12-1 Mixing of neutral B mesons

The time evolution of the flavor eigenstates B^0 and $\bar{B}^0$ is governed by a phenomenological Schrödinger eq. (Wigner–Weisskopf approximation):

$$i\frac{d}{dt}\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = H\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = \left(M-\frac{i}{2}\Gamma\right)\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix}$$

Note that the 2×2 effective Hamiltonian H is not Hermitian, since otherwise the mesons would only oscillate and not decay. On the otherhand, the mass matrix M and the decay matrix Γ are t-independent 2×2 Hermitian matrices.

The mass eigenstates are defined as the eigenvectors of H as

$$\begin{pmatrix}|B_L\rangle\\|B_H\rangle\end{pmatrix} = \begin{pmatrix}p & q\\ p & -q\end{pmatrix}\begin{pmatrix}|B\rangle\\|\bar{B}\rangle\end{pmatrix} \quad \text{with } |p|^2+|q|^2=1$$

Note that, in general, $|B_L\rangle$ and $|B_H\rangle$ are not orthogonal to each other.

(If CP is a symmetry of H, $|q/p|=1 \Rightarrow \langle B_L|B_H\rangle = 0$.)

$$\begin{pmatrix}|B(t)\rangle\\|\bar{B}(t)\rangle\end{pmatrix} = \frac{1}{2pq}\begin{pmatrix}q & q\\ p & -p\end{pmatrix}\begin{pmatrix}|B_L(t)\rangle\\|B_H(t)\rangle\end{pmatrix}$$
$$= \frac{1}{2pq}\begin{pmatrix}q & q\\ p & -p\end{pmatrix}\begin{pmatrix}e^{-(iM_L+\Gamma_L/2)t}\,|B_L\rangle\\ e^{-(iM_H+\Gamma_H/2)t}\,|B_H\rangle\end{pmatrix}$$

$$\Rightarrow\begin{cases}|B(t)\rangle = g_+(t)\,|B\rangle + \frac{q}{p}\,g_-(t)\,|\bar{B}\rangle\\ |\bar{B}(t)\rangle = \frac{p}{q}\,g_-(t)\,|B\rangle + g_+(t)\,|\bar{B}\rangle\end{cases}$$

where
$$g_\pm(t) = \frac{1}{2}\left[e^{-(iM_L+\Gamma_L/2)t} \pm e^{-(iM_H+\Gamma_H/2)t}\right]$$

$$\longrightarrow\begin{cases}g_+(t) = e^{-(im+\Gamma/2)t}\left[\cos\frac{\Delta m}{2}t\cosh\frac{\Delta\Gamma}{4}t - i\sin\frac{\Delta m}{2}t\sinh\frac{\Delta\Gamma}{4}t\right]\\ g_-(t) = e^{-(im+\Gamma/2)t}\left[-\cos\frac{\Delta m}{2}t\sinh\frac{\Delta\Gamma}{4}t + i\sin\frac{\Delta m}{2}t\cosh\frac{\Delta\Gamma}{4}t\right]\end{cases}$$

where
$$m=\frac{M_H+M_L}{2},\quad \Gamma=\frac{\Gamma_H+\Gamma_L}{2},\quad \Delta m = M_H-M_L,\quad \Delta\Gamma=\Gamma_L-\Gamma_H$$

The time dependent probability to observe the $B^0/\bar{B}^0$ meson for $\Delta\Gamma\approx 0$:

$$P(B^0\to B^0) = |\langle B|B(t)\rangle|^2 = |g_+(t)|^2 = e^{-\Gamma t}\left(\cos\frac{\Delta m}{2}t\right)^2 = \frac{1}{2}e^{-\Gamma t}(1+\cos\Delta m t)$$
$$P(B^0\to\bar{B}^0) = |\langle\bar{B}|B(t)\rangle|^2 = \left|\frac{q}{p}\right|^2|g_-(t)|^2 = \underbrace{e^{-\Gamma t}}_{|\frac{q}{p}|^2}\left(\sin\frac{\Delta m}{2}t\right)^2 = \underbrace{\frac{1}{2}}_{|\frac{q}{p}|^2}e^{-\Gamma t}(1-\cos\Delta m t)$$

($*$ $P(\bar{B}^0\to\bar{B}^0) = P(B^0\to B^0)$ ← CPT

$P(\bar{B}^0\to B^0) \neq P(B^0\to\bar{B}^0)$ if $|\frac{q}{p}|\neq 1$ → $\cancel{CP}$, $\cancel{T}$)

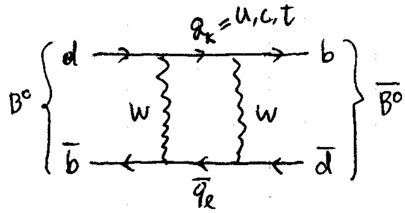
The mixing asymmetry is

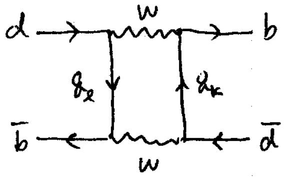
$$A = \frac{P(B^0\to B^0)-P(B^0\to\bar{B}^0)}{P(B^0\to B^0)+P(B^0\to\bar{B}^0)} = \frac{(1+\cos\Delta m t)-|\frac{q}{p}|^2(1-\cos\Delta m t)}{(1+\cos\Delta m t)+|\frac{q}{p}|^2(1-\cos\Delta m t)}$$

$$\xrightarrow[|\frac{q}{p}|\approx 1]{} \cos(\Delta m t)$$

12-2 GIM-suppression

$B^0 - \bar{B}^0$ mixing :





The corresponding transition amplitudes are given by

Inami-Lin func.

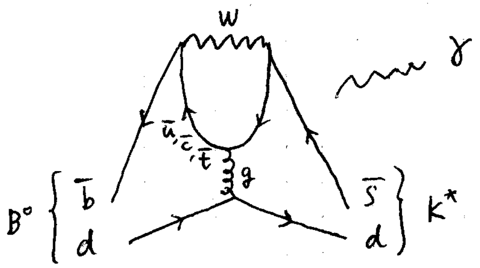
$$\sum_{q_k, q_l} A(q_k, q_l) = \sum_{q_k, q_l} (V_{q_k d} V^*_{q_k b})(V^*_{q_l d} V_{q_l b}) \cdot f(m_{q_k}, m_{q_l}) \cdot A_0$$

In case of equal quark masses, $m = m_u = m_c = m_t$:

$$\sum_{q_k, q_l} A(q_k, q_l) = f(m, m)\, A_0 \underbrace{\sum_{q_k} V_{q_k d} V^*_{q_k b}}_{=0} \underbrace{\sum_{q_l} V^*_{q_l d} V_{q_l b}}_{=0}$$

since the unitarity condition $\sum_i V_{ij} V^*_{ik} = \delta_{ik}$.

12-3 Penguin decays



※ $B^0 \cdots J^P = 0^-$
$K^* \cdots J^P = 1^-$
$\gamma \cdots J^P = 1^-$

$$0^- \not\rightarrow 0^- + \gamma,\ 0^+ + \gamma$$

because of the angular momentum conservation.