Experimental tests of QED

1. From the matrix element to the measurement
2. e^+e^- annihilation experiments
3. Casimir effect
4. Anomalous magnetic moment

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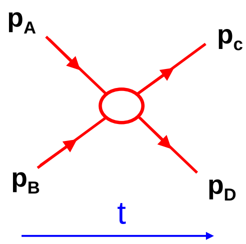
1. From matrix element to measurement

1.1 Transition rate and cross section

$$A + B \rightarrow C + D$$



Diff. transition rate / unit volume

Phase space factor

$$dw_{fi} = \frac{(2\pi)^4}{V^4}\delta^4(p_A + p_B - p_C - p_D)\cdot\left|M_{fi}\right|^2 d\rho_f$$

$$\left|\langle f\,|H'|\,i\rangle\right|^2$$

$$d\rho_f(C,D) = \frac{Vd^3p_C}{2E_C(2\pi)^3}\,\frac{Vd^3p_D}{2E_D(2\pi)^3}$$

$$dw_{fi} = \frac{(2\pi)^4}{V^4}\delta^4(p_A + p_B - p_C - p_D)\cdot\left|M_{fi}\right|^2\cdot\frac{Vd^3p_c}{2E_C(2\pi)^3}\cdot\frac{Vd^3p_D}{2E_D(2\pi)^3}$$

(Nachtmann, S.71)

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Phase space density

$\hbar = 1$

Quantum mechanics restricts the number of final states of particle in a volume V with momentum $\in [\vec{p}, \vec{p} + d\vec{p}]$

$$d\rho_f = \frac{Vd^3p}{h^3} = \frac{Vd^3p}{\hbar^3(2\pi)^3} = \frac{Vd^3p}{(2\pi)^3}$$

With $\langle p | p \rangle = 2p^0 V$ i. e. probability to find particle in volume V is $2p^0$ one has to introduce a factor $1/2E = 1/2p^0$ to normalize probality to 1:

$$d\rho_f = \frac{Vd^3p}{2E(2\pi)^3}$$

$$\langle p' | p \rangle = \langle 0 | a(p') a^+(p) | 0 \rangle = \langle 0 | \{a(p'), a^+(p)\} | 0 \rangle = (2\pi)^3 2p^0 \delta^3(\vec{p} - \vec{p}')$$

For particle C and D scattered into momentum elements d^3p_C and d^3p_D

$$d\rho_f(C,D) = \frac{Vd^3p_C}{2E_C(2\pi)^3} \frac{Vd^3p_D}{2E_D(2\pi)^3}$$

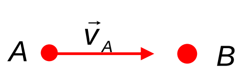
Cross section = $\dfrac{\text{Transition rate / unit volume}}{\text{incident particle flux F}}$

Differential cross section: $$d\sigma = \frac{dw_{fi}}{F}$$

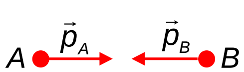
Incident particle flux F

Choose system where particle B is at rest

F = (flux density A) × (density B)



$$F = |\vec{v}_A| \frac{2E_A}{V} \cdot \frac{2E_B}{V} \quad \text{with } \vec{v}_A = \frac{\vec{p}_A}{E_A}$$



$\vec{p}_A = -\vec{p}_B = \vec{p}_i$

$$F = \frac{4}{V^2} |\vec{p}_i| \cdot (E_A + E_B) = \frac{4}{V^2} |\vec{p}_i| \sqrt{s}$$

J. Pawlowski / U. Uwer

1.2 Lorentz invariant phase space factor

Putting everything together $\quad d\sigma = \frac{dw_{fi}}{F}$

$$d\sigma = \frac{(2\pi)^4}{V^4}\delta^4(p_A + p_B - p_C - p_D)\cdot|M_{fi}|^2 \cdot \frac{Vd^3p_c}{2E_C(2\pi)^3}\cdot\frac{Vd^3p_D}{2E_D(2\pi)^3}\cdot\frac{V^2}{|\vec{v}_A| 2E_A 2E_B}$$

$$= \frac{|M_{fi}|^2}{|\vec{v}_A| 2E_A 2E_B}\cdot(2\pi)^4\delta^4(p_A + p_B - p_C - p_D)\frac{d^3p_c}{2E_C(2\pi)^3}\cdot\frac{d^3p_D}{2E_D(2\pi)^3}$$

Particle flux ***F*** $\qquad$ Lorentz invariant 2-particle phase space factor $d\Phi_2$

Remark: unit volume V drops out !

$$d\Phi_n(P, \underbrace{p_1, p_2, \ldots, p_n}_{\text{Final state}}) = \delta^4(P - (p_1 + p_2 + \ldots + p_n))\prod_{final}\frac{d^3p_f}{(2\pi)^3 2E_f}$$

See also PDG
http://pdg.lbl.gov/2007/reviews/kinemarpp.pdf

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Phase space factor **dΦ** for two-particles final-state

CM System:
$\vec{p}_A = -\vec{p}_B \quad \vec{p}_C = -\vec{p}_D$

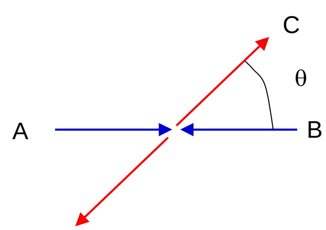
$$d\Phi_2 \xrightarrow{\int} \int d\Phi_2 = \frac{1}{4\pi^2}\int \delta^3(\vec{p}_C + \vec{p}_D)\delta(E_A + E_B - E_C - E_D)\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}$$

$$\int d\Phi = \frac{1}{4\pi^2}\int \frac{|\vec{p}_f|}{4\sqrt{s}}\, d\Omega$$

$\vec{p}_f = \vec{p}_C = -\vec{p}_D$

$s = (E_A + E_B)^2$

$d\Omega = d\cos\theta\, d\varphi$



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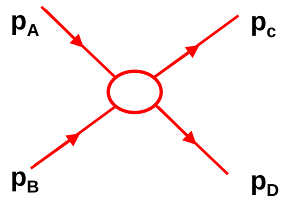
1.3 Differential cross section …putting everything together

$$A + B \rightarrow C + D$$



$$d\sigma = \frac{|M_{fi}|^2}{F} d\Phi = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |M_{fi}|^2 d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |M_{fi}|^2$$

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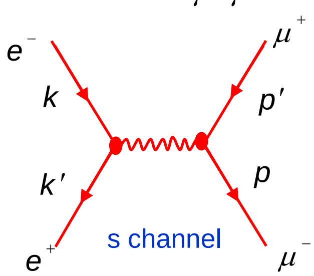
2. e^+e^- annihilation experiments

2.1 Myon pair production

$$e^+e^- \rightarrow \mu^+\mu^-$$



$$M = -\frac{e^2}{q^2} \bar{v}_e(k')\gamma_\mu u_e(k) \cdot \bar{u}_\mu(p)\gamma^\mu v_\mu(p')$$

Spinors decribe a specific spin state of the fermions

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections $\Rightarrow$ need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{|M|^2} = \frac{1}{(2s_e+1)(2s_e+1)} \cdot \sum_{s_e, s'_e} \sum_{s_\mu, s'_\mu} |M|^2$$

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$$\overline{|M|^2}_{e^+e^-\to\mu^+\mu^-}(t,s,u) = 2e^4\,\frac{t^2+u^2}{s^2}$$

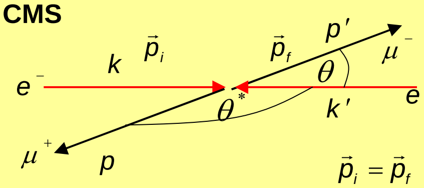
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2}\cdot\frac{1}{s}\cdot\frac{t^2+u^2}{s^2} = \frac{e^4}{64\pi^2}\cdot\frac{1}{s}\cdot(1+\cos^2\theta)$$

$e^2 = 4\pi\alpha$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

Kinematics for high-relativistic particles

CMS



$\vec{p}_i = \vec{p}_f$

$$s = (k+k')^2 \approx 4E_i^2$$

$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*) \approx -\frac{s}{2}(1+\cos\theta)$$

$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta) \approx -\frac{s}{2}(1-\cos\theta)$$

← 1/s dependence from flux factor

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$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86\,\text{nb}\,\text{GeV}^2}{s}$$

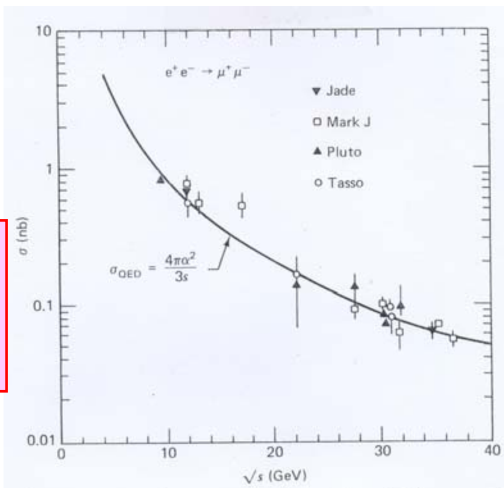

Fig. 6.6 The total cross section for $e^-e^+\to\mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

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2.2 Experimental methods

e^+e^- accelerator (selection)

Accelerator	Lab	$\sqrt{s}$	L_{int} / Exper.
SPEAR	SLAC	2 – 8 GeV	
PEP	SLAC	→29 GeV	220 - 300 pb^{-1}
PETRA	DESY	12 - 47 GeV	~20 pb^{-1}
TRISTAN	KEK	50 – 60 GeV	~20 pb^{-1}
LEP	CERN	90 GeV	~200 pb^{-1}

Cross section (experimental definition)

$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon\, L_{int}}$$

- N_{ff} number of detected $e^+e^- \to ff$ events
- b background fraction
- ε acceptance / efficiency
- L_{int} integrated luminosity of collider

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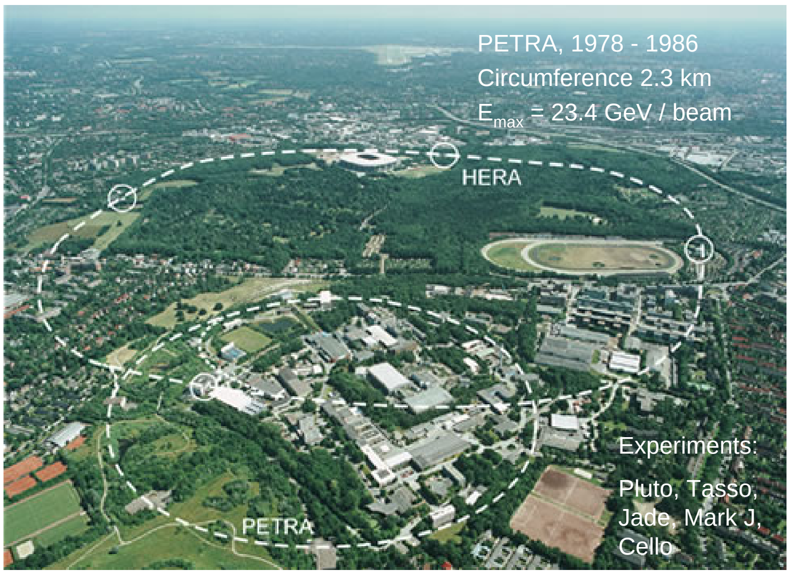

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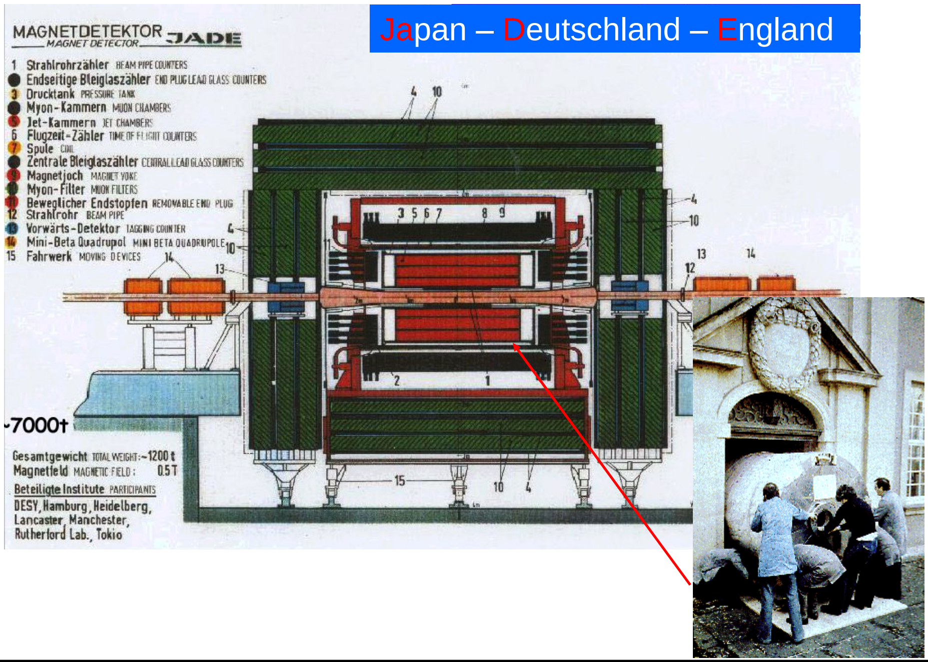
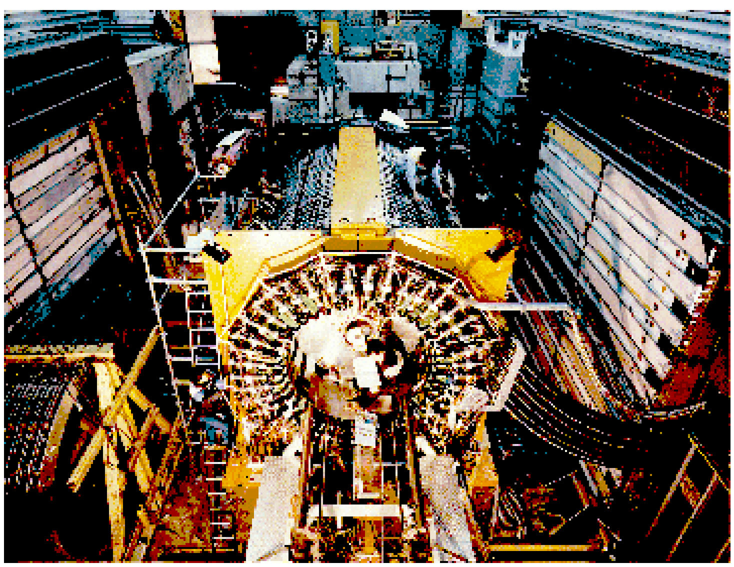
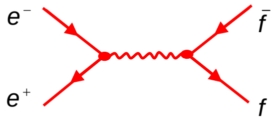
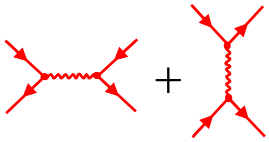

Japan – Deutschland – England
MAGNETDETEKTOR JADE
MAGNET DETECTOR
1 Strahlrohrzähler BEAM PIPE COUNTERS
Endseitige Bleiglaszähler END PLUG LEAD GLASS COUNTERS
3 Drucktank PRESSURE TANK
Myon-Kammern MUON CHAMBERS
5 Jet-Kammern JET CHAMBERS
6 Flugzeit-Zähler TIME OF FLIGHT COUNTERS
7 Spule COIL
Zentrale Bleiglaszähler CENTRAL LEAD GLASS COUNTERS
9 Magnetjoch MAGNET YOKE
10 Myon-Filter MUON FILTERS
11 Beweglicher Endstopfen REMOVABLE END PLUG
12 Strahlrohr BEAM PIPE
13 Vorwärts-Detektor TAGGING COUNTER
14 Mini-Beta Quadrupol MINI BETA QUADRUPOLE
15 Fahrwerk MOVING DEVICES
~7000t
Gesamtgewicht TOTAL WEIGHT: ~1200 t
Magnetfeld MAGNETIC FIELD: 0.5 T
Beteiligte Institute PARTICIPANTS
DESY, Hamburg, Heidelberg, Lancaster, Manchester, Rutherford Lab., Tokio



Experimental Signatures:

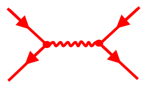


$f\,\bar{f} = e^-e^+$

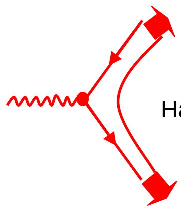


$\mu^-\mu^+$

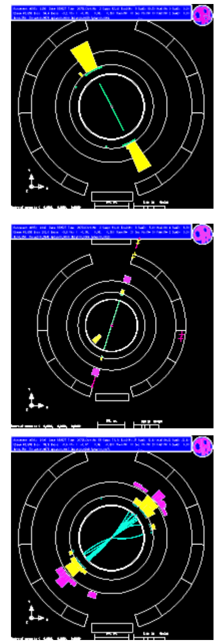
$\tau^-\tau^+$



$q\,\bar{q}$ mit
$q = u,d,s,c,b,(t)$



OPAL / LEP



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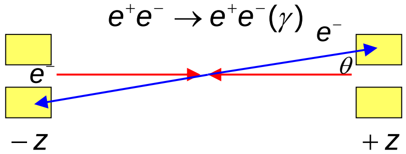
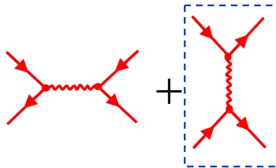
Determination of integrated luminosity

$$L_{int} = \int L_{ee}(t)dt$$

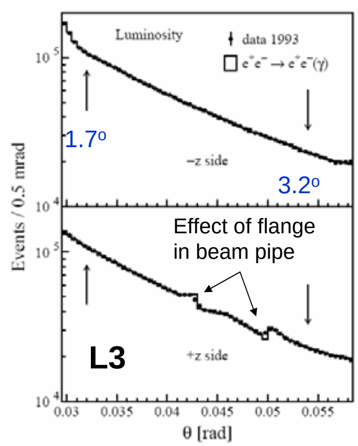
small angle Bhabha scattering
(low momentum transfer, QED works !!):

$$e^+e^- \to e^+e^-(\gamma)$$





$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon\, L_{int}}$$



Small angle Bhabha scattering is t channel dominated: theoretical cross section σ_{theo} well known.

$$L_{int} = \frac{N_{ee}}{\sigma_{theo}\varepsilon}$$

At LEP:
typ. errors < 0.5%

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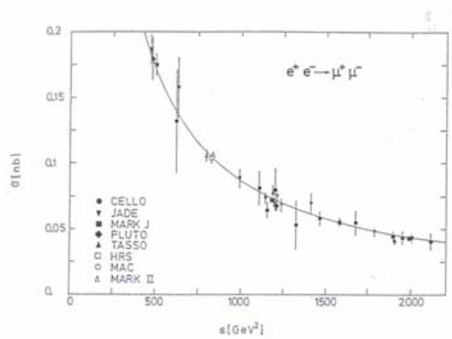
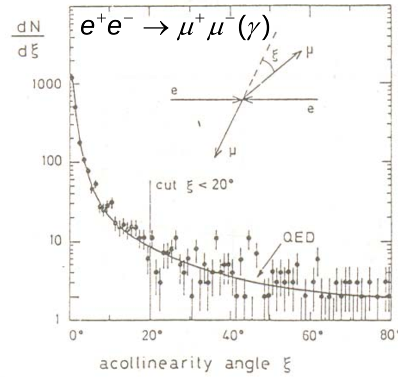
2.3 $e^+e^- \to \mu^+\mu^-$



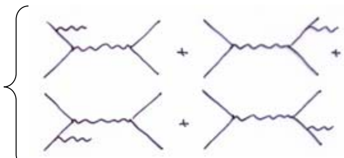


Good agreement with QED!

Quantitative limit for new physics ?

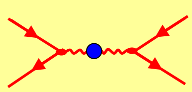
Effect of bremsstrahlung:

There will be always additional photons



<u>Possible deviation from QED:</u>

- additonal heavy photon



$$\boxed{\frac{1}{q^2} \to \frac{1}{q^2} - \frac{1}{q^2-\Lambda^2}} = \frac{1}{q^2}\left(1-\frac{q^2}{q^2-\Lambda^2}\right)$$

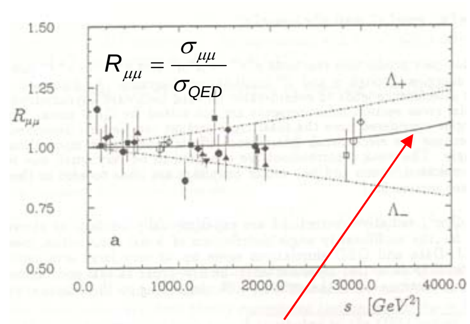
$$\approx \frac{1}{q^2}\left(1+\frac{q^2}{\Lambda^2}\right)$$

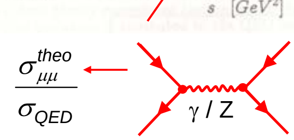
Λ corresponds to the mass of new photon

To also account for possible lower cross sections:

$$\frac{1}{q^2} \to \frac{1}{q^2}\left(1 \mp \frac{q^2}{q^2-\Lambda_\pm^2}\right)$$

Meaning of Λ_- not clear





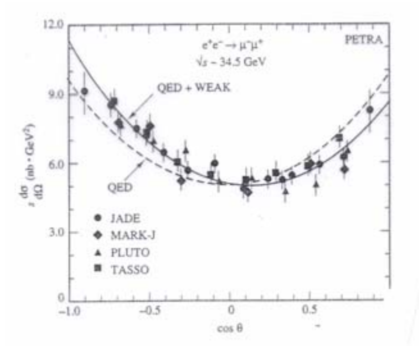
Additional heavy photon:

$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{s}\left(1 \mp \frac{s}{s-\Lambda_\pm}\right)^2$$

$\to \Lambda_\pm > 200\,\text{GeV}$

Effect of Z boson exchange in angular distribtion

„heavy photon w/ different couplings"



$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)\Big|_{QED}$$

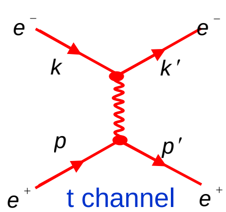
$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{4s}\cdot(1+\cos^2\theta+A\cos\theta)\Big|_{\gamma+Z}$$

Clear deviation from QED:

⇒Effect of electro-weak γ/Z interference

2.4 Bhabha scattering $e^+e^-\to e^+e^-$

M =



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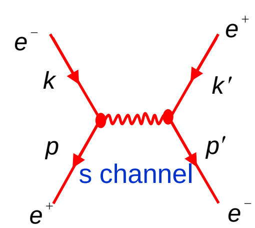
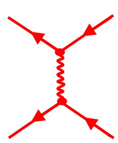
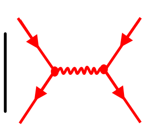
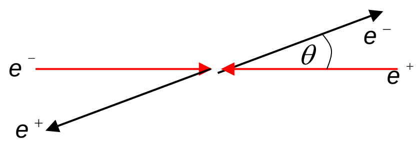


$\overline{|M|^2}$ = $\Big|$  $\Big|^2$ + interference + $\Big|$  $\Big|^2$

$e^-\mu^-\to e^-\mu^-$ $\qquad$ $e^+e^-\to\mu^+\mu^-$

$$\frac{d\sigma}{d\Omega}=\frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2}+\frac{2u^2}{ts}+\frac{t^2+u^2}{s^2}\right)$$

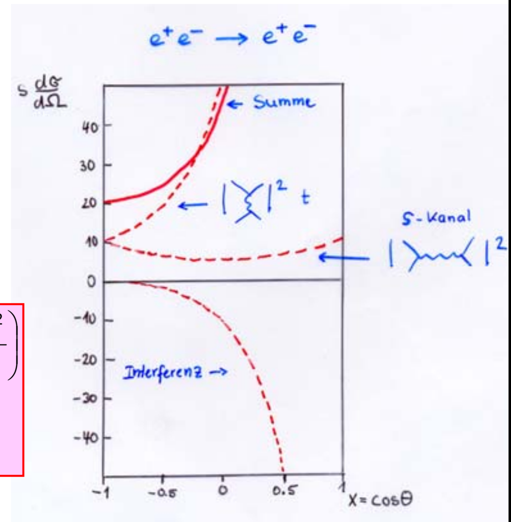
CM system:



with x=cosθ

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{4+(1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2}\right)$$

$$= \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

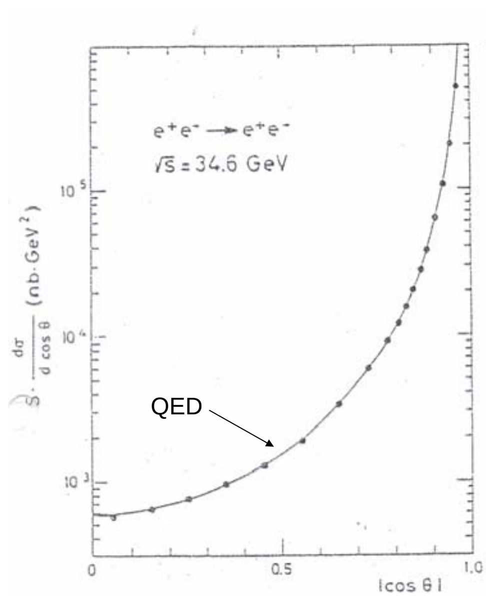
divergent for cosθ→1

Additional „heavy photon":

$$\frac{1}{q^2} \to \frac{1}{q^2}\left(1 \mp \frac{q^2}{q^2-\Lambda_\pm^2}\right)$$

⇒ Can be described by form factor:

$$F(q^2) = 1 \mp \frac{q^2}{q^2-\Lambda_\pm^2}$$



Form factor modifies differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{u^2+s^2}{t^2}|F(t)|^2 + \frac{2u^2}{ts}|F(t)F(s)| + \frac{u^2+t^2}{s^2}|F(s)|^2\right)$$

Fit to combined PETRA e^+e^- data:
Λ+ >435 GeV @ 95% CL
Λ- > 590 GeV

In the "space picture" form factor corresponds to modified Coulomb potential at small distances:

$$\frac{1}{r} \rightarrow \frac{1}{r}(1-e^{-\Lambda r})$$

i.e. Λ measures point-like nature of eγ interaction (size of electron).

Λ> ~500 GeV ⇔ r_e< 0.197/500 fm

Electr. substructure $< 0.5 \times 10^{-18}$ m

Tasso: Λ_+ >370 GeV

Λ_- > 190 GeV