## IV. Quantum Chromodynamics (QCD)

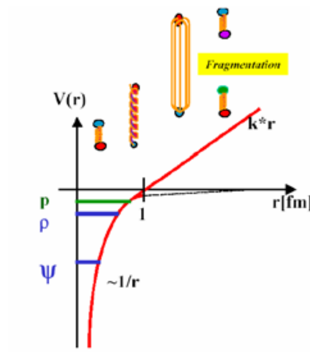
Theory of strong interaction provides the nuclear forces that keep nuclear cores together.

Peculiar properties: $\alpha_s = \dfrac{g_s^2}{4\pi}$

(1) asymptotic freedom: $\alpha_s(p^2 \to \infty) \to 0$

Interaction between quarks at very small distances, i.e. at large momentum transfer, theory looks more like a free field theory w/o interaction (justification of the parton model).

Gross, Wilczek, Politzer (Nobel Prize 2004)

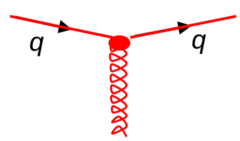


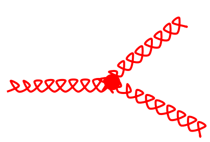

(2) Confinement: no free quarks

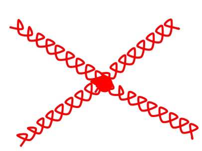
For large distances $V_{q\bar{q}}(r) \sim r$


(3) Octet of massless color charged vector gluon fields interacting with a color triplet of spin ½ quarks

(4) self-interaction:





(non abelian gauge theory SU(3) )

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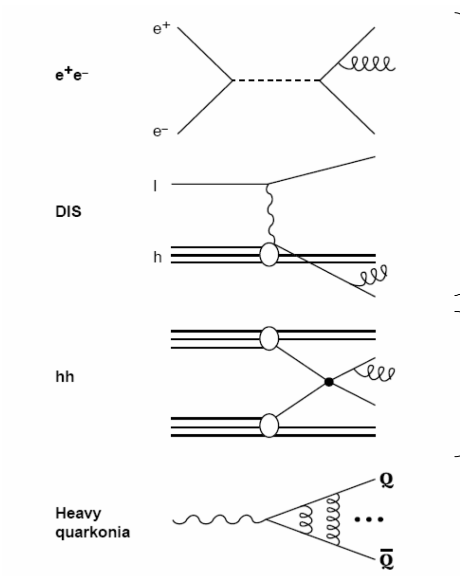
## Experimental Tests of QCD




Discussed

Tevatron / LHC
(not discussed)

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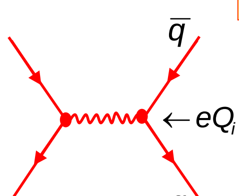
## 1. Test of QCD in $e^+e^-$ annihilation

### 1.1 Quarks carry color charge

Additional color factor $N_C$

$e^+e^- \rightarrow$ hadrons




$$Q_i = \begin{cases} +\frac{2}{3} \\ -\frac{1}{3} \end{cases}$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{ee\rightarrow hadrons} = \frac{\alpha^2}{4s} \cdot N_C \cdot \underbrace{\sum_{quarks\ i}}\, Q_i^2 (1+\cos^2\theta)$$

Sum over all possible quarks: $4m_q^2 < s$

| $\sqrt{s}$ | Quarks |
|---|---|
| < ~3 GeV | uds |
| < ~10 GeV | udsc |
| < ~350 GeV | udscb |
| > ~350 GeV | udscbt |

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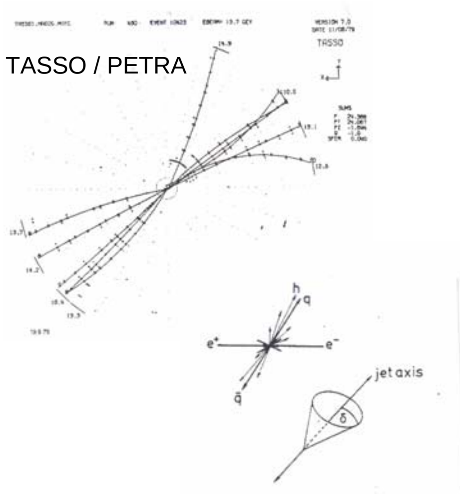
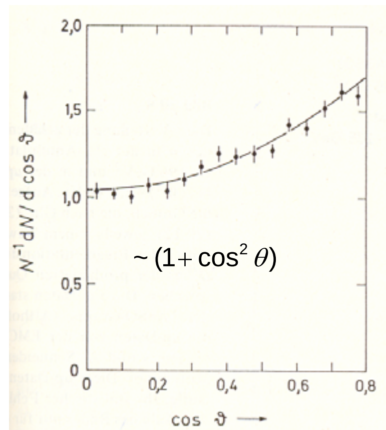
**Spin ½ Quarks:**

$$\frac{d\sigma}{d\Omega} \sim \frac{\alpha^2}{4s} \cdot (1+\cos^2\theta)$$

Jet-like events: Jet axis approximate quark direction

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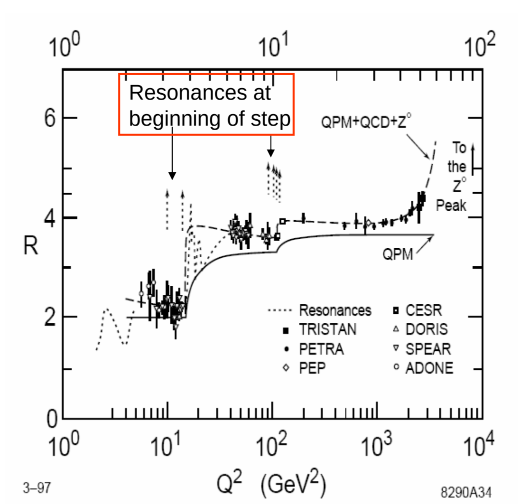
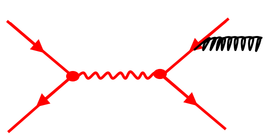
**Definition:**

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = \overset{N_C}{3} \cdot \sum_i Q_i^2$$

| $\sqrt{s}$ | Quarks | $R_{had} = 3 \cdot \sum_i Q_i^2$ |
|---|---|---|
| < ~3 GeV | uds | 3·6/9=2.00 |
| < ~10 GeV | udsc | 3·10/9=3.33 |
| < ~350 GeV | udscb | 3·11/9=3.67 |
| > ~350 GeV | udscbt | 3·15/9=5.00 |




Data lies systematically higher that the prediction from Quark Parton Model (QPM) → gluon bremsstrhl.



$$\sigma(s) = \sigma_{QED}(s)\left[1 + \underbrace{\frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots}_{\sim 7\%}\right]$$

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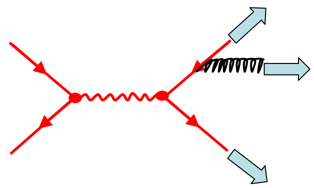
## 1.2 Discovery of the gluon

Discovery of 3-jet events by the TASSO collaboration (PETRA) in 1977:



3-jet events are interpreted as quark pairs with an additional hard gluon.

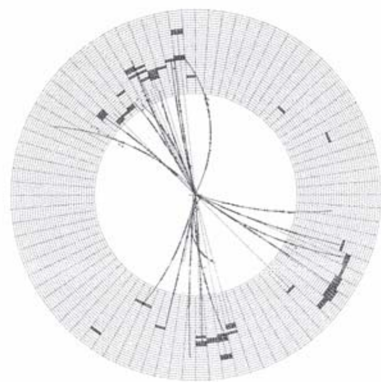


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

$$\frac{\#3-\text{jet events}}{\#2-\text{jet events}} \approx 0.15 \sim \alpha_s$$ ⇒ $\alpha_s$ is large

at $\sqrt{s}$=20 GeV

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## 1.3 Spin of the gluon

Ellis-Karlinger angle

Ordering of 3 jets: $E_1>E_2>E_3$

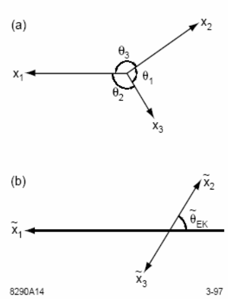


Figure 8: (a) Representation of the momentum vectors in a three-jet event, and (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: $\theta_{EK}$

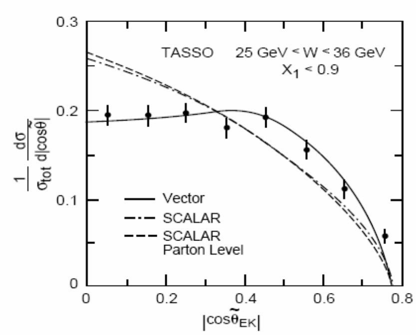


Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

Gluon spin J=1

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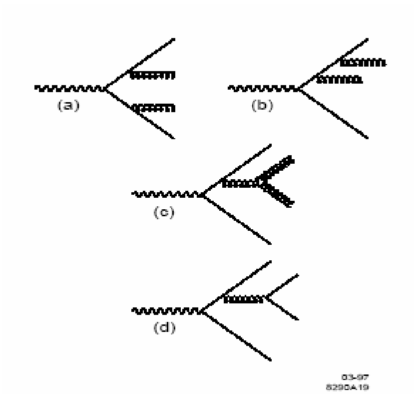
## 1.4 Multi-jet events and gluon self coupling

Non-abelian gauge theory (SU(3))

**4-jet events**



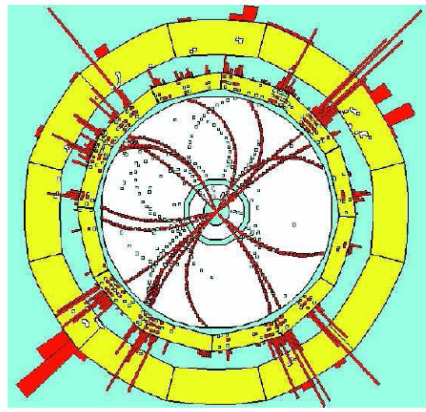



Figure 1: Hadronic event of the type $e^+e^- \to 4$ jets recorded with the ALEPH detector at LEP-I.

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## Multiple jets and jet algorithm

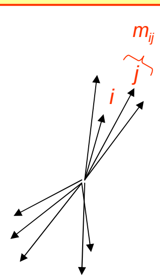
**Jet Algorithm**

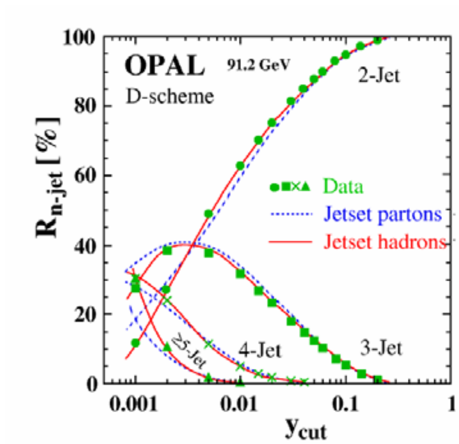
Hadronic particles are i and j grouped to a pseudo particle k as long as the invariant mass is smaller than the **jet resolution parameter**:

$$\frac{m_{ij}^2}{s} < y_{cut}$$

$m_{ij}$ is the invariant mass of i and j.

Remaining pseudo particles are **jets.**

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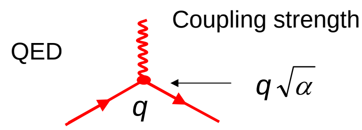
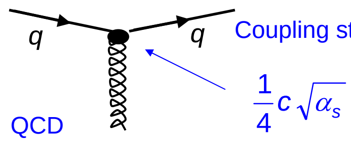
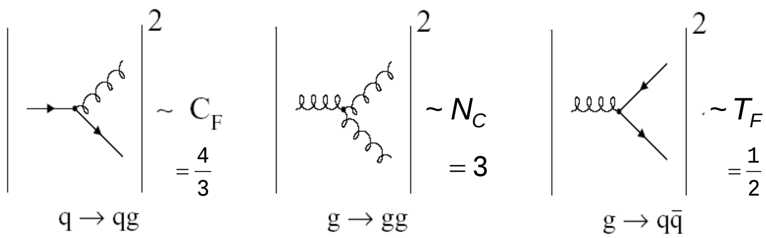
## Color Factor




Color factor: $C_F = c^2$

**Color factors**

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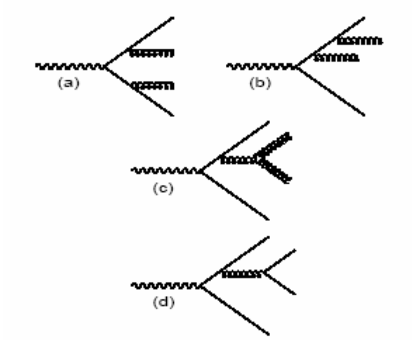
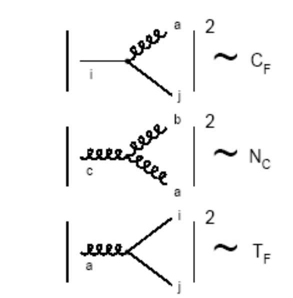
## 4-jet events




Color factors:




4-jet cross section:

$$\frac{1}{\sigma_0}d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[F_A + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right)F_B + \frac{N_C}{C_F}F_C\right]$$
$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[\frac{T_F}{C_F}N_f F_D + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right)F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions

| Group | $N_C$ | $C_F$ | $T_F$ |
|---|---|---|---|
| U(1) | 0 | 1 | 1 |
| $U(1)_3$ | 0 | 1 | 3 |
| SU(N) | $N$ | $(N^2-1)/2N$ | 1/2 |
| SU(3) | 3 | 4/3 | 1/2 |

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## Angular correlation of jets in 4-jet events

Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle

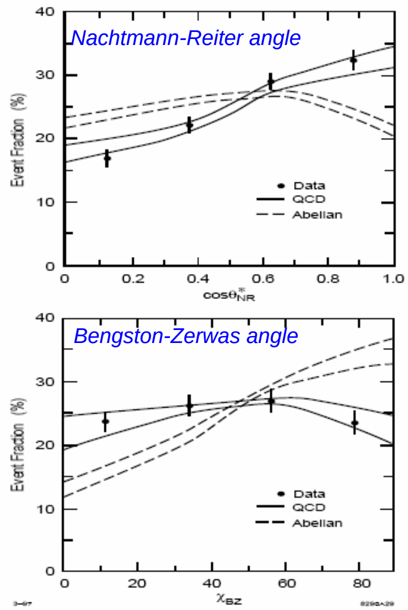
$$\cos\chi_{BZ} \propto (\vec{p}_1 \times \vec{p}_2)\cdot(\vec{p}_3 \times \vec{p}_4)$$

- Nachtmann-Reiter angle

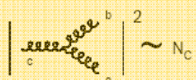
$$\cos\theta_{NR} \propto (\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)$$

Allows to measure the ratios $T_F/C_F$ and $N_C/C_F$

SU(3) predicts: $T_F/C_F = 0.375$ and $N_C/C_F = 2.25$




If $N_C/C_F \neq 0$ $\rightarrow$ contribution from gluon self-coupling in the 4-jet events

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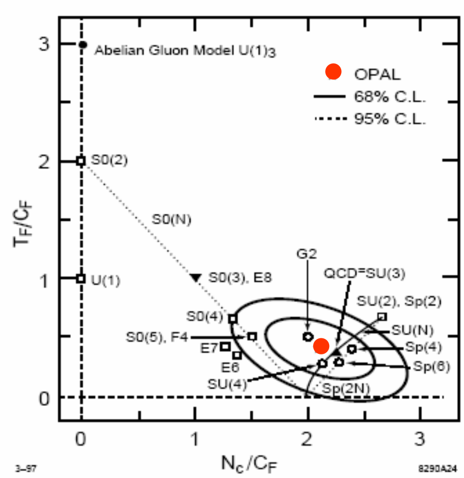


Confirms QCD prediction (SU(3)) and gluon self-coupling:

$T_F/C_F = 0.375$ and $N_C/C_F = 2.25$

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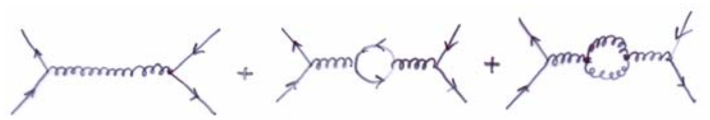
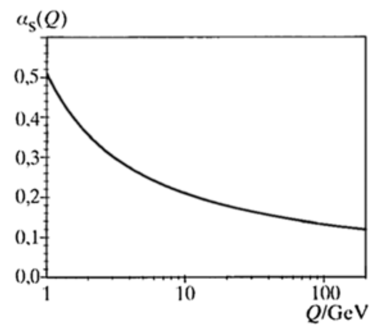
## 2. Running of $\alpha_s$



$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \log\frac{Q^2}{\mu^2}}$$




$\Rightarrow$ Asymptotic freedom

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## Measurement of strong coupling $\alpha_s$

➡ $\alpha_s$ measurements are done at given scale $Q^2$: $\alpha_s(Q^2)$

### a) $\alpha_s$ from total hadronic cross section

$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = 3\sum Q_q^2 \left\{1 + \frac{\alpha_s}{\pi} + 1.411\frac{\alpha_s^2}{\pi^2} + \ldots\right\}$$

Not very precise.

**➡ $\alpha_s(s)$**

### b) $\alpha_s$ from hadronic event shape variables

3-jet rate: $R_3 \equiv \frac{\sigma_{3-jet}}{\sigma_{had}}$ depends on $\alpha_s$

3-jet rate is measured as function of a jet resolution parameter $y_{cut}$

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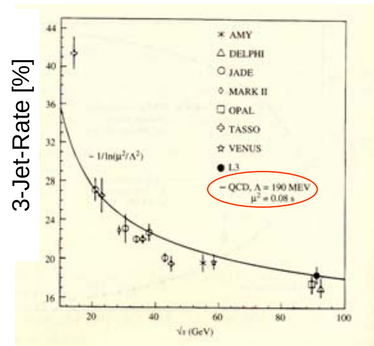
QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s, y_{cut})$

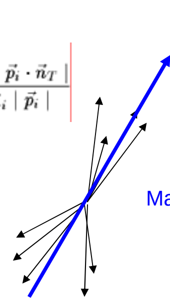
$\rightarrow$ fit $R_3^{theo}(\alpha_s, y_{cut})$ to the data to determine $\alpha_s$

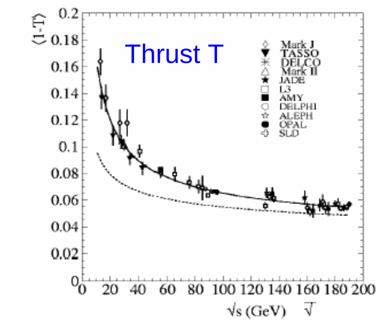
Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for $\alpha_s$

➡ $\alpha_s(s)$




$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$

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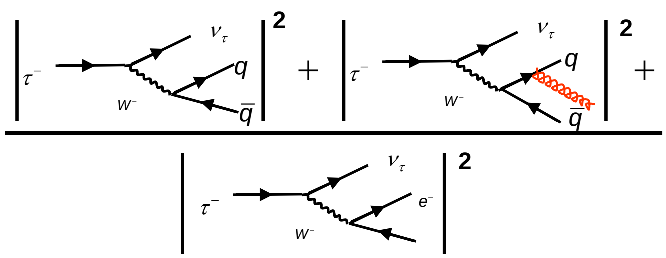
c) $\alpha_s$ from hadronic $\tau$ decays

$$R^{\tau}_{had} = \frac{\Gamma(\tau \rightarrow \nu_\tau + Hadrons)}{\Gamma(\tau \rightarrow \nu_\tau + e\bar{\upsilon}_e)} \sim f(\alpha_s)$$

$R^{\tau}_{had} =$




$$R^{\tau}_{had} = R^{\tau,0}_{had}\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots\right)$$

d) $\alpha_s$ from DIS (deep inelastic scattering)

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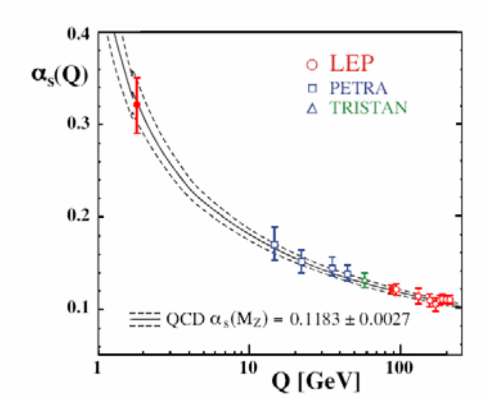
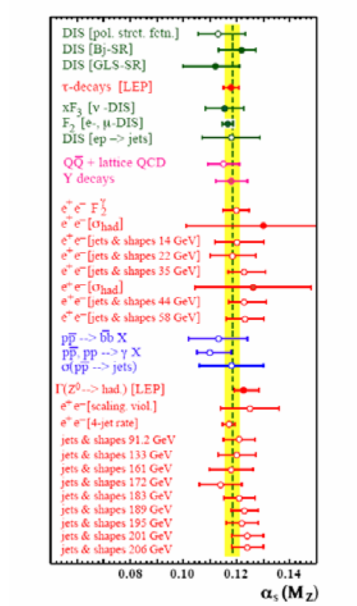
Running of $\alpha_s$ and asymptotic freedom

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \log\frac{Q^2}{\mu^2}}$$

$$\Rightarrow \quad \alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}$$

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