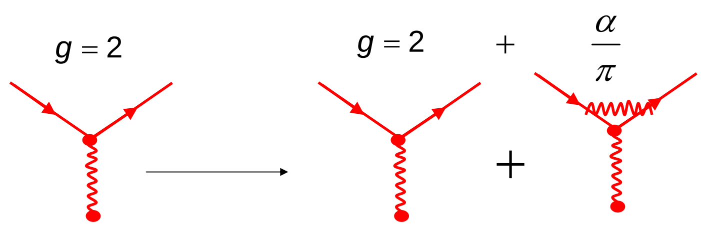
4. Anomalous magnetic moment

Magnetic moment $\vec{\mu} = -g \cdot \mu_B \cdot \vec{s}$



1st order: $\vec{\mu} = -(2 + \frac{\alpha}{\pi}) \cdot \mu_B \cdot \vec{s}$

$$g = 2 + \frac{\alpha}{\pi}$$
$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}$$

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Higher order corrections to g-2

Radiative corrections g-2 are calculated to the 4-loop level:

Feynman Graphs		
O(α)		1
O(α²)		7
O(α³)	analytically	72
O(α⁴)	numerically	891
til O(α⁴)		971

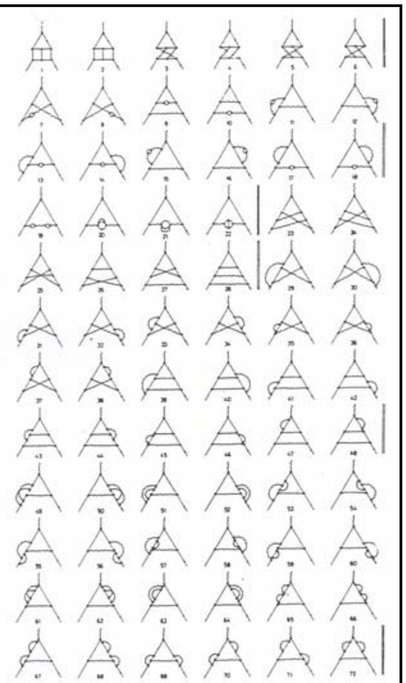


Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α³ corrections to the lepton magnetic moments (after Lautrup et al. 1972).

Most precise QED prediction.

T. Kinoshita et al.

Kinoshita 2006 $a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.505....\left(\frac{\alpha}{\pi}\right)^4$

Kinoshita 2007 $a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.9144....\left(\frac{\alpha}{\pi}\right)^4$

Electron g-2

Experimental method:

Storage of **single** electrons in a Penning trap (electrical quadrupole + axial B field)

$\Rightarrow$ complicated electron movement (cyclotron and magnetron precessions).

Idea: bound electron (**geonium**)

$a_{e^-} = 0.001159\,652\,188\,4\,(43)$
$a_{e^+} = 0.001159\,652\,187\,9\,(43)$

H. Dehmelt et al. 1987

$a_e = 0.001159\,652\,180\,85\,(76)$

G. Gabrielse et al. 2006

Using $a_e \Rightarrow$ most precise value of α:

$\alpha^{-1}(a_e) = 137.035\,999\,710\,(96)$

For comparison α from Quanten Hall

$\alpha^{-1}(qH) = 137.036\,003\,00\,(270)$

Phys. Rev. Lett. ***97****, 030801 (2006)*
Phys. Rev. Lett. ***97****, 030802 (2006)*

Theory *Kinoshita 2006*

$a_e = 0.001159\,652\,133\,(290)$

$a_e = 0.001159\,652\,180\,85\,(76)$

Experimental determination of muon g-2

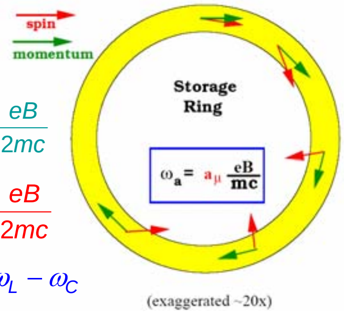
Principle:

- store polarized muons in a storage ring; revolution with cyclotron frequency ω_c
- measure spin precession around the magnetic dipole field relative to the direction of cyclotron motion

$$\omega_C = 2\frac{eB}{2mc}$$

$$\omega_L = g\frac{eB}{2mc}$$

$$\omega_a = \omega_L - \omega_C$$



Precession:

$$\vec{\omega}_a = -\frac{e}{m_\mu c}\left[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}\right]$$

Difference between Lamor and cyclotron frequency

Effect of electrical focussing fields (relativistic effect).

$= 0$ for $\gamma = 29.3$

$\Leftrightarrow p_\mu = 3.094$ GeV/c

First measurements:

CERN 70s

$a_{\mu^-} = 0.001165937(12)$

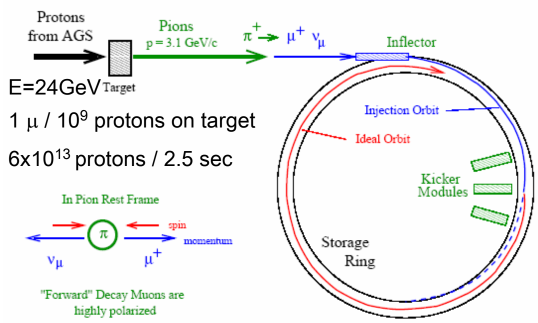
$a_{\mu^+} = 0.001165911(11)$

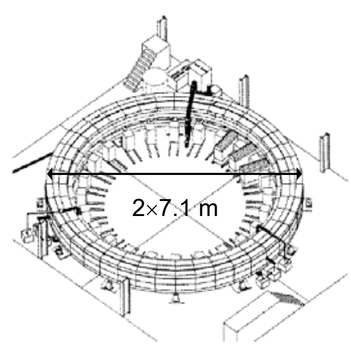
(g-2)$_\mu$ Experiment at BNL

E=24GeV

1 μ / 10^9 protons on target

6×10^{13} protons / 2.5 sec

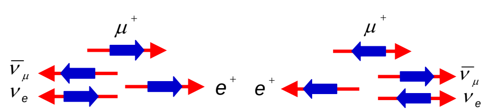


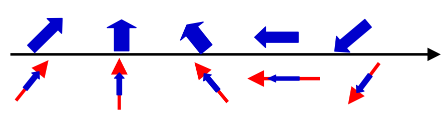


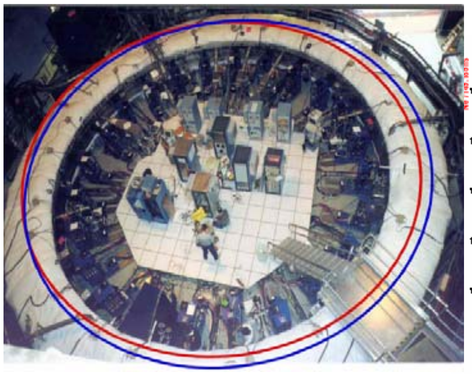
"V-A" structure of weak decay:

Use high-energy e⁺ from muon decay to measure the muon polarization

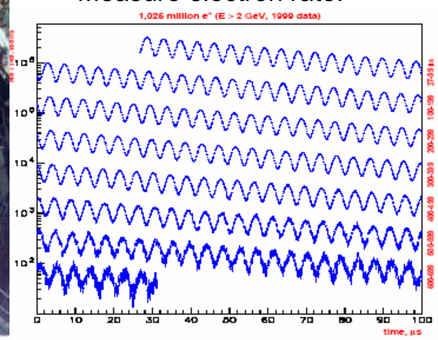
Weak charged current couples to LH fermions (RH anti-fermions)



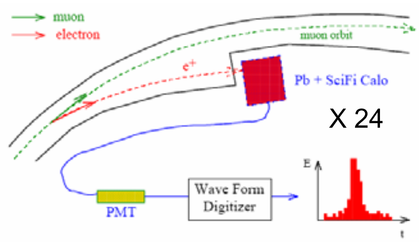




Measure electron rate:



$$N(t) = N_0\, e^{-\lambda t}\left[1 + A\cos(\omega_a t + \varphi)\right]$$



X 24

$$\frac{\omega_a}{2\pi} = 229023.59(16)\,\text{Hz}$$

(0.7ppm)

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu c}\langle B\rangle} \; ?$$

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From ω_a to a_μ - How to measure the B field

$<B>$ is determined by measuring the proton nuclear magnetic resonance (NMR) frequency ω_p in the magnetic field.

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu c}\langle B\rangle} = \frac{\omega_a}{\frac{e}{m_\mu c}\frac{\hbar\tilde{\omega}_p}{2\mu_p}} = \frac{\omega_a}{\frac{4\mu_\mu}{\hbar g_\mu}\frac{\hbar\tilde{\omega}_p}{2\mu_p}} = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p}(1+a_\mu)$$

$$\Downarrow$$

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

$\mu_{\mu+}/\mu_p = 3.183\,345\,39(10)$

W. Liu *et al.*, Phys. Rev. Lett. **82**, 711 (1999).

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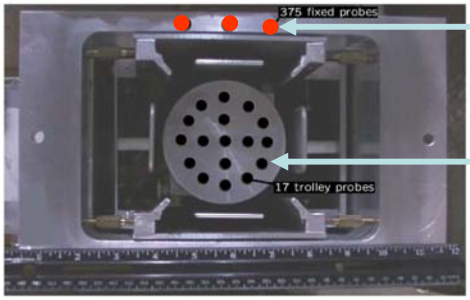
NMR trolley



375 fixed NMR probes around the ring

17 trolley NMR probes

$\tilde{\omega}_p/2\pi$ = 61 791 400(11) Hz (0.2ppm)

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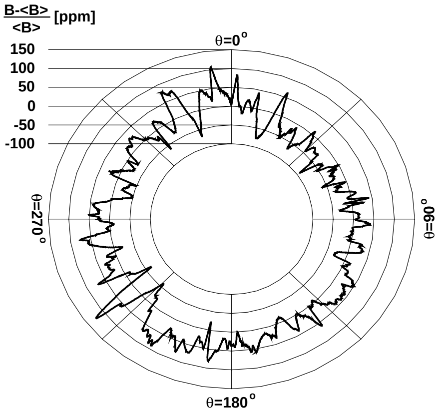
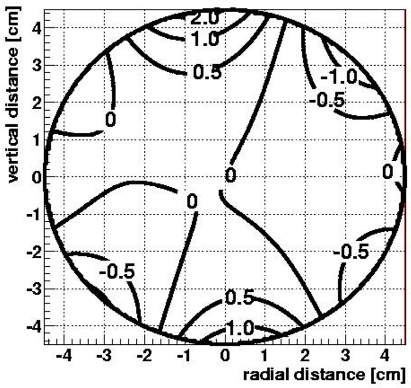
B field determination



The *B* field variation at the center of the storage region.
<B>≈1.45 T



The *B* field averaged over azimuth.

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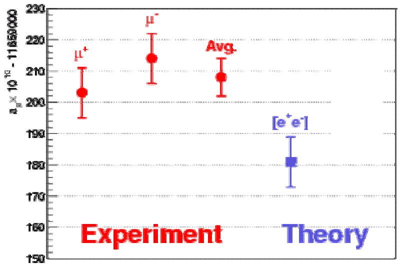
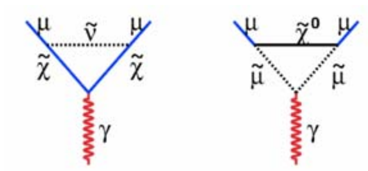
$$a_{\mu^+} = 11659\,203(8) \times 10^{-10} (0.7 ppm)$$
$$a_{\mu^-} = 11659\,214(8) \times 10^{-10} (0.7 ppm)$$
$$a_{\mu} = 11659\,208(6) \times 10^{-10} (0.5 ppm)$$



About 2.6σ deviation:

- Often interpreted as sign of New Physics: SUSY contributions
- careful: "Theory" has uncertainties!



Potential SUSY contributions:

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Remarks: Theoretical prediction of a_μ

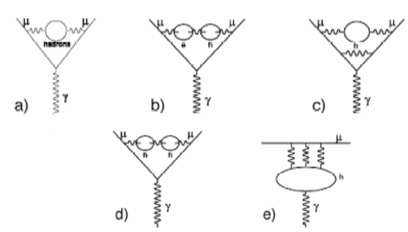
Beside pure QED corrections there are weak corrections (W, Z) exchange and „hadronic corrections“

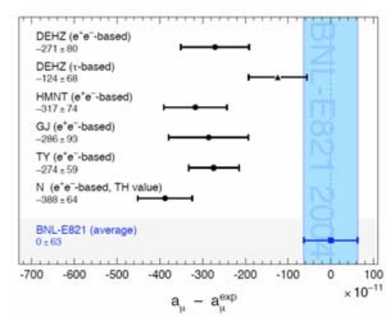
$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW}$$

(For the electron with much lower mass the hadronic and weak corrections are suppressed, and can be neglected.)

→ Determination of hadronic corrections is difficult and is in addition based on data: hot discussion amongst theoreticians how to correctly use the data.

Hadronic corrections





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