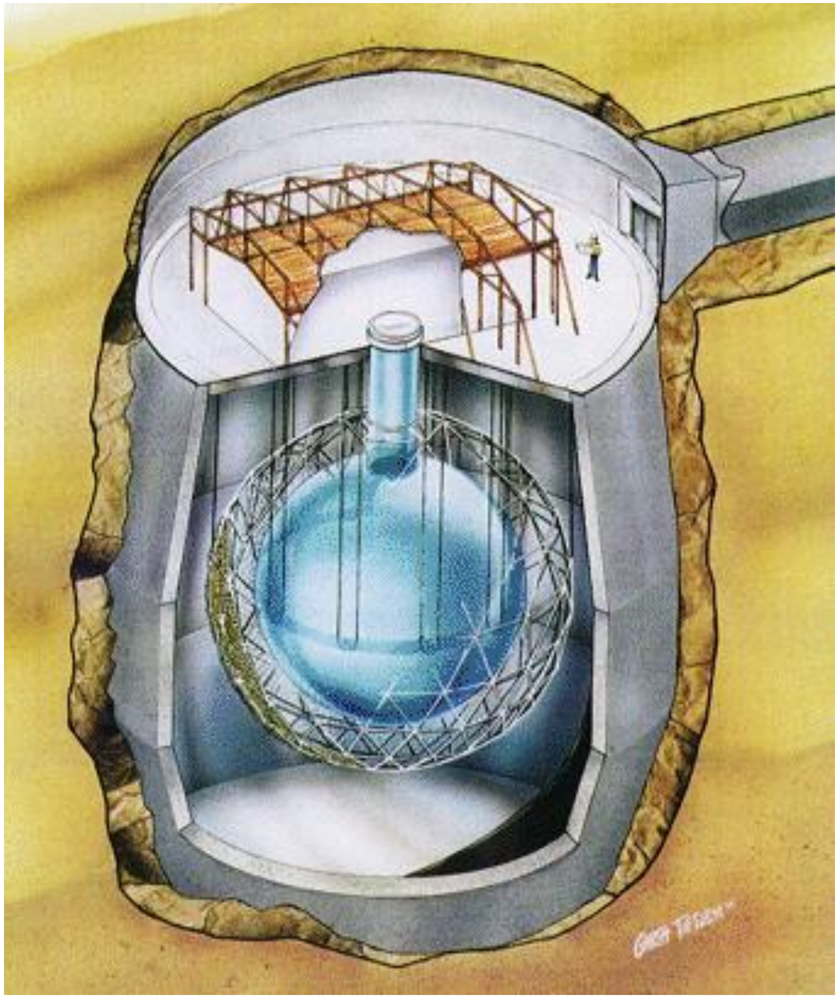
Can one measure the oscillated neutrinos ??

Sudbury Neutrino Observatory

➡ Try to measure the "oscillated" neutrinos of different flavor

- 6 m radius transparent acrylic vessel
- 1000 t of heavy water (D_2O)
- 9456 inward looking photo multipliers
- Add 2 t of NaCl to detect neutrons





Solar Neutrino (^{8}B) detection with SNO

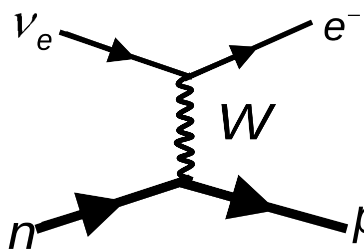
Low energy neutrinos!

Charged current

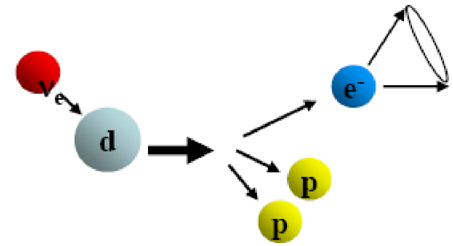
$$\sigma(\nu_\mu) = \sigma(\nu_\tau) = 0$$

(kinematics)

$$\phi_{CC} = \phi_{\nu_e}$$



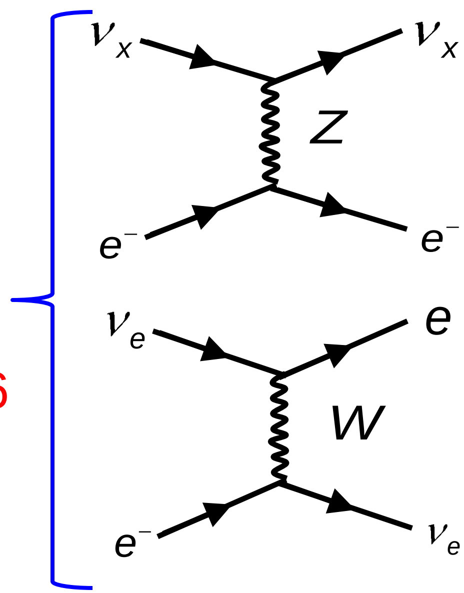
Cherenkov Light



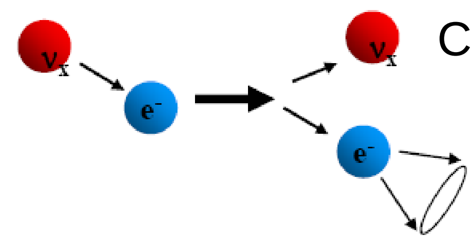
Elastic scattering

$$0.154 \cdot \sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$$

$$\phi_{ES} = \phi_{\nu_e} + (\phi_{\nu_\mu} + \phi_{\nu_\tau}) / 6$$



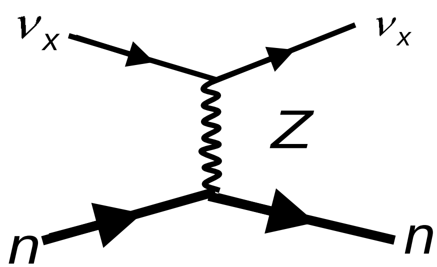
Cherenkov Light

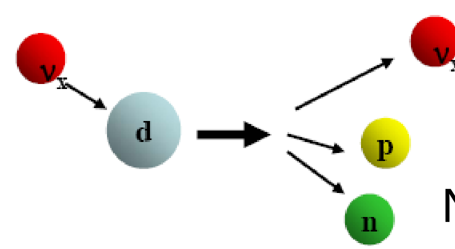


Neutral current

$$\sigma(\nu_e) = \sigma(\nu_\mu) = \sigma(\nu_\tau)$$

$$\phi_{NC} = \phi_{\nu_e} + \phi_{\nu_\mu} + \phi_{\nu_\tau}$$

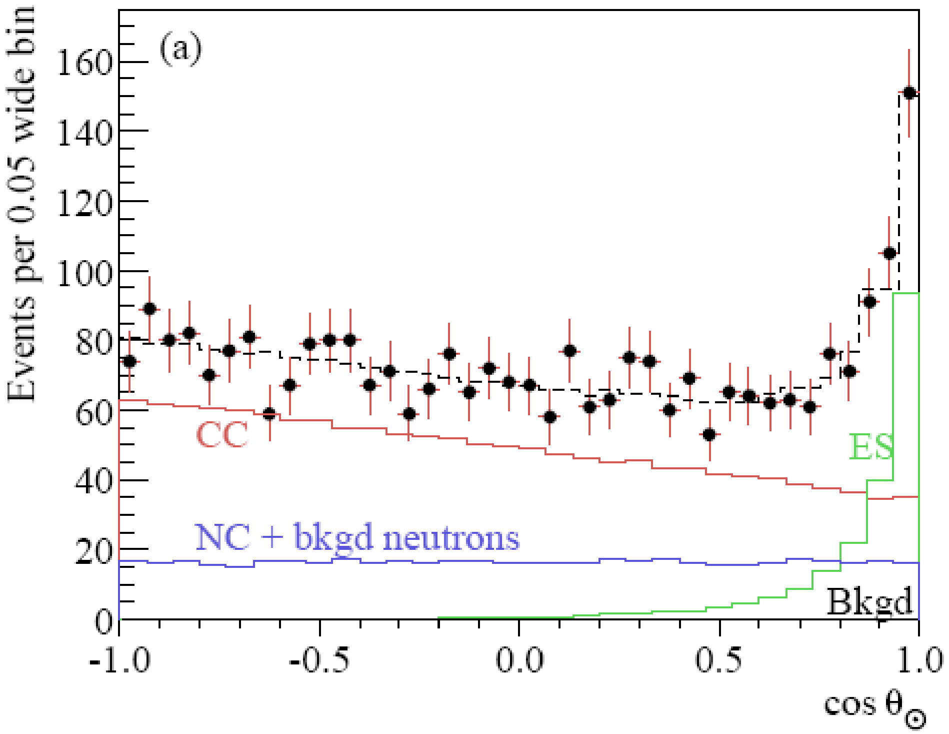




Neutron

$^{35}Cl\,(n,\gamma)^{36}Cl$

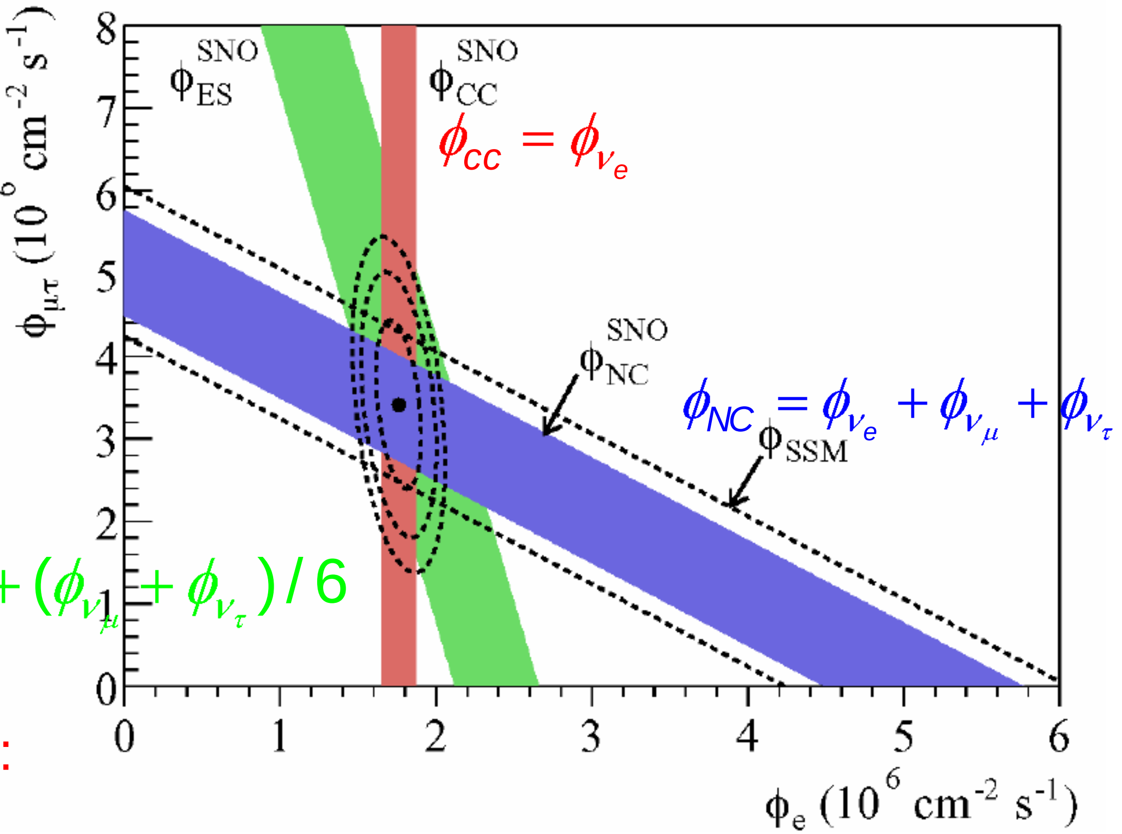
Separation between CC, ES and NC events:



SNO Evidence for Neutrino Oscillation

$$\phi_{CC}^{SNO} = 1.76^{+0.06}_{-0.05}(\text{stat})^{+0.09}_{-0.09}(\text{syst}),$$
$$\phi_{ES}^{SNO} = 2.39^{+0.24}_{-0.23}(\text{stat})^{+0.12}_{-0.12}(\text{syst}),$$
$$\phi_{NC}^{SNO} = 5.09^{+0.44}_{-0.43}(\text{stat})^{+0.46}_{-0.43}(\text{syst}).$$

Phys. Rev.Lett. 89 (2002) 011301



$\phi_{ES} = \phi_{\nu_e} + (\phi_{\nu_\mu} + \phi_{\nu_\tau})/6$

Electron neutrino flux is too low:

$P_{\nu e \nu e} = (35 \pm 2)\%$

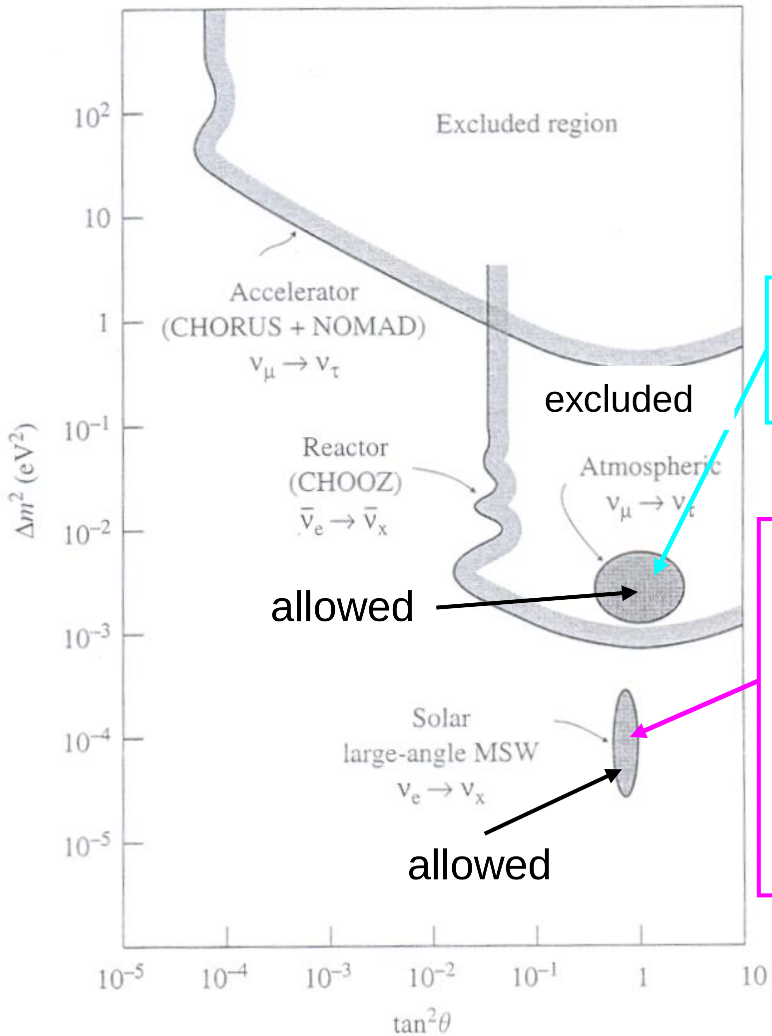
Total flux of neutrinos is correct!!

Interpreted as
$\nu_e \leftrightarrow \nu_\mu$ or ν_τ oscillation

But in case of simple “vacuum oszillation”: $P_{\nu e \nu e} \geq 1 - \frac{1}{2}\sin^2 2\theta \geq 50\%$?

Vacuum oscillation effect is enhanced by matter inside the sun: MSW effect.
Mikheyev–Smirnov–Wolfenstein

1.3 Status of oscillation measurements



Atmos
$\nu_\mu \rightarrow \nu_x$

$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}$ eV
$\sin^2 2\theta > 0.92$ @ 90% *C.L.*

Solar+KamLAND ← Long baseline reactor experiment

$\nu_e \rightarrow \nu_x$

LMA = large mixing angle + matter effect :
MSW effect*)

$\Delta m^2 = (8.2 \pm 0.6) \times 10^{-5}$ eV
$\sin^2 2\theta \approx 0.83$

Different oscillation pattern for different neutrinos – what can we learn about the masses ??

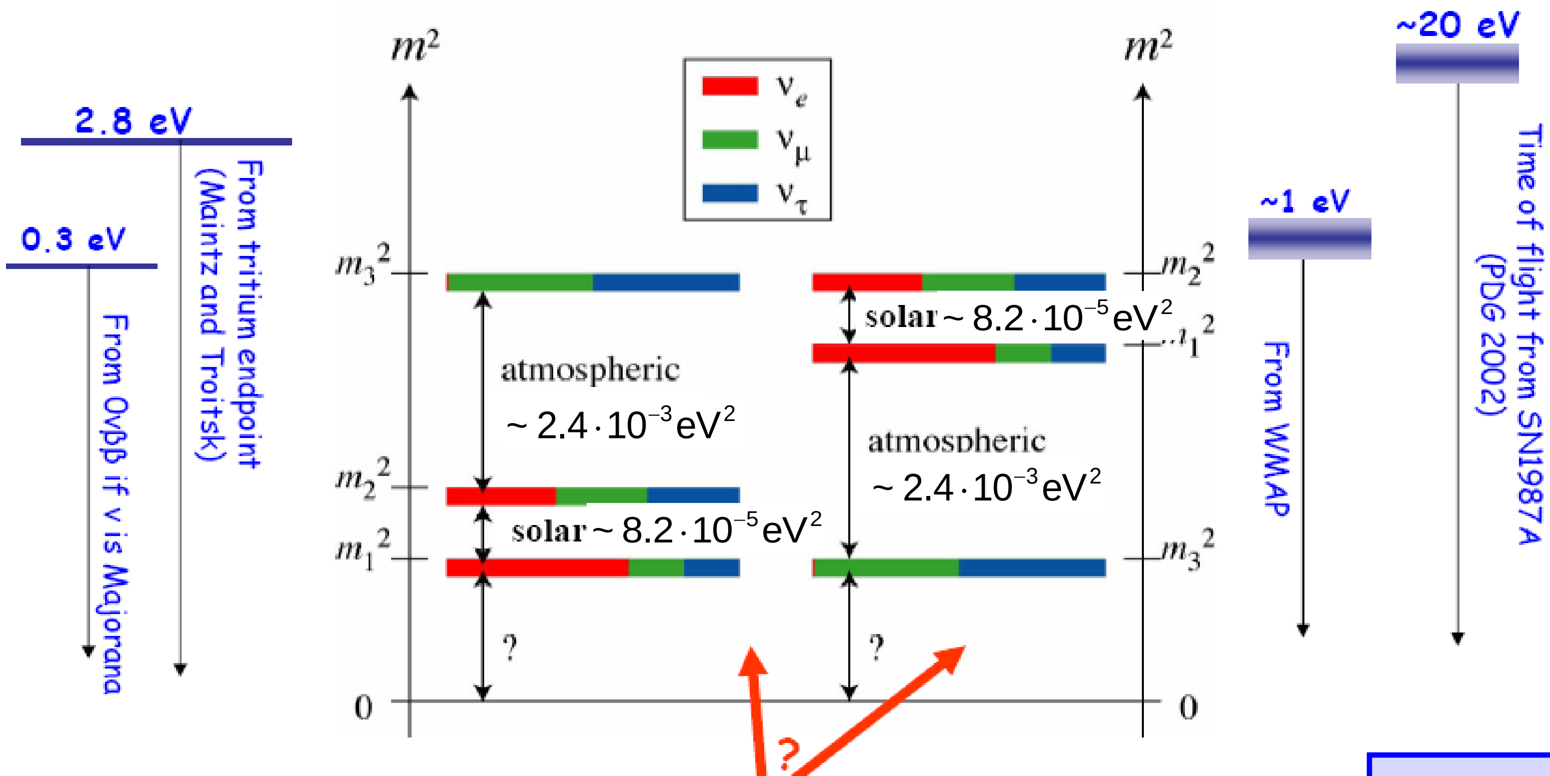
**) Mikhaev, Smirnov (1986), Wolfenstein (1976)*

1.4 Neutrino masses

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13} & c_{12}c_{23} - s_{12}s_{23}s_{13} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13} & -c_{12}s_{23} - s_{12}c_{23}s_{13} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

$\theta_{12} \equiv \theta_{sol}$ $\theta_{23} \equiv \theta_{atm}$ $\theta_{13} \approx 0$



Sign of Δm^2_{13} (Future accelerator experiments)

Absolute neutrino masses are not known !

Outlook:

CP Violation in Neutrino Mixing ?