Introduction to state of the art calculations for LHC

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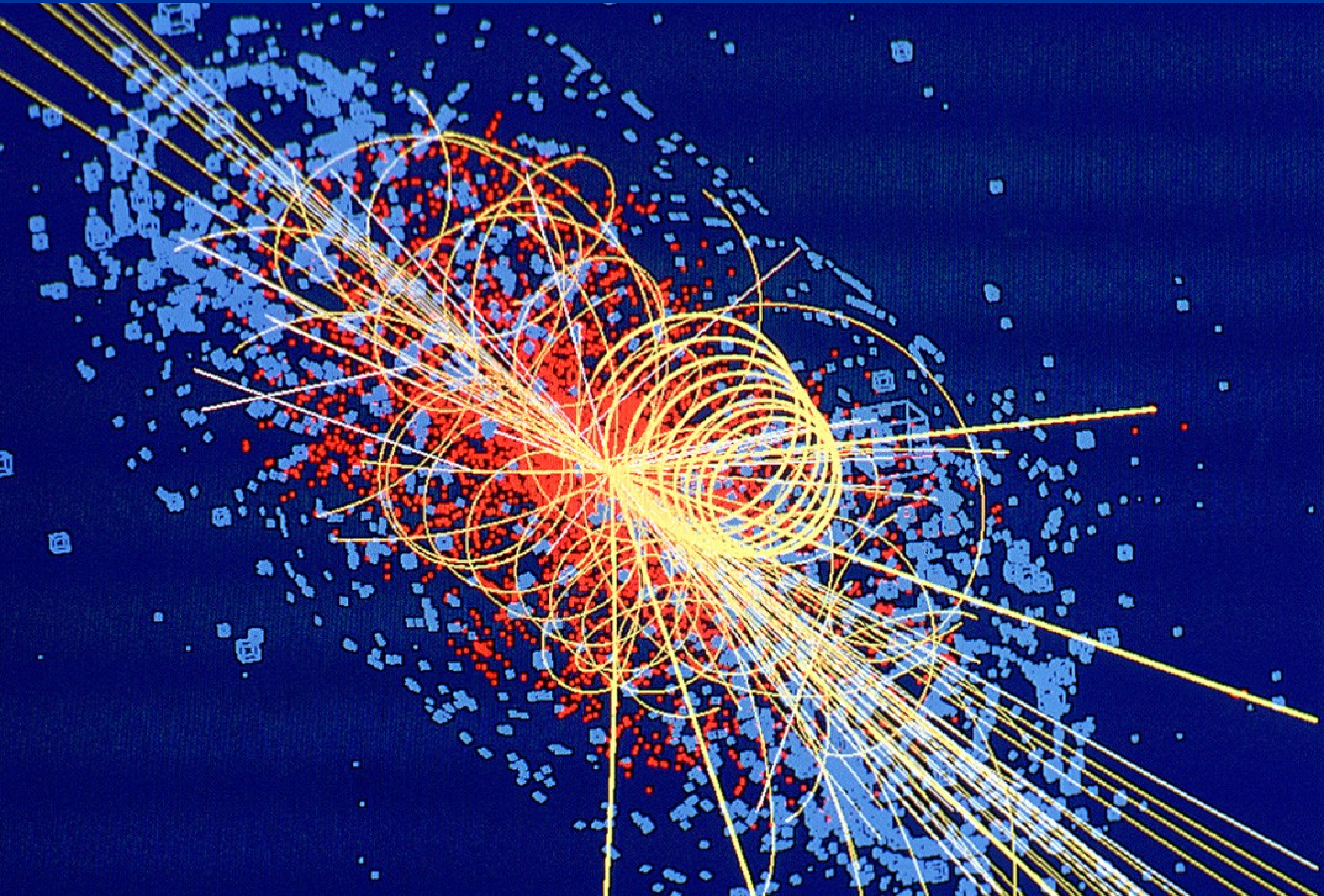
Deutsche Forschungsgemeinschaft DFG

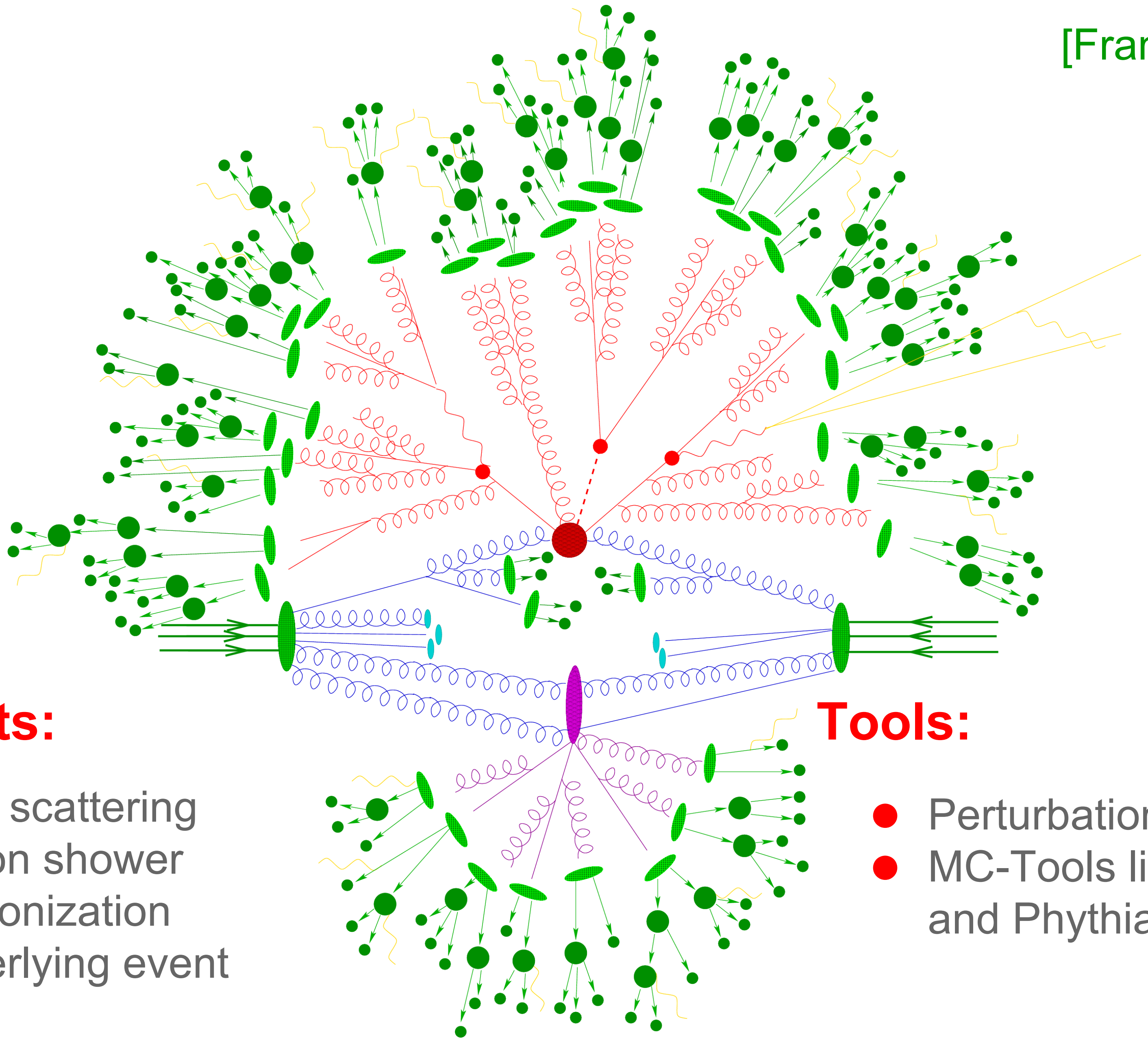
Contents

What we will see at the LHC…



… and how we understand it

[Frank Krauss]



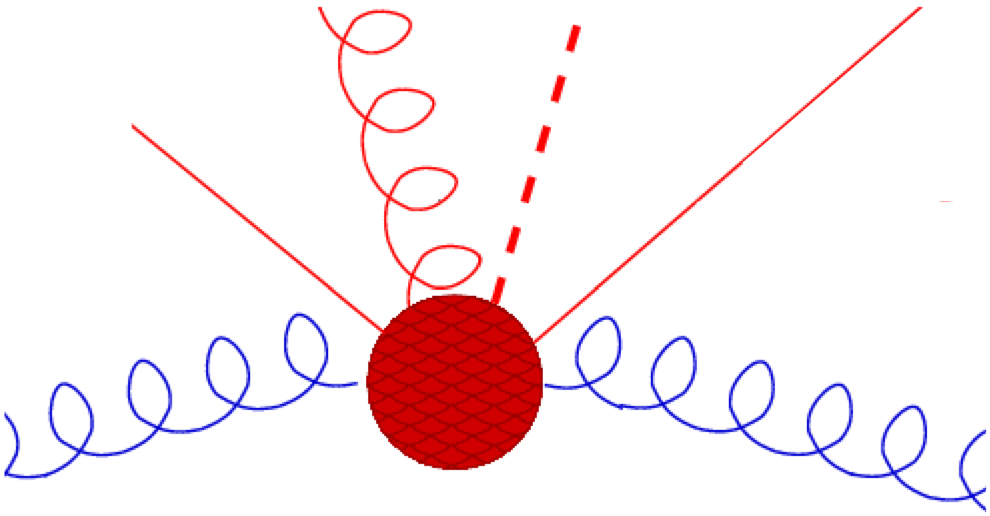
Aspects:

- Hard scattering
- Parton shower
- Hadronization
- Underlying event

Tools:

- Perturbation theory
- MC-Tools like Herwig and Phythia

The perturbative part



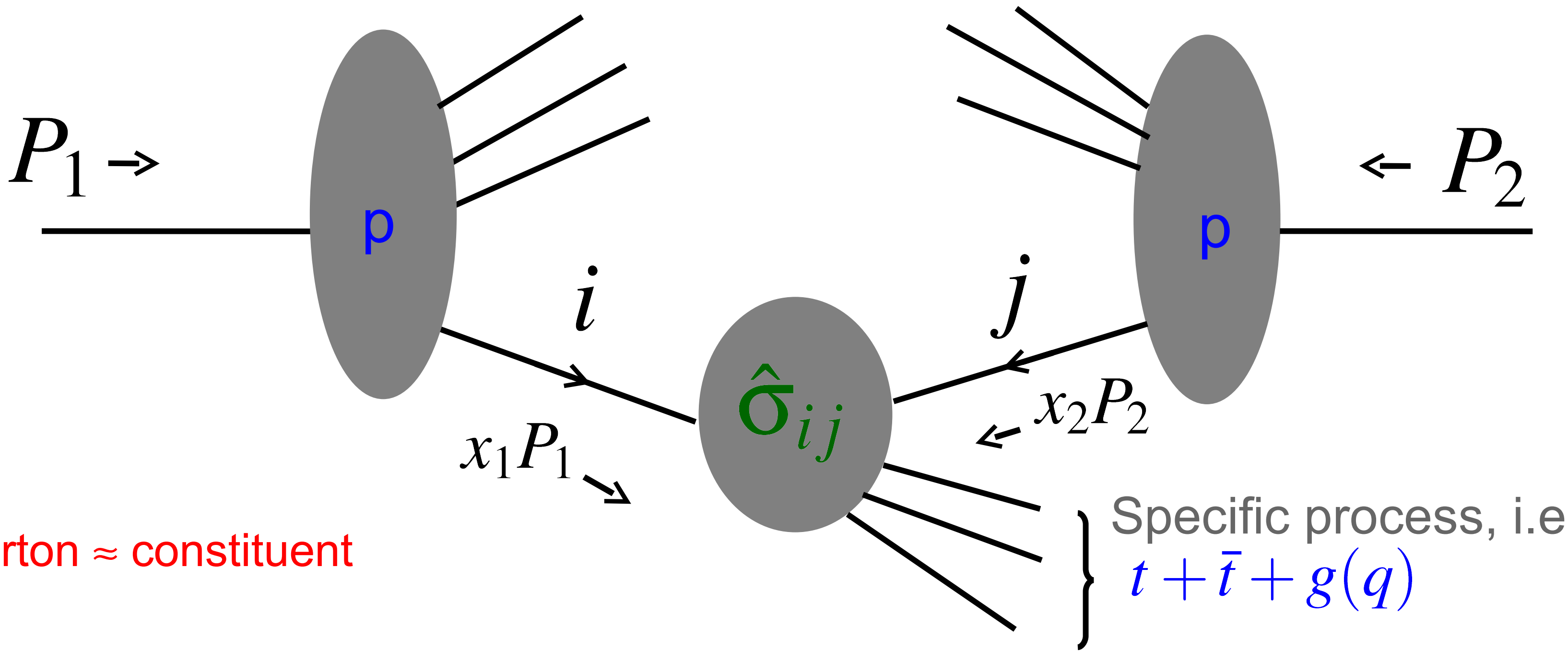
Parton-parton scattering

- The matrix elements describing the transition ij → X are calculable in perturbation theory

How do we calculate the corresponding hadronic cross sections

?

Simplified picture of the hadronic cross section



Parton ≈ constituent

$$d\sigma(p + p \to t\bar{t} + 1\text{jet} + X) =$$

$$\sum_{ij} \int dx_1 dx_2 F_i(x_1, \mu^2) F_j(x_2, \mu^2)$$

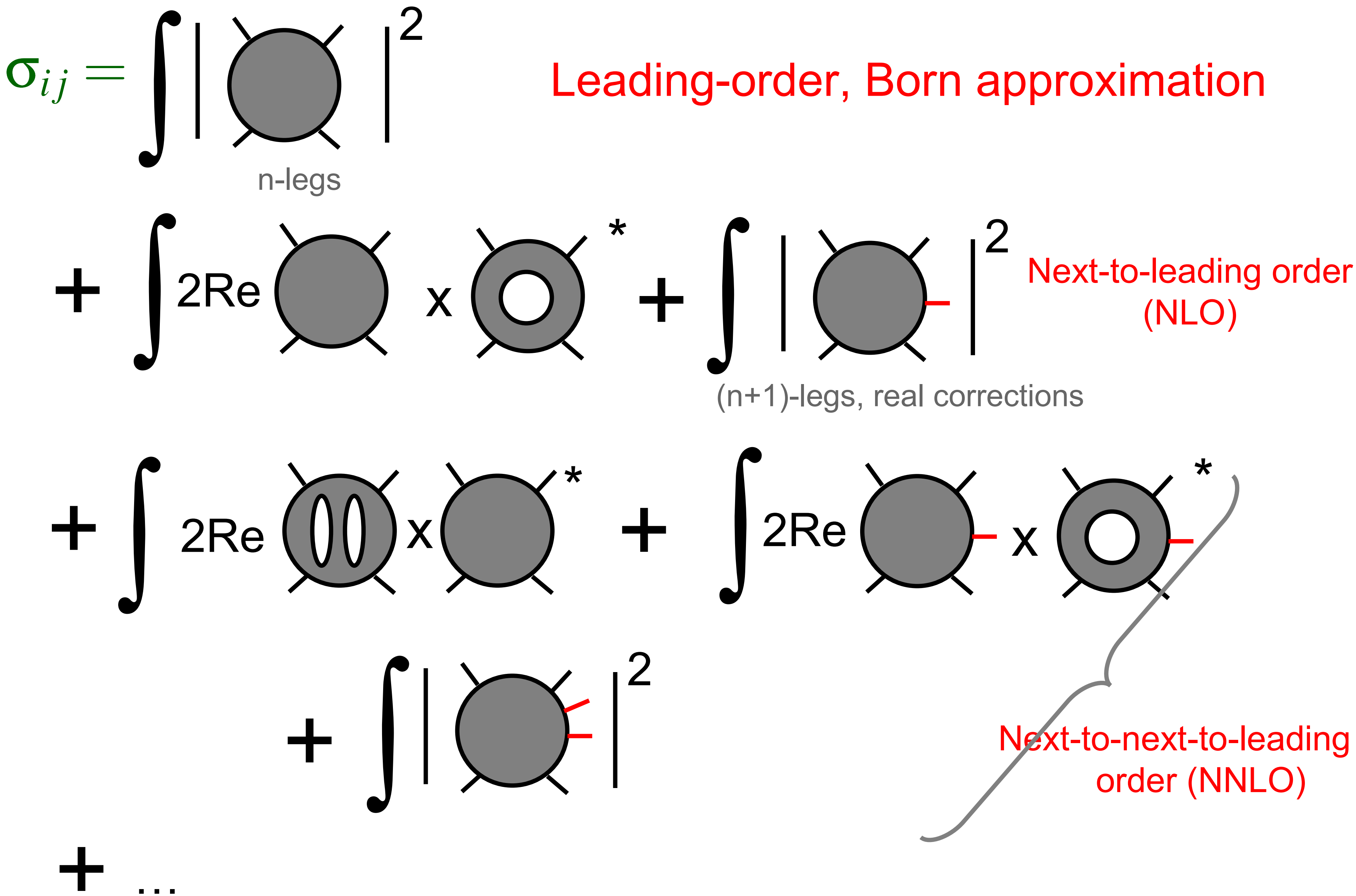
$$\times \; d\hat{\sigma}_{ij}(i(x_1 P_1) + j(x_2 P_2) \to t\bar{t}g(q))$$

Parton distribution functions (PDF)
(non-perturbativ
→ experiment, lattice)

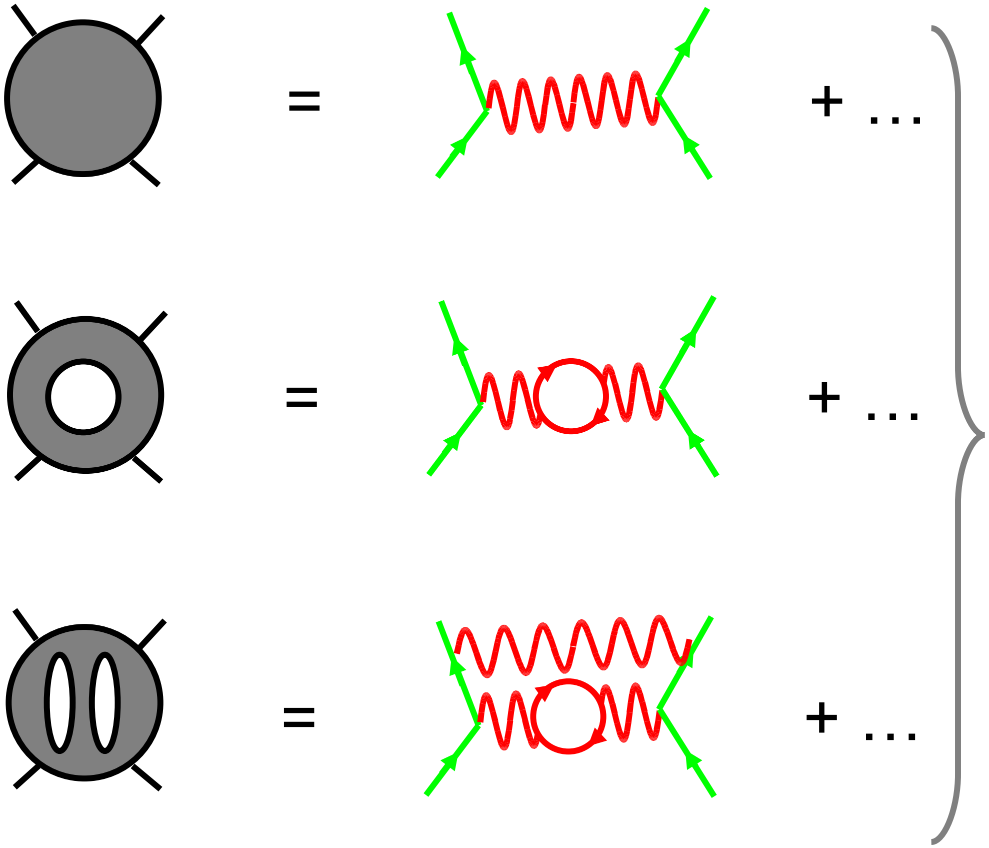
Partonic cross section

QCD improved parton model

Partonic cross section



Pictorial representation of amplitudes



Born approximation

1-loop approximation

Complex functions of the kinematics

2-loop approximation

$$\int = \frac{1}{\mathcal{N}}\frac{1}{2s}\int \delta(p_a + p_b - (p_1 + p_2 \ldots)) \prod_i \frac{d^3 p_i}{2E_i}$$

Phase space integral

Current state of the art

- Leading-order:

  2 → 8 + n processes calculable in automated way

  Drawback: matrix element evaluation and phase space evaluation might be slow

  Note: many phase space points needed for good accuracy (high dim. phase space integrals)

- Next-to-leading:

  2 → 3 processes feasible with current technology, no true 2 → 4 process @ NLO currently available for LHC

- Next-to-next-to-leading order:

  2→1 processes can be done, do we need NNLO for 2→2?

Les Houches wishlist

Physics at TeV Colliders

Les Houches, 11-29 June 2007

Les Houches 07 wishlist

NLO (items 1–7)

process ($V \in \{Z, W, \gamma\}$)	# groups working on
1. $pp \to V\,V$ jet ✓	2
2. $pp \to t\bar{t}\,b\bar{b}$	1
3. $pp \to t\bar{t} + 2$ jets	
4. $pp \to W\,W\,W$	1
5. $pp \to V\,V\,b\bar{b}$	
6. $pp \to V\,V + 2$ jets	
7. $pp \to V + 3$ jets	
8. $b\bar{b}b\bar{b}$	1
9. $gg \to W^*W^*$ (NLO, 2 loops)	?
10. EW corrections to VBF	1
11. NNLO to VBF, $t\bar{t}$, Z/γ+jet, W+jet	

Summary of activities in NLO multi-leg working group – p.

[Heinrich 07]

→ High demand for one-loop calculations for the LHC

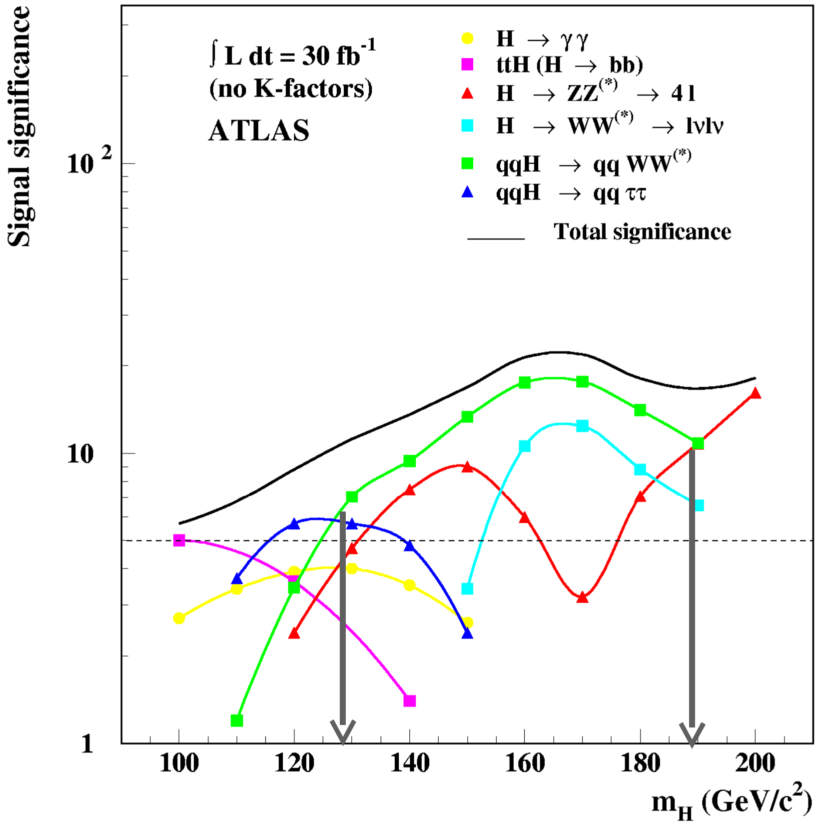
A concrete example:

$$pp \rightarrow t\,\bar{t} + 1\ \text{Jet}$$

@ NLO

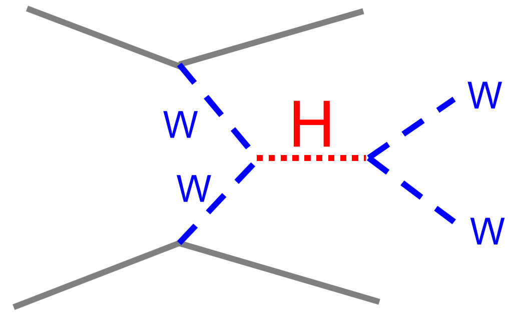
Motivation: Topquark as background for Higgs search

Higgs search at LHC



[Atlas]

"Weak Boson Fusion" (WBF)



Background processes:

channel	$e^{\pm}\mu^{\mp}$	$e^{\pm}\mu^{\mp}$ w/minijet veto	$e^{\pm}e^{\mp}, \mu^{\pm}\mu^{\mp}$	$e^{\pm}e^{\mp}, \mu^{\pm}\mu^{\mp}$ w/minijet veto
$70 < m_h < 300$ GeV	1.90	1.69	1.56	1.39
SM, $m_h = 155$ GeV	5.60	4.98	4.45	3.96
$t\bar{t}$	0.086	0.025	0.086	0.025
$t\bar{t}j$	7.59	2.20	6.45	1.87
$t\bar{t}jj$	0.83	0.24	0.72	0.21
single-top (tbj)	0.020	0.015	0.016	0.012
$b\bar{b}jj$	0.010	0.003	0.003	0.001
QCD $WWjj$	0.448	0.130	0.390	0.113
EW $WWjj$	0.269	0.202	0.239	0.179
QCD $\tau\tau jj$	0.128	0.037	0.114	0.033
EW $\tau\tau jj$	0.017	0.013	0.016	0.012
QCD $\ell\ell jj$	–	–	0.114	0.033
EW $\ell\ell jj$	–	–	0.011	0.008
total bkg	9.40	2.87	8.04	2.49
S/B	1/5.0	1/1.7	1/5.1	1/1.8
$\mathcal{L}_{5\sigma}^{\text{obs}}[\text{fb}^{-1}]$	65	25	82	32

[Alves, Eboli, Plehn, Rainwater '04]

→ Precise predictions for pp → t $\bar{t}$ + 1-Jet are important

Side remark: New physics search at the LHC

LHC-Physics = Standardmodell + X

new physics

X = LHC-Physics – Standardmodell

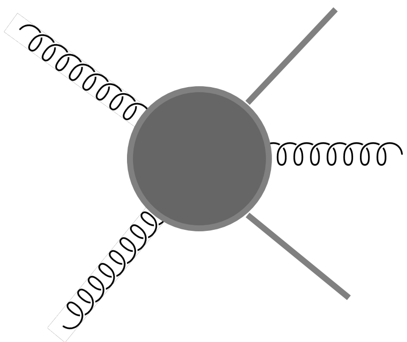
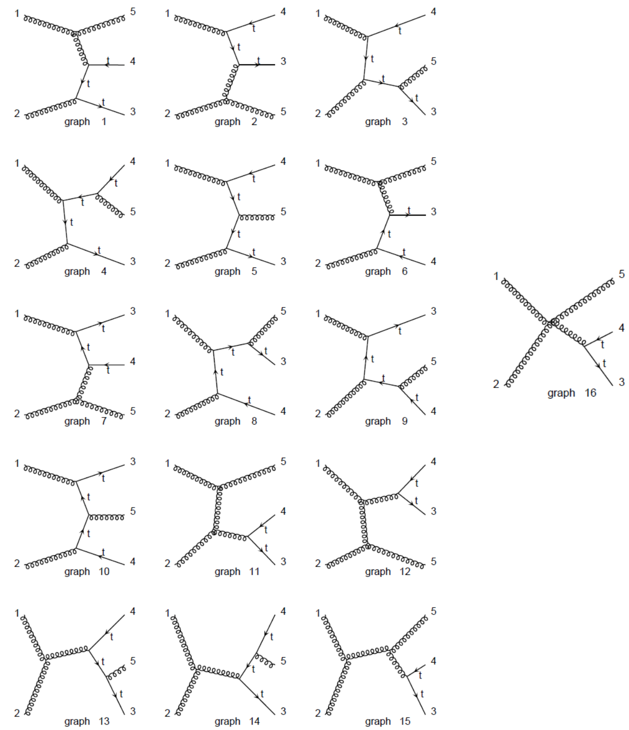
Experiment

Theory prediction

Scattering amplitudes for ij → t $\bar{t}$ + 1Jet

$$d\hat{\sigma}_{ij} \sim \delta(p_i + p_j - (k_t + k_{\bar{t}} + k_g)) \frac{d^3k_t}{2E_t} \frac{d^3k_{\bar{t}}}{2E_{\bar{t}}} \frac{d^3k_{g(q)}}{2E_{g(q)}} |\mathcal{A}_{ij\to t\bar{t}g(q)}|^2$$

$$gg \to t\bar{t}g, \quad q\bar{q} \to t\bar{t}g, \quad qg \to t\bar{t}q, \quad g\bar{q} \to t\bar{t}\bar{q}$$

$\mathcal{A}_{gg\to t\bar{t}g} =$  $=$ 

$$= \ldots = \alpha_s(\mu)^{3/2} A(\{p_i, \lambda_i\})$$

strong coupling α_s

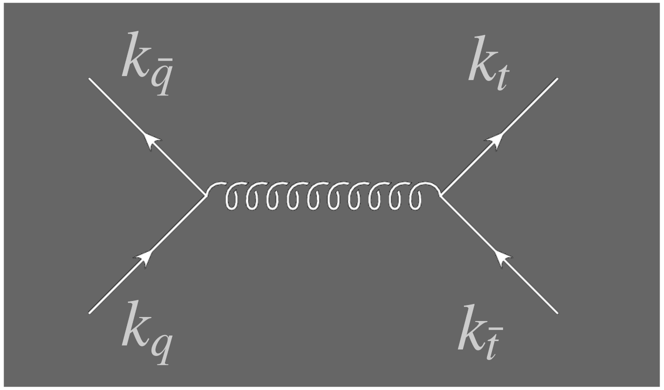
complexe function of momenta and polarisation

Methods to calculate scattering amplitudes (LO)

1. Analytically by hand on a piece of paper
2. Analytically using computer algebra
3. purely numerical

Lets take a closer look to see how it works by hand and why we don’t want to do it that way

A simple example how to do it by hand



$$
\begin{aligned}
i\mathcal{T} &= \bar{u}(k_t)\,(-i)g_s\gamma_\mu T^a\, v(k_{\bar{t}}) \\
&\quad \frac{-ig^{\mu\nu}\delta^{ab}}{(k_t+k_{\bar{t}})^2+i\varepsilon} \\
&\quad \bar{v}(k_{\bar{q}})\,(-i)g_s\gamma_\nu T^b\, u(k_q) \\
&= ig_s^2\frac{1}{s}\bar{u}(k_t)\,\gamma_\mu T^a\, v(k_{\bar{t}})\bar{v}(k_{\bar{q}})\,\gamma^\mu T^a\, u(k_q)
\end{aligned}
$$

$$
\begin{aligned}
|\mathcal{T}|^2 &= g_s^4\frac{1}{s^2}\bar{u}(k_t)\,\gamma_\mu T^a\, v(k_{\bar{t}})\bar{v}(k_{\bar{q}})\,\gamma^\mu T^a\, u(k_q) \\
&\times\; \bar{v}(k_{\bar{t}})\,\gamma_\mu T^b u(k_t)\bar{u}(k_q)\gamma^\mu T^b v(k_{\bar{q}})
\end{aligned}
$$

Color is not observerd → average over incoming color, sum over outgoing

$$
\frac{1}{N\cdot N}\sum_{t\bar{t},q,\bar{q}}(T^a)_{t\bar{t}}(T^b)_{\bar{t}t}(T^a)_{\bar{q}q}(T^b)_{q\bar{q}} = \frac{1}{N^2}Tr[T^aT^b]Tr[T^aT^b] =
$$
$$
\frac{1}{N^2}\frac{1}{2}\delta_{ab}\frac{1}{2}\delta_{ab} = \frac{1}{4N^2}\delta_{aa} = \frac{1}{4N^2}(N^2-1)
$$

A simple example how to do it by hand (cont'd)

If spin is not observed: average over incoming sum over outgoing

Use:

$$\sum_s u_\alpha(k_t,s)\bar{u}_\beta(k_t,s) = (\not{k}_t + m)_{\alpha\beta}$$

$$\sum_s v_\alpha(k_{\bar{t}},s)\bar{v}_\beta(k_{\bar{t}},s) = (\not{k}_{\bar{t}} - m)_{\alpha\beta}$$

$$\sum|\mathcal{T}|^2 \sim g_s^4 \frac{1}{s^2}(\not{k}_t + m)_{\alpha\beta}(\gamma_\nu)_{\beta\alpha'}(\not{k}_{\bar{t}} - m)_{\alpha'\beta'}(\gamma_\mu)_{\beta'\alpha}(\not{k}_{\bar{q}})_{\rho\gamma}(\gamma^\nu)_{\gamma\varepsilon}(\not{k}_q)_{\varepsilon\delta}(\gamma^\mu)_{\delta\rho}$$

$$= g_s^4 \frac{1}{s^2} Tr[(\not{k}_t + m)\gamma_\nu(\not{k}_{\bar{t}} - m)\gamma_\mu] Tr[\not{k}_{\bar{q}}\gamma^\nu \not{k}_q \gamma^\mu]$$

Calculating the traces gives:

$$\sum|\mathcal{T}|^2 \sim g_s^4 \tfrac{1}{s^2} 4(2 + (z^2 - 1)\beta^2)s^2$$

z cosine of the scattering angle, $\beta = \sqrt{1 - \frac{4m^2}{s}}$ velocity

A simple example how to do it by hand (cont'd)

Last step to obtain total cross section: phase space integral

$$\delta(k_q + k_{\bar{q}} - (k_t + k_{\bar{t}}))\frac{d^3k_t}{2E_t}\frac{d^3k_{\bar{t}}}{2E_{\bar{t}}} = \frac{1}{16\pi}\beta dz$$

The differential (partonic) cross section becomes:

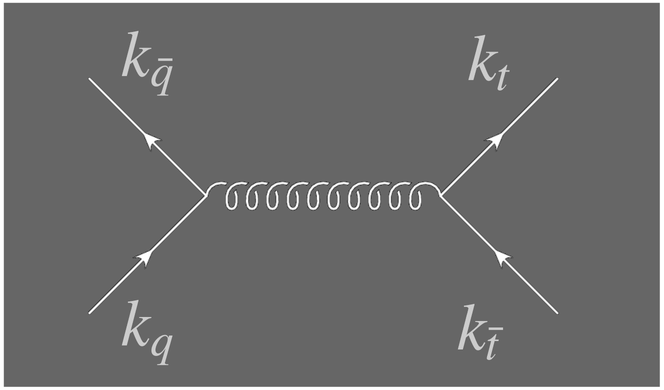
$$\begin{aligned} d\sigma_{q\bar{q}} &= \frac{1}{2s}\frac{1}{2\cdot 2}\frac{N^2-1}{4N^2}g_s^4\frac{1}{s^2}4(2+(z^2-1)\beta^2)s^2\frac{1}{16\pi}\beta dz \\ &= \frac{1}{9}\pi\alpha_s^2\beta(2+(z^2-1)\beta^2)dz \end{aligned}$$

$$\alpha_s = \frac{g_s}{4\pi},\ z = \cos(\theta),\ \beta = \sqrt{1-\frac{4m^2}{s}}$$

What are the problems when going to more complicated processes

?

A simple example how to do it by hand



$$i\mathcal{T} = \bar{u}(k_t)(-i)g_s\gamma_\mu T^a v(k_{\bar{t}}) \frac{-ig^{\mu\nu}\delta^{ab}}{(k_t+k_{\bar{t}})^2+i\varepsilon} \bar{v}(k_{\bar{q}})(-i)g_s\gamma_\nu T^b u(k_q)$$

$$= ig_s^2\frac{1}{s}\bar{u}(k_t)\gamma_\mu T^a v(k_{\bar{t}})\bar{v}(k_{\bar{q}})\gamma^\mu T^a u(k_q)$$

more diagrams, longer expressions

$$|\mathcal{T}|^2 = g_s^4\frac{1}{s^2}\bar{u}(k_t)\gamma_\mu T^a v(k_{\bar{t}})\bar{v}(k_{\bar{q}})\gamma^\mu T^a u(k_q) \times \bar{v}(k_{\bar{t}})\gamma_\mu T^b u(k_t)\bar{u}(k_q)\gamma^\mu T^b v(k_{\bar{q}})$$

many terms i.e. ~16x16 = 256

Color is not observerd → average over incoming color, sum over outgoing

$$\frac{1}{N\cdot N}\sum_{t\bar{t},q,\bar{q}}(T^a)_{t\bar{t}}(T^b)_{\bar{t}t}(T^a)_{\bar{q}q}(T^b)_{q\bar{q}} = \frac{1}{N^2}Tr[T^aT^b]Tr[T^aT^b] = \frac{1}{N^2}\frac{1}{2}\delta_{ab}\frac{1}{2}\delta_{ab} = \frac{1}{4N^2}\delta_{aa} = \frac{1}{4N^2}(N^2-1)$$

more structures

A simple example how to do it by hand (cont'd)

If spin is not observed: average over incoming sum over outgoing

Use:
$$\sum_s u_\alpha(k_t,s)\bar{u}_\beta(k_t,s) = (\not{k}_t + m)_{\alpha\beta}$$
$$\sum_s v_\alpha(k_{\bar{t}},s)\bar{v}_\beta(k_{\bar{t}},s) = (\not{k}_{\bar{t}} - m)_{\alpha\beta}$$

$$\sum |\mathcal{T}|^2 \sim g_s^4 \frac{1}{s^2} (\not{k}_t + m)_{\alpha\beta} (\gamma_\nu)_{\beta\alpha'} (\not{k}_{\bar{t}} - m)_{\alpha'\beta'} (\gamma_\mu)_{\beta'\alpha} (\not{k}_{\bar{q}})_{\rho\gamma} (\gamma^\nu)_{\gamma\varepsilon} (\not{k}_q)_{\varepsilon\delta} (\gamma^\mu)_{\delta\rho}$$
$$= g_s^4 \frac{1}{s^2} Tr[(\not{k}_t + m)\gamma_\nu(\not{k}_{\bar{t}} - m)\gamma_\mu] Tr[\not{k}_{\bar{q}}\gamma^\nu \not{k}_q \gamma^\mu]$$

more complicated traces → more γ matrices

Calculating the trace gives:

$$\sum |\mathcal{T}|^2 \sim g_s^4 \frac{1}{s^2} 4(2 + (z^2 - 1)\beta^2) s^2$$

function of many variables

z cosine of the scattering angle, $\beta = \sqrt{1 - \frac{4m^2}{s}}$ velocity

A simple example how to do it by hand (cont'd)

Last step to obtain cross section: phase space integral

$$\delta(k_q + k_{\bar{q}} - (k_t + k_{\bar{t}})) \frac{d^3 k_t}{2E_t} \frac{d^3 k_{\bar{t}}}{2E_{\bar{t}}} = \frac{1}{16\pi} \beta dz$$

more particles → phase space more complicated

The differential (partonic) cross section becomes:

$$d\sigma_{q\bar{q}} = \frac{1}{2s} \frac{1}{2 \cdot 2} \frac{N^2 - 1}{4N^2} g_s^4 \frac{1}{s^2} 4(2 + (z^2 - 1)\beta^2) s^2 \frac{1}{16\pi} \beta dz$$

$$= \frac{1}{9} \pi \alpha_s^2 \beta (2 + (z^2 - 1)\beta^2) dz$$

$$\alpha_s = \frac{g_s}{4\pi}, \; z = \cos(\theta), \; \beta = \sqrt{1 - \frac{4m^2}{s}}$$

One solution: Use computer algebra

Generate diagrams

(→Topologies)

[QGRAF, Feynarts]

↓ Feynman rules

Algebraic expressions
(→ Maple, Mathematica, Form)

↓ explicit representation of the spinors and ε's

Analytic expressions for amplitudes for specific helicty configurations

↓ evaluate $\bar{u}(k)u(k)$ numerically

Evaluate amplitude numerically
as complex number using C/C++ or Fortran,
calculate the square numerically

Another approach: Completely numerical approach

Two common approaches for amplitude calculations:

1. Feynman diagram based i.e. Madgraph,… [Long, Stelzer '94]
2. Use recurrence relation i.e. Alpgen,… [Mangano et al]

In 1. for every diagram a code is generated to evaluate it numerically

In 2. amplitudes are calculated from simpler objects via recurrence relation

→ some progress recently from string inspired methods

Want to use it as a black box → don't care what is inside!

We care about speed and numerical accuracy!

Example Madgraph

uwer on ttpodo: /users/ttp/uwer

```
uwer@ttpodo:~>madgraph


  "$o    o$"              "$     o""""$                                  "$
   $"o  o"$     ooo     ooo $    $       "    oo ooo     ooo    oo ooo    $ ooo
   $  "o" $    "    $  $    "$   $           $"   "     "    $  $"    $   $"    $
   $   "  $   o""""$   $     $   $    ""$"   $          o""""$  $     $   $     $
  o$o    o$o  "ooo"$o  "ooo"$o    "oooo"    o$oo        "ooo"$o  $"ooo"  o$o   o$o
                                                                 $
                                                                """

     o                        o               oooooooooooooooooooooooooo
      ""o                   o""                            $
         ""oo           oo""            o                  $
            o"""""""""""o             oo$oo                $
         o""              ""o           $                  $
      oo"                    "oo                           $
     "                         "              """"""""""""""""""""""""""


 Standard Model particles include:
     Quarks:   d u s c b t d~ u~ s~ c~ b~ t~
     Leptons:  e- mu- ta- e+ mu+ ta+ ve vm vt ve~ vm~ vt~
     Bosons:   g a z w+ w- h

 Enter process you would like calculated in the form  e+ e- -> a.
 (<return> to exit MadGraph.)
g g -> t t~ g

Attempting Process: g g  -> t t~ g

Enter the number of QCD vertices between 3 and 3 (3):

The number of QFD vertices is 0
No QFD possible all QCD ok?:

Enter a name to identify process (gg_ttbg):


 Generating diagrams for              5  external legs
 There are           18  graphs.
 Writing Feynman graphs in file gg_ttbg.ps
 Reduced color matrix             15           18
 Writing function GG_TTBG in file gg_ttbg.f.
```

Example Madgraph – Output

uwer on ttpodo: /users/ttp/uwer

```
      REAL*8 FUNCTION SGG_TTBG(P1, P2, P3, P4, P5)
C
C FUNCTION GENERATED BY MADGRAPH
C RETURNS AMPLITUDE SQUARED SUMMED/AVG OVER COLORS
C AND HELICITIES
C FOR THE POINT IN PHASE SPACE P1,P2,P3,P4,...
C
C FOR PROCESS : g g  -> t t~ g
C
      IMPLICIT NONE
C
C CONSTANTS
C
      INTEGER    NEXTERNAL,   NCOMB
      PARAMETER (NEXTERNAL=5, NCOMB= 32)
C
C ARGUMENTS
C
      REAL*8 P1(0:3),P2(0:3),P3(0:3),P4(0:3),P5(0:3)

C
C LOCAL VARIABLES
C
      INTEGER NHEL(NEXTERNAL,NCOMB),NTRY

      REAL*8 T
      REAL*8 GG_TTBG

      INTEGER IHEL
      LOGICAL GOODHEL(NCOMB)
      DATA GOODHEL/NCOMB*.FALSE./
      DATA NTRY/0/
      DATA (NHEL(IHEL,  1),IHEL=1,5) / -1, -1, -1, -1, -1/
      DATA (NHEL(IHEL,  2),IHEL=1,5) / -1, -1, -1, -1,  1/
      DATA (NHEL(IHEL,  3),IHEL=1,5) / -1, -1, -1,  1, -1/
      DATA (NHEL(IHEL,  4),IHEL=1,5) / -1, -1, -1,  1,  1/
      DATA (NHEL(IHEL,  5),IHEL=1,5) / -1, -1,  1, -1, -1/
      DATA (NHEL(IHEL,  6),IHEL=1,5) / -1, -1,  1, -1,  1/
      DATA (NHEL(IHEL,  7),IHEL=1,5) / -1, -1,  1,  1, -1/
      DATA (NHEL(IHEL,  8),IHEL=1,5) / -1, -1,  1,  1,  1/
      DATA (NHEL(IHEL,  9),IHEL=1,5) / -1,  1, -1, -1, -1/
```

Example Madgraph – Output

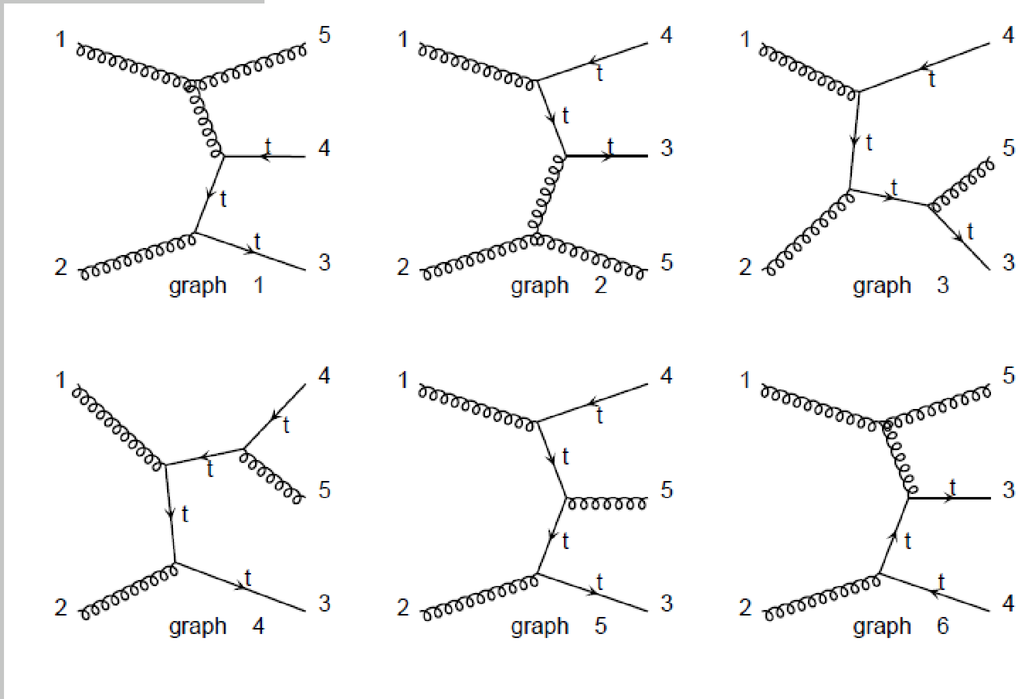
```
      PARAMETER (ZERO=0D0)
C
C ARGUMENTS
C
      REAL*8 P1(0:3),P2(0:3),P3(0:3),P4(0:3),P5(0:3)

      INTEGER NHEL(NEXTERNAL)

C
C LOCAL VARIABLES
C
      INTEGER I,J
      REAL*8 EIGEN_VAL(NEIGEN), EIGEN_VEC(NGRAPHS,NEIGEN)
      COMPLEX*16 ZTEMP
      COMPLEX*16 AMP(NGRAPHS)
      COMPLEX*16 W1(6)  , W2(6)  , W3(6)  , W4(6)  , W5(6)
      COMPLEX*16 W6(6)  , W7(6)  , W8(6)  , W9(6)  , W10(6)
      COMPLEX*16 W11(6) , W12(6) , W13(6) , W14(6) , W15(6)
      COMPLEX*16 W16(6) , W17(6) , W18(6) , W19(6) , W20(6)
      COMPLEX*16 W21(6) , W22(6) , W23(6)
C
C GLOBAL VARIABLES
C
      REAL*8            GG(2), G
      COMMON /COUPQCD/ GG,    G
      REAL*8             FMASS(12), FWIDTH(12)
      COMMON /FERMIONS/ FMASS,      FWIDTH
C
C COLOR DATA
C
      DATA EIGEN_VAL(1  )/       1.0416666666666660D-01 /
      DATA EIGEN_VEC(1  ,1  )/   0.0000000000000000D+00 /
      DATA EIGEN_VEC(2  ,1  )/   0.0000000000000000D+00 /
      DATA EIGEN_VEC(3  ,1  )/  -4.0824829046386341D-01 /
      DATA EIGEN_VEC(4  ,1  )/  -4.0824829046386268D-01 /
      DATA EIGEN_VEC(5  ,1  )/  -4.0824829046386313D-01 /
      DATA EIGEN_VEC(6  ,1  )/   0.0000000000000000D+00 /
```

Input: QCD coupling
+ masses and widths

Example Madgraph – Output

uwer on ttpodo: /users/ttp/uwer

```
      DATA EIGEN_VEC(14 ,6 )/ -1.1941938778965616D-01 /
      DATA EIGEN_VEC(15 ,6 )/ -2.3883877557931230D-01 /
      DATA EIGEN_VEC(16 ,6 )/  2.3883877557931230D-01 /
      DATA EIGEN_VEC(17 ,6 )/  2.6099752752803979D-01 /
      DATA EIGEN_VEC(18 ,6 )/ -4.9983630310735233D-01 /
C ----------
C BEGIN CODE
C ----------
      CALL VXXXXX(P1  , ZERO,NHEL(1  ),-1,W1  )
      CALL VXXXXX(P2  , ZERO,NHEL(2  ),-1,W2  )
      CALL OXXXXX(P3  ,FMASS(11 ),NHEL(3  ), 1,W3  )
      CALL IXXXXX(P4  ,FMASS(11 ),NHEL(4  ),-1,W4  )
      CALL VXXXXX(P5  , ZERO,NHEL(5  ), 1,W5  )
      CALL FVOXXX(W3  ,W2  ,GG,FMASS(11 ),FWIDTH(11 ),W6  )
      CALL JIOXXX(W4  ,W6  ,GG,ZERO,ZERO,W7  )
      CALL GGGXXX(W5  ,W1  ,W7  ,G,AMP(1  ))
      CALL FVIXXX(W4  ,W1  ,GG,FMASS(11 ),FWIDTH(11 ),W8  )
      CALL JGGXXX(W5  ,W2  ,G,W9  )
      CALL IOVXXX(W8  ,W3  ,W9  ,GG,AMP(2  ))
      CALL FVOXXX(W3  ,W5  ,GG,FMASS(11 ),FWIDTH(11 ),W10 )
      CALL IOVXXX(W8  ,W10 ,W2  ,GG,AMP(3  ))
      CALL FVOXXX(W6  ,W1  ,GG,FMASS(11 ),FWIDTH(11 ),W11 )
      CALL IOVXXX(W4  ,W11 ,W5  ,GG,AMP(4  ))
      CALL IOVXXX(W8  ,W6  ,W5  ,GG,AMP(5  ))
      CALL FVIXXX(W4  ,W2  ,GG,FMASS(11 ),FWIDTH(11 ),W12 )
      CALL JGGXXX(W5  ,W1  ,G,W13 )
      CALL IOVXXX(W12 ,W3  ,W13 ,GG,AMP(6  ))
      CALL FVOXXX(W3  ,W1  ,GG,FMASS(11 ),FWIDTH(11 ),W14 )
      CALL JIOXXX(W4  ,W14 ,GG,ZERO,ZERO,W15 )
      CALL GGGXXX(W5  ,W2  ,W15 ,G,AMP(7  ))
      CALL IOVXXX(W12 ,W10 ,W1  ,GG,AMP(8  ))
      CALL FVOXXX(W14 ,W2  ,GG,FMASS(11 ),FWIDTH(11 ),W16 )
      CALL IOVXXX(W4  ,W16 ,W5  ,GG,AMP(9  ))
      CALL IOVXXX(W12 ,W14 ,W5  ,GG,AMP(10 ))
      CALL JIOXXX(W4  ,W3  ,GG,ZERO,ZERO,W17 )
      CALL GGGXXX(W13 ,W2  ,W17 ,G,AMP(11 ))
      CALL GGGXXX(W1  ,W9  ,W17 ,G,AMP(12 ))
      CALL JGGXXX(W1  ,W2  ,G,W18 )
      CALL FVIXXX(W4  ,W18 ,GG,FMASS(11 ),FWIDTH(11 ),W19 )
      CALL IOVXXX(W19 ,W3  ,W5  ,GG,AMP(13 ))
      CALL FVOXXX(W3  ,W18 ,GG,FMASS(11 ),FWIDTH(11 ),W20 )
      CALL IOVXXX(W4  ,W20 ,W5  ,GG,AMP(14 ))
      CALL GGGXXX(W5  ,W17 ,W18 ,G,AMP(15 ))
      CALL JGGGXX(W1  ,W2  ,W5  ,G,W21 )
      CALL IOVXXX(W4  ,W3  ,W21 ,GG,AMP(16 ))
      CALL JGGGXX(W5  ,W1  ,W2  ,G,W22 )
      CALL IOVXXX(W4  ,W3  ,W22 ,GG,AMP(17 ))
      CALL JGGGXX(W2  ,W5  ,W1  ,G,W23 )
      CALL IOVXXX(W4  ,W3  ,W23 ,GG,AMP(18 ))
      GG_TTBG = 0.D0
      DO I = 1, NEIGEN
          ZTEMP = (0.D0,0.D0)
          DO J = 1, NGRAPHS
              ZTEMP = ZTEMP + EIGEN_VEC(J,I)*AMP(J)
          ENDDO
          GG_TTBG =GG_TTBG+ZTEMP*EIGEN_VAL(I)*CONJG(ZTEMP)
      ENDDO
C      CALL GAUGECHECK(AMP,ZTEMP,EIGEN_VEC,EIGEN_VAL,NGRAPHS,NEIGEN)
```



Postscript figure also produced by Madgraph

What about phase space integration ?

- High dimensional for multiparton processes (i.e. 5 for 2→3)
- Want to include arbitrary cuts / observables

→ Do integration numerically using Monte Carlo techniques

Basic idea:

$$\int d^n x f(\vec{x}) = \int d^n x \frac{f(\vec{x})}{\rho(\vec{x})} \rho(\vec{x}) = \langle \frac{f(\vec{x})}{\rho(\vec{x})} \rangle_\rho \approx \frac{1}{N} \sum_i \frac{f(\vec{x}_i)}{\rho(\vec{x}_i)}$$

ρ with $\int d^n x \rho(\vec{x}) = 1$ can be tuned to the integrand

→ Computer Code (F77) i.e. Vegas by Lepage

call vegas(ndim, fxn, avg, sd, chi2)

integrates fxn over $[0,1]^{ndim}$

Missing piece: mapping $[0,1]^n \rightarrow$ dLIPS

dLIPS = lorentz invariant phase space measure

$$\text{dLIPS} \sim \delta(p_i + p_j - (\sum_i k_i)) \prod_i \frac{d^3 k_i}{2E_i}$$

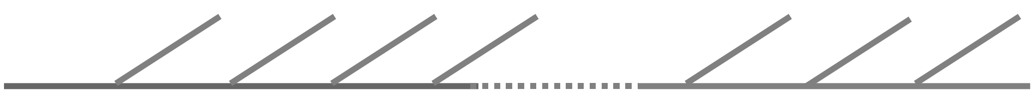
- Flat mapping:

RAMBO by Ellis, Kleiss, Stirling

SUBROUTINE RAMBO(N,ET,XM,P,WT)

disadvantage: flat and $[0,1]^{4n} \rightarrow$ dLIPS

- Sequential splitting



$[0,1]^{3n-4} \rightarrow$ dLIPS [Book: Byckling,Kajantie p. 273]

- Multi channel algorithms

Adopt MC to structure of the integrand by using different mappings in parallel

Last missing piece: Parton distribution functions

Remember:

$$d\sigma(p+p \to t\bar{t} + 1\text{jet} + X) =$$

$$\sum_{ij}\int dx_1 dx_2 F_i(x_1,\mu^2) F_j(x_2,\mu^2)$$

$$\times\ d\hat{\sigma}_{ij}(i(x_1 P_1) + j(x_2 P_2) \to t\bar{t}g(q))$$

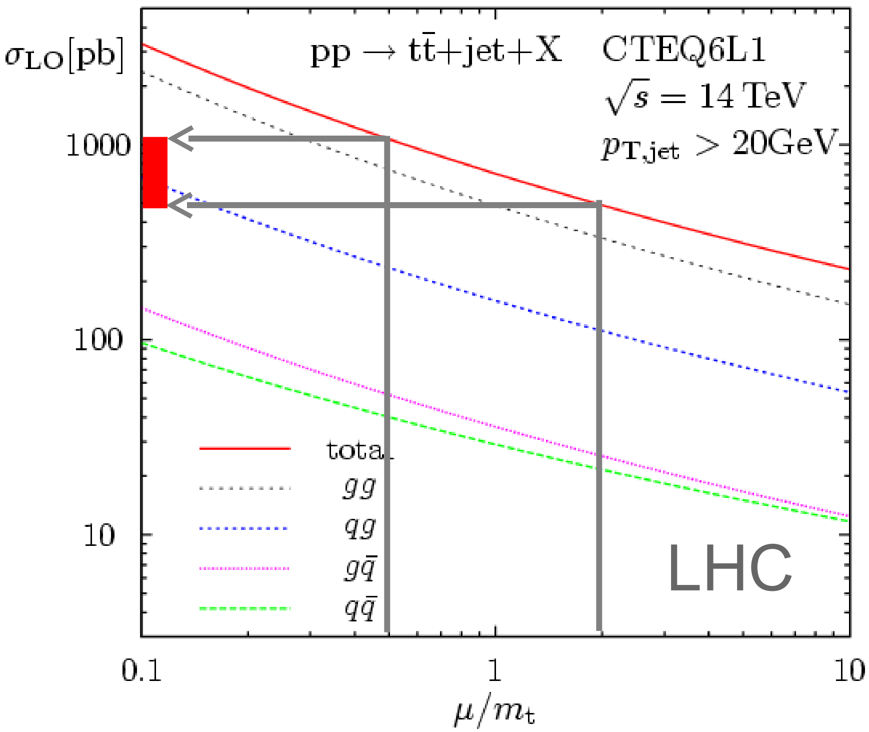
→ 2 additional integration over x_1, x_2, no problem in MC approach

How to evaluate the PDF's ?

→ use LHAPDF, MRST/MSTW or CTEQ code

CTEQ: Subroutine SetCtq6 (Iset) / Function Ctq6Pdf (Iparton, X, Q) } Cteq6Pdf-2007.f

Topquark pair production + 1 Jet (Born)



Large scale dependence (~100%)

→ we need NLO

$$\mu \frac{d\alpha_s(\mu)}{d\mu} = \alpha_s(\mu)\,\beta(\alpha_s(\mu)) \quad \text{but} \quad \frac{d\sigma}{d\mu} = 0$$

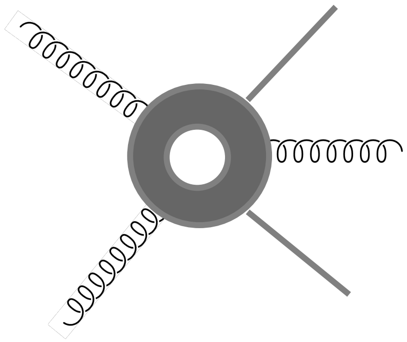
Perturbation theory:

$$\sigma = \alpha_s(\mu)^3 a_0 + \alpha_s(\mu)^4 \left(a_1^0 + a_1^1 \ln\left(\frac{\mu}{m}\right)\right) + \cdots$$

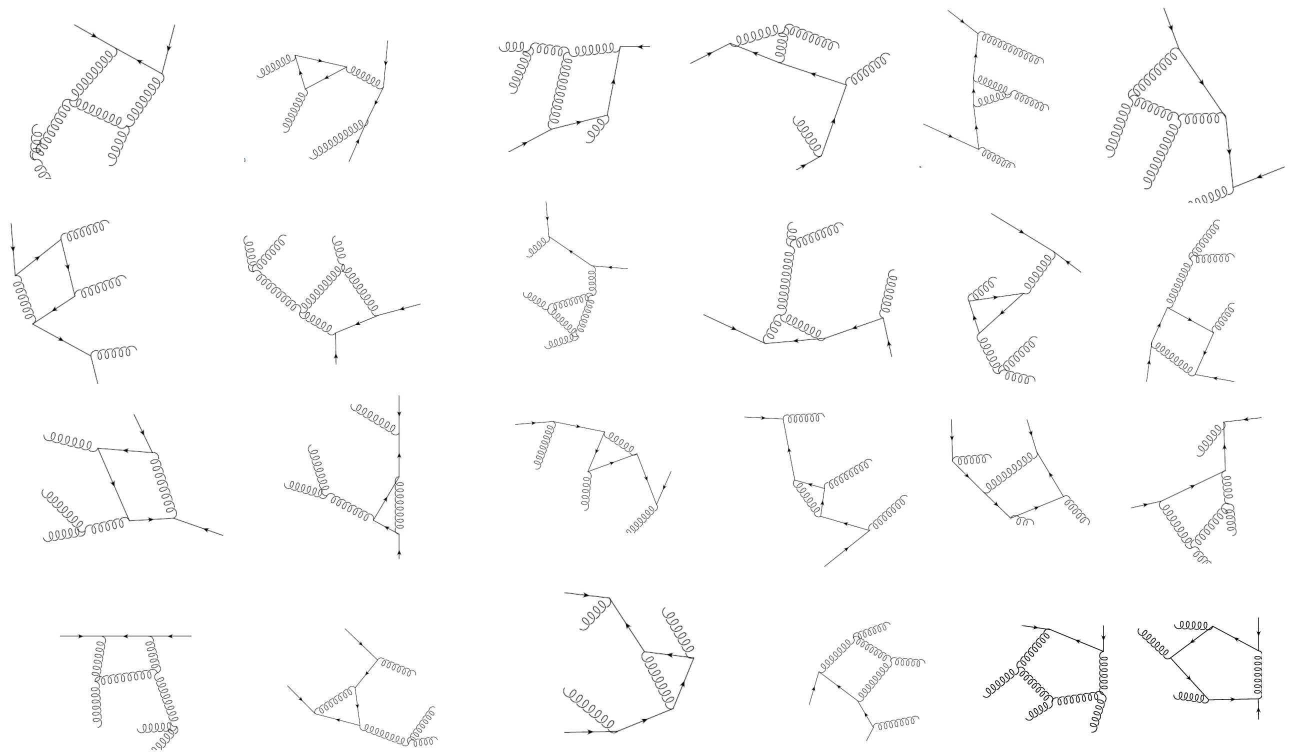
Born

one-loop corrections

One-loop diagrams



=



⋮

→ ~350 diagrams

Computer-Algebra + numerical methods

Diagram generation with QGRAF

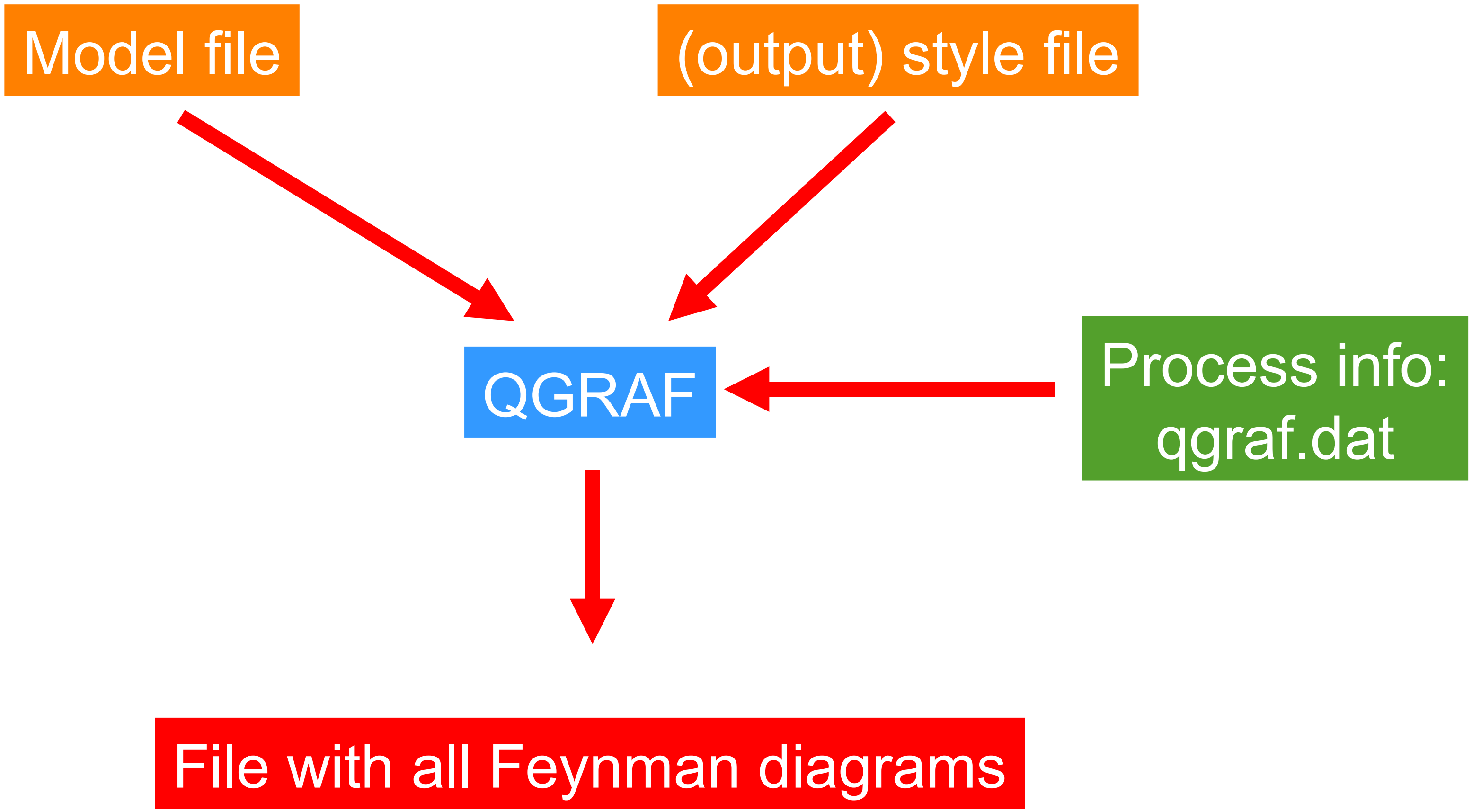


Diagram generation with QGRAF: Input

uwer on ttpodo: /...

```
uwer@ttpodo:include>more qcd-1.1
* propagators

[q,qb,-]
[t,tb,-]
[g,g,+]

* vertices

[qb,q,g]
[tb,t,g]
[g,g,g]
[g,g,g,g]

uwer@ttpodo:include>
```

Model file

uwer on ttpodo: /users/ttp/uwer/projekt...

```
uwer@ttpodo:include>more myform-1.0.sty
<prologue>
*
* file generated by <program>
*
* style-file: myform-1.0.sty
*
<prologue_loop>* <data>
<end>*
<diagram>
  <back>
l a<diagram_index>:=
(<sign><symmetry_factor>)*
<leg_loop> e(<leg_momentum>,i{50+(<field_index>)})*
<end><propagator_loop> <field>prop(i{50+(<field_index>)},
<back>i{50+(<dual_field_index>)},<momentum>)*
<end><vertex_loop> vrtx(<sub_loop>i{50+(<field_index>)},
<back><end><back>)*
<end><back><back>;
<epilogue>

* end

<exit>

uwer@ttpodo:include>
```

style file

Diagram generation with QGRAF: Output LO

uwer on ttpodo: /users/ttp/uwer/proj...

```
uwer@ttpodo:auto>more ggttbg.qgraf
*
* file generated by qgraf 2.0
*
* style-file: myform-1.0.sty
*
* output = './include/auto/ggttbg.qgraf' ;
* style  = './include/myform-1.0.sty' ;
* model = './include/qcd-1.1'   ;
* in =  g[p1], g[p2] ;
* out = g[p3], t[kq], tb[kqb];
* loops = 0 ;
* loop_momentum = l ;
* options = notadp,onshell;
*

l a1:=
(+1)*
 e(p1,i{50+(-1)})*
 e(p2,i{50+(-3)})*
 e(p3,i{50+(-2)})*
 e(kq,i{50+(-4)})*
 e(kqb,i{50+(-6)})*
 gprop(i{50+(1)},i{50+(2)},kq+kqb)*
 vrtx(i{50+(-4)},i{50+(-6)},i{50+(1)})*
 vrtx(i{50+(-1)},i{50+(-3)},i{50+(-2)},i{50+(2)});

l a2:=
(+1)*
 e(p1,i{50+(-1)})*
 e(p2,i{50+(-3)})*
 e(p3,i{50+(-2)})*
 e(kq,i{50+(-4)})*
```

Repetition of input

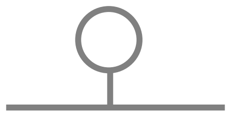
Output

Diagram generation with QGRAF: Output NLO

uwer on ttpodo: /users/ttp/uwer/projekt...

```
uwer@ttpodo:auto>more diags.qgraf
*
* file generated by qgraf 2.0
*
* style-file: myform-1.0.sty
*
* output = './include/auto/diags.qgraf' ;
* style  = './include/myform-1.0.sty' ;
* model = './include/qcd-1.1'   ;
* in =  g[p1], g[p2] ;
* out = g[p3], t[kq], tb[kqb];
* loops = 1 ;
* loop_momentum = l ;
* options = notadp,onshell;
*

l a1:=
(+1/2)*
 e(p1,i{50+(-1)})*
 e(p2,i{50+(-3)})*
 e(p3,i{50+(-2)})*
 e(kq,i{50+(-4)})*
 e(kqb,i{50+(-6)})*
 gprop(i{50+(1)},i{50+(2)},kq+kqb)*
 gprop(i{50+(3)},i{50+(4)},-kq-kqb)*
 gprop(i{50+(5)},i{50+(6)},l1)*
 vrtx(i{50+(-4)},i{50+(-6)},i{50+(1)})*
 vrtx(i{50+(-1)},i{50+(-3)},i{50+(-2)},i{50+(3)})*
 vrtx(i{50+(4)},i{50+(2)},i{50+(5)},i{50+(6)});

l a2:=
(+1/2)*
 e(p1,i{50+(-1)})*
--More--(0%)
```

No tadpols

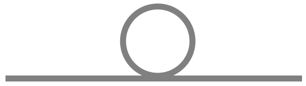


No corrections
On external lines

dummy index i49

polarisation vectors

propagators, vertices



"snail"

a1 could be suppressed by option nosnail

Diagram generation with QGRAF: Output NLO

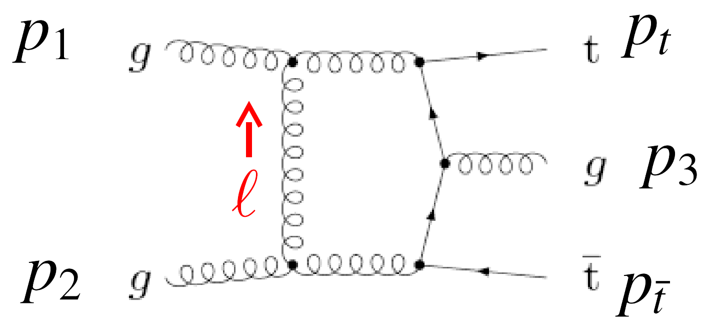
uwer on ttpodo: /users/ttp/uwer/projekt...

```
 e(kq,i{50+(-4)})*
 e(kqb,i{50+(-6)})*
 tprop(i{50+(1)},i{50+(2)},l1)*
 tprop(i{50+(3)},i{50+(4)},l1-p1)*
 gprop(i{50+(5)},i{50+(6)},-l1+kq)*
 gprop(i{50+(7)},i{50+(8)},l1-p1+kqb)*
 gprop(i{50+(9)},i{50+(10)},-l1+p3+kq)*
 vrtx(i{50+(2)},i{50+(3)},i{50+(-1)})*
 vrtx(i{50+(-4)},i{50+(1)},i{50+(5)})*
 vrtx(i{50+(4)},i{50+(-6)},i{50+(7)})*
 vrtx(i{50+(-2)},i{50+(6)},i{50+(9)})*
 vrtx(i{50+(-3)},i{50+(8)},i{50+(10)});

l a354:=
(+1)*
 e(p1,i{50+(-1)})*
 e(p2,i{50+(-3)})*
 e(p3,i{50+(-2)})*
 e(kq,i{50+(-4)})*
 e(kqb,i{50+(-6)})*
 gprop(i{50+(1)},i{50+(2)},-l1)*
 gprop(i{50+(3)},i{50+(4)},l1-p1)*
 tprop(i{50+(5)},i{50+(6)},-l1+kq)*
 tprop(i{50+(7)},i{50+(8)},-l1+p1-kqb)*
 tprop(i{50+(9)},i{50+(10)},-l1+p3+kq)*
 vrtx(i{50+(-1)},i{50+(1)},i{50+(3)})*
 vrtx(i{50+(-4)},i{50+(5)},i{50+(2)})*
 vrtx(i{50+(8)},i{50+(-6)},i{50+(4)})*
 vrtx(i{50+(6)},i{50+(9)},i{50+(-2)})*
 vrtx(i{50+(10)},i{50+(7)},i{50+(-3)});

* end

uwer@ttpodo:auto>
```

5 propagators containing the loop momenta → pentagon diagram

pentagon diagrams are the most complicated once

More on pentagon diagrams



loop momenta appears
in numerator → tensor integrale

$$\mathcal{A}_i = \int d^d\ell \frac{u(p_1, p_2, p_3, p_t, p_{\bar{t}}, \ell)}{(\ell^2 + i\varepsilon)((\ell + p_1)^2 + i\varepsilon)((\ell + p_1 - p_t)^2 - m_t^2 + i\varepsilon)}$$

$$\times \frac{1}{((\ell - p_1 + p_{\bar{t}})^2 - m_t^2 + i\varepsilon)} \frac{1}{((\ell - p_2)^2 + i\varepsilon)}$$

loop integration needs to be done in d dimensions to regulate UV and IR singularities

→complicated complex function of 5 variables, i.e. $s_{ij} = 2p_i \cdot p_j$

How to calculate the loop diagrams ?

many diagrams many topologies

↓

many different tensor integrals

↓

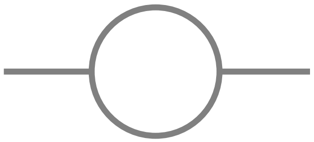
we cannot calculate every tensor integral analytically by hand

Solution:

Tensor integrals can be expressed in terms of a small set of scalar “master integrals”

Tensor reduction à la Passarino & Veltman

Passarino-Veltman



$$\int d^d\ell \frac{\ell_\mu}{(\ell^2 - m_0^2 + i\varepsilon)((\ell+p)^2 - m_1^2 + i\varepsilon)} = p_\mu B_1$$

Contract with p

Terms in red add up to zero

$$\frac{1}{2}\int d^d\ell \frac{\ell^2 + 2p\ell + p^2 - m_1^2 - (\ell^2 - m_0^2) + m_1^2 - m_0^2 - p^2}{(\ell^2 - m_0^2 + i\varepsilon)((\ell+p)^2 - m_1^2 + i\varepsilon)} = p^2 B_1$$

$$\Rightarrow \quad p^2 B_1 = \frac{1}{2}\left(A(m_0) - A(m_1) + (m_1^2 - m_0^2 - p^2) B_0\right)$$

Scalar integrals:

$$A(m) = \int d^d\ell \frac{1}{\ell^2 - m^2 + i\varepsilon} \qquad B_0 = \int d^d\ell \frac{1}{(\ell^2 - m_0^2 + i\varepsilon)((\ell+p)^2 - m_1^2 + i\varepsilon)}$$

Passarino-Veltman reduction (cont'd)

$$B_1 = \frac{1}{2p^2}\left(A(m_0) - A(m_1) + (m_1^2 - m_0^2 - p^2)B_0\right)$$

→ problematic for $p^2 \to 0$

Analytically the limit "0/0" can be taken, numerically it might result in severe instabilities

General problem:

Numerical stable and efficient calculation of tensor integrals

Basic version of Passarino-Veltman implemented in LoopTools

Improvement of Passarino-Veltman

[Denner, Dittmaier and others]

- Derive special reduction formulae for problematic phase space regions

- Special reductions for 5- and 6-point tensor integrals

Remark about scalar integrals:

- Only 1-,2-,3-,and 4-point scalar integrals needed, higher point integrals can be reduced
- Evaluation of scalar integrals can be assumed as solved

Alternative reduction procedure – first step

From Schwinger or Feynman parametrization of tensor integrals:

$$\int d\ell \frac{\ell_{\mu_1} \ldots \ell_{\mu_r}}{((\ell+q_1)^2 - m_1^2)((\ell+q_2)^2 - m_2^2)\ldots((\ell+q_n)^2 - m_n^2)}$$

$$= \sum_{\lambda, z_1, \ldots, z_n} \delta(2\lambda + \sum_i z_i - r) \left(-\frac{1}{2}\right) z_1! \ldots z_n! \{g^\lambda q_1^{z_1} \ldots q_n^{z_n}\}^{\mu_1 \ldots \mu_r}$$

$$\times \; I(d + 2(m-\lambda), \{1 + z_i\})$$

[Davydychev]

→ Reduction of tensor integrals to scalar integrals with raised powers of the propagators and in higher dimensions!

Alternative reduction procedure – second step

Integration-by-parts (IBP)

[Chetyrkin, Kataev, Tkachov]

$$\int d^d\ell \frac{\partial}{\partial \ell_\mu} \frac{\{\ell^\mu, p^\mu\}}{(\ell^2 - m_0^2 + i\varepsilon)((\ell + p)^2 - m_1^2 + i\varepsilon)\cdots} = 0$$

→ Linear relation between different scalar integrals with raised powers of the propagators

Problematic phase points can be studied systematically

General feature of the reduction

$$\int d^d\ell \frac{\ell^\mu \ell^\nu \ell^\rho \ldots}{(\ell^2 + i\varepsilon)((\ell + p_1)^2 + i\varepsilon)((\ell + p_1 - p_t)^2 - m_t^2 + i\varepsilon)}$$

$$\times \frac{1}{((\ell - p_1 + p_{\bar{t}})^2 - m_t^2 + i\varepsilon)} \frac{1}{((\ell - p_2)^2 + i\varepsilon)}$$

$$= \sum \{p_1 p_2 p_t p_{\bar{t}}\}_i^{\mu\nu\rho\ldots} I_i$$

→ apart from the presence of I_i calculation is similar to leading-order calculation

Same techniques:

helicity basis, numerical evalualtion of spinor products, numerical evaluation of amplitude

For pp → t t + 1Jet we used:

1.) Impoved Passarino-Veltman reduction, Feynarts, F77

2.) 2-loop inspired techniques (IBP), QGRAF, C++

→ F77/C++ library to calculate tensor integrals

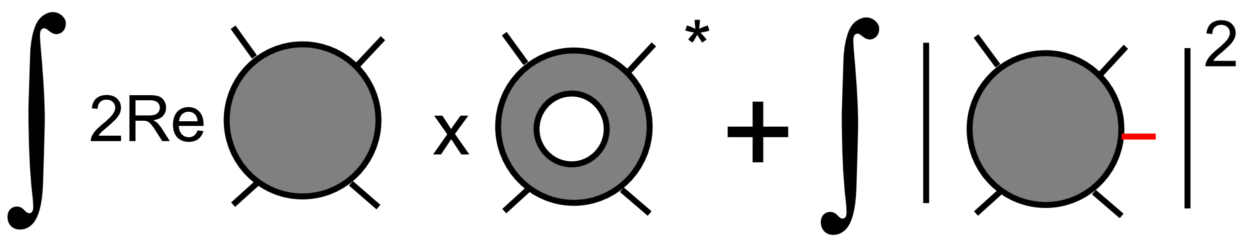
Methods completely general, also applicable to other processes

Real corrections

Note: Virtual corrections contain UV and IR singularities

UV singularities are cancelled via the renormalization procedure

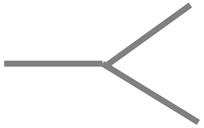
IR singularities are cancelled by real corrections



(n+1)-legs, real corrections

divergent

divergent



$$\sim \frac{1}{(p_1+p_2)^2} = \frac{1}{2p_1p_2} = \frac{1}{2E_1E_2(1-\cos(\theta_{12}))}$$

Real corrections (cont'd)

- In the real corrections the singularity is produced by the phase space integration over soft and collinear regions
- When we use dimensional regularization for the virtual corrections the same has to be done for the real corrections
- d dimensional integration of the phase space integrals in general not feasible

Solution:

Subtraction Method

[Catani,Seymour,…]

Real corrections: Dipole subtraction method

→ Add and subtract a counterterm which is easy enough to be integrated analytically:

$$\begin{aligned} & \int_0^\alpha dx \frac{1}{x} f(x) x^\epsilon \\ = \quad & \int_0^\alpha dx \frac{1}{x} (f(x) - f(0)) x^\epsilon + \frac{1}{x} f(0) x^\epsilon \\ = \quad & + \frac{1}{\epsilon} \alpha^\epsilon + \int_0^\alpha \frac{1}{x} (f(x) - f(0)) + O(\epsilon) \end{aligned}$$

Can be done numerically

Construction of subtraction for real corrections more involved,

Fortunately a general solution exists:

→Dipole subtraction formalism

Dipole subtraction method (2)

How it works in practise:

[Frixione,Kunszt,Signer ´95, Catani,Seymour ´96, Nason,Oleari 98, Phaf, Weinzierl, Catani,Dittmaier,Seymour, Trocsanyi ´02]

$$\sigma_{\rm NLO} = \int_{m+1} \sigma_{\rm real} + \int_m \sigma_{\rm virt.} + \int dx \int_m \sigma_{\rm fact.}(x)$$

$$\sigma_{\rm NLO} = \underbrace{\int_{m+1} [\sigma_{\rm real} - \sigma_{\rm sub}]}_{\rm finite} + \underbrace{\int_m [\sigma_{\rm virt.} + \bar{\sigma}^1_{\rm sub}]}_{\rm finite} + \underbrace{\int dx \int_m [\sigma_{\rm fact.}(x) + \bar{\sigma}_{\rm sub}(x)]}_{\rm finite}$$

Requirements:

$$0 = -\int_{m+1} \sigma_{\rm sub} + \int_m \bar{\sigma}^1_{\rm sub} + \int dx \int_m \bar{\sigma}_{\rm sub}(x)$$

$\sigma_{\rm sub} \longrightarrow \sigma_{\rm real}$ in all single-unresolved regions

Due to universality of soft and collinear factorization, general algorithms to construct subtractions exist

Recently: NNLO algorithm [Daleo, Gehrmann, Gehrmann-de Ridder, Glover, Heinrich, Maitre]

Dipole subtraction method (3)

Universal structure:

$$\sigma_{\text{sub}} = \sum_{\text{dipoles}} \mathcal{D}_{ij,k}(p_i, p_j, p_k)$$

Generic form of individual dipol:

Leading-order amplitudes
Vector in color space

$$\mathcal{D}_{ij;k} = -\frac{1}{(p_i+p_j)-m_{ij}^2} \langle \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \left| \frac{\mathbf{T}_a \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}} V_{ij,k} \right| \ldots, \widetilde{ij}, \ldots, \tilde{k}, \ldots \rangle$$

universal

Color charge operators, induce color correlation !

Spin dependent part, induces spin correlation !

Example gg→ttgg: 6 different colorstructures in LO, $(T^a T^b T^c)_{ij}$
36 (singular) dipoles $\mathcal{D}_{g_1 g_3, t}, \ldots$

Example

For gg→ttgg the LO amplitude gg→ttg is required:

$$|p_1, p_2, p_t, p_{\bar{t}}, p_3\rangle = \begin{pmatrix} (T^{a_1}T^{a_2}T^{a_3})_{\bar{t}t} A(p_1, \lambda_1, p_2, \lambda_2, p_3, \lambda_3, p_t, s_t, p_{\bar{t}}, s_{\bar{t}}) \\ (T^{a_1}T^{a_3}T^{a_2})_{\bar{t}t} A(p_1, \lambda_1, p_3, \lambda_3, p_2, \lambda_2, p_t, s_t, p_{\bar{t}}, s_{\bar{t}}) \\ \vdots \end{pmatrix}$$

→ Six component vector in color space

Dipole subtraction method — implementation

emacs@pcth188.cern.ch

File Edit Options Buffers Tools C++ Help

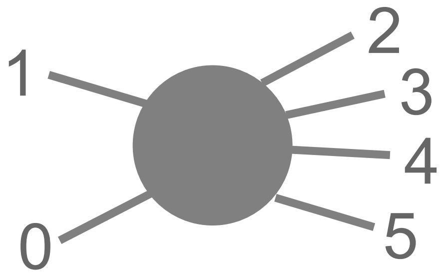
```
double ggttgg_counterterm(const vector<FourMomentum>& momenta) {

  static ggttg ggttgamplitude;
  static Correlator correlator(ggttgamplitude);

  static Dipole d[36] = {
    // FinalFinal:
    Dipole(2,4,3), Dipole(2,4,5), Dipole(2,5,3), Dipole(2,5,4),
    Dipole(3,4,2), Dipole(3,4,5), Dipole(3,5,2), Dipole(3,5,4),
    Dipole(4,5,2), Dipole(4,5,3),
    //FinalInitial:
    Dipole(2,4,0), Dipole(2,4,1), Dipole(2,5,0), Dipole(2,5,1),
    Dipole(3,4,0), Dipole(3,4,1), Dipole(3,5,0), Dipole(3,5,1),
    Dipole(4,5,0), Dipole(4,5,1),
    // InitialFinal:
    Dipole(0,4,2), Dipole(0,4,3), Dipole(0,4,5), Dipole(0,5,2),
    Dipole(0,5,3), Dipole(0,5,4), Dipole(1,4,2), Dipole(1,4,3),
    Dipole(1,4,5), Dipole(1,5,2), Dipole(1,5,3), Dipole(1,5,4),
    // IntialIntial:
    Dipole(0,4,1), Dipole(0,5,1), Dipole(1,4,0), Dipole(1,5,0)
  };


  SplittingKernels splittings(momenta,particles);


  double sum = 0.;

  for (int i = 0; i<36; i++) {
    splittings.Kernel(d[i]);
    correlator.EvalfDipole(d[i]);
    sum += d[i].value;
  }

  return( sum );

}
```

$\longrightarrow \mathcal{D}_{15,0}$

--:-- Dipoles.cpp 3:44PM 0.02 (C++ Abbrev)--L26--C0--All-----------------------

LO – amplitude, with colour information, i.e. correlations

List of dipoles we want to calculate

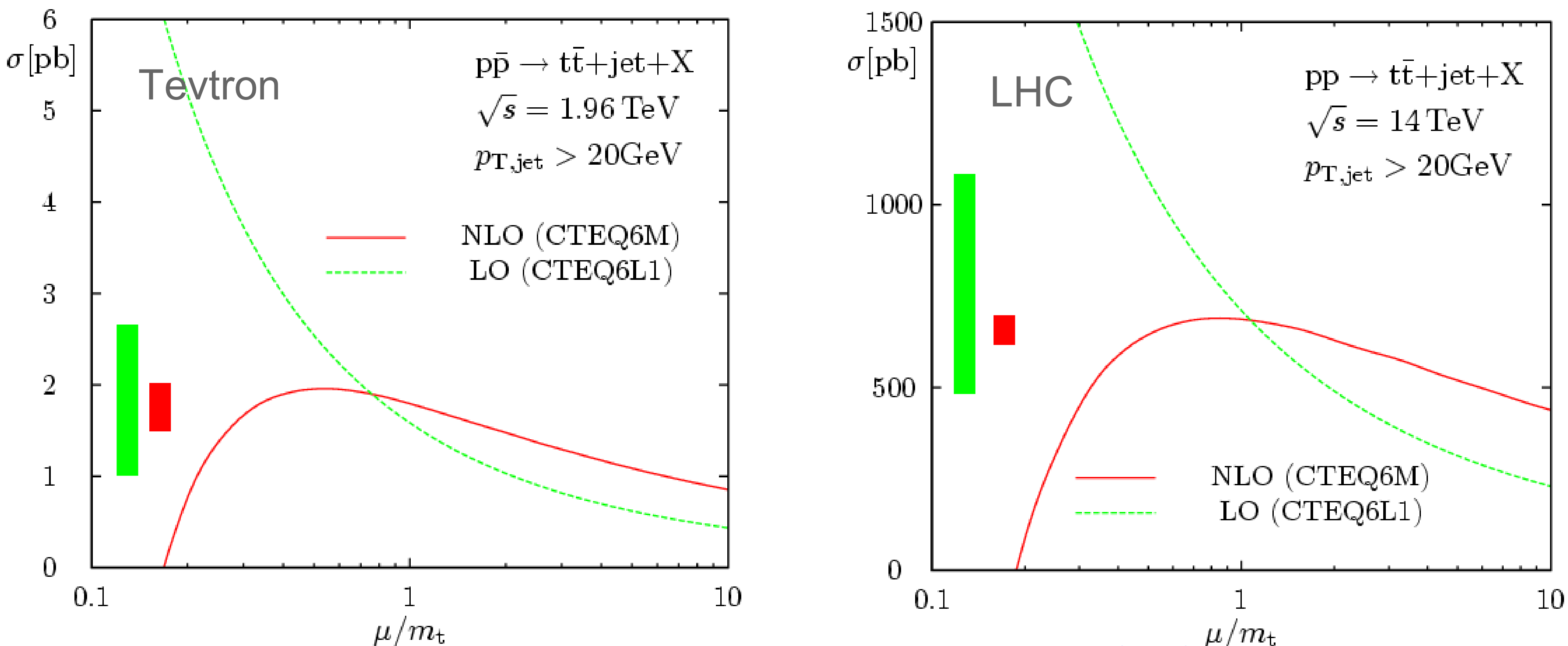


reduced kinematics, “tilde momenta”

Dipole d_i

Topquarkpaar + 1-Jet-Production (NLO)

[Dittmaier, Uwer, Weinzierl, Phys. Rev. Lett. 98:262002, '07]



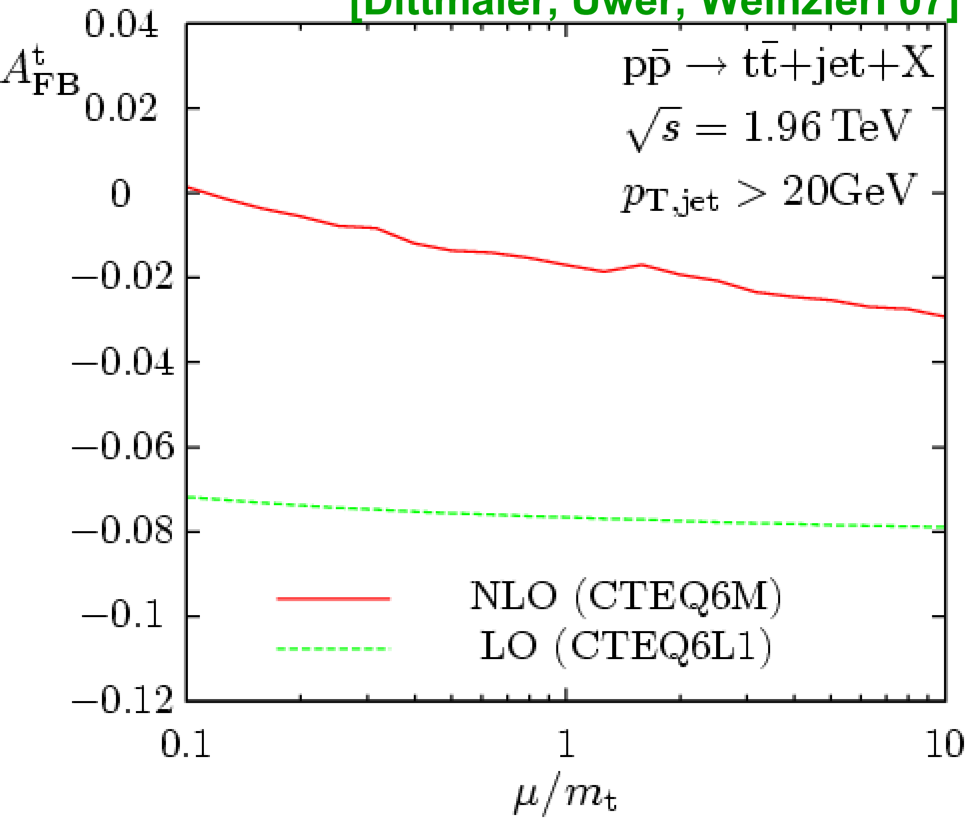
$$\sigma = \alpha(\mu)^3 a_0 + \alpha(\mu)^4 \left(a_1^0 + a_1^1 \ln\left(\frac{\mu}{m}\right)\right) + \cdots$$

- scale dependence is improved
- tools are completely general: arbitrary infrared save observables are calculable (→work in progress)

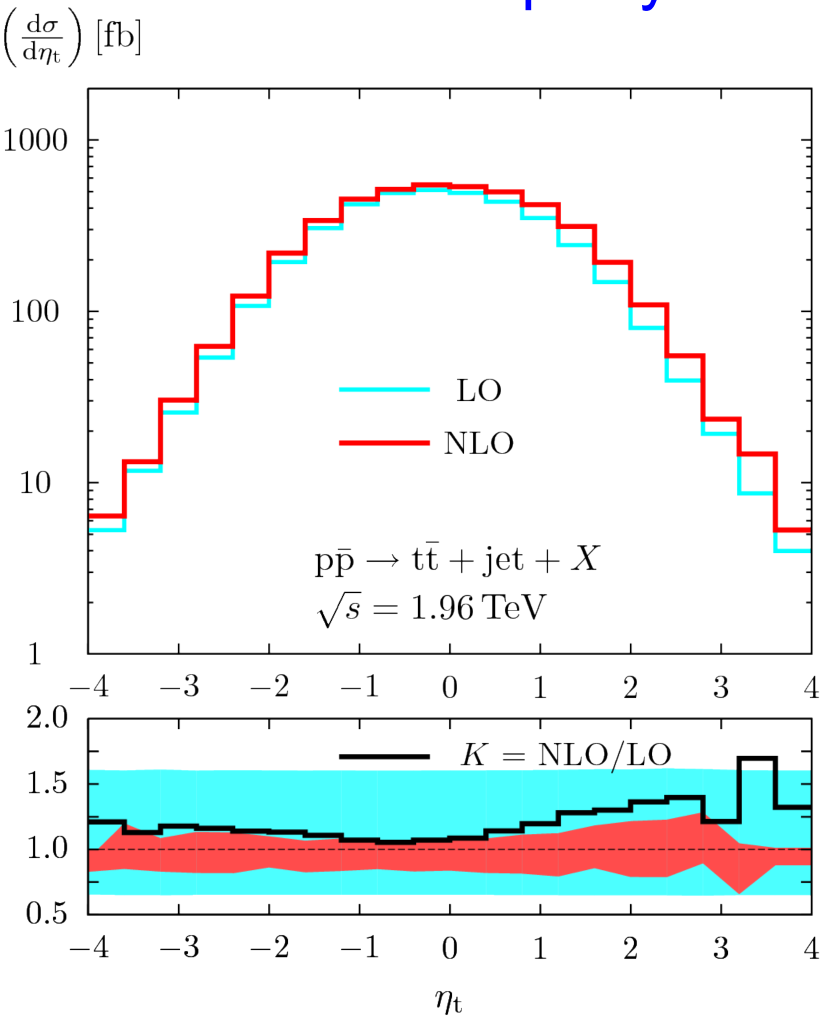
Differential distributions

$$A_{FB}^{t} = \frac{\sigma(\eta_t > 0) - \sigma(\eta_t < 0)}{\sigma(\eta_t > 0) + \sigma(\eta_t < 0)}$$

[Dittmaier, Uwer, Weinzierl 07]



Pseudo rapidity



→ currently studied at the Tevatron