V. Test of Electro-weak Interaction

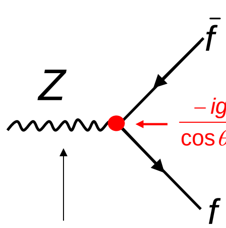
1. Physics of the Z Boson
2. Higgs-Mechanism

1. Physics of the Z boson

(LEP and SLC)

~4.5M Z decays / experiment

~0.5M Z decays



$$\frac{-ig}{\cos\theta_w}\gamma^\mu \frac{1}{2}\left(g_V^f - g_A^f\gamma^5\right)$$

$$g = \frac{e}{\sin\theta_w}$$

$$\frac{-i\left(g_{\mu\nu} - q_\mu q_\nu / M^2\right)}{q^2 - M^2}$$

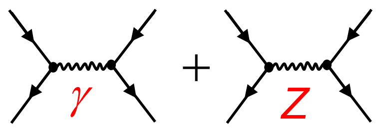
Standard Model	g_V	g_A
ν	$\frac{1}{2}$	$\frac{1}{2}$
ℓ^-	$-\frac{1}{2} + 2\sin^2\theta_W$	$-\frac{1}{2}$
$u-quark$	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_W$	$\frac{1}{2}$
$d-quark$	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_W$	$-\frac{1}{2}$

$g_V = T_3 - 2Q\sin^2\theta_W$ and $g_A = T_3$

$$g_L = \frac{1}{2}(g_V + g_A) \qquad g_R = \frac{1}{2}(g_V - g_A)$$

Cross section for $e^+e^- \rightarrow \gamma / Z \rightarrow f\bar{f}$

$$|M|^2 = \Big| \quad + \quad \Big|^2$$



for $e^+e^- \rightarrow \mu^+\mu^-$

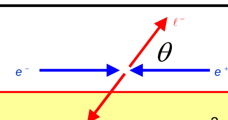
$$M_\gamma = -ie^2(\bar{u}_\mu \gamma^\nu v_\mu)\frac{g_{\rho\nu}}{q^2}(\bar{v}_e \gamma^\rho u_e)$$

$$M_Z = -i\frac{g^2}{\cos^2\theta_W}\left[\bar{u}_\mu \gamma^\nu \frac{1}{2}(g_V^\mu - g_A^\mu \gamma^5)v_\mu\right]\underbrace{\frac{g_{\rho\nu} - q_\rho q_\nu / M_Z^2}{(q^2 - M_Z^2) + iM_Z\Gamma_Z}}_{\text{Z propagator considering a finite Z width}}\left[\bar{v}_e \gamma^\rho \frac{1}{2}(g_V^e - g_A^e \gamma^5)u_e\right]$$

Z propagator considering a finite Z width

With a “little bit” of algebra similar as for M_γ ….

… one finds for the differential cross section:



$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s}\left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta)\frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} + F_Z(\cos\theta)\frac{s^2}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}\right]$$

γ   γ/Z interference   Z

Vanishes at $\sqrt{s} \approx M_Z$

$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2(1+\cos^2\theta) = (1+\cos^2\theta)$$

symmetric in cosθ

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W}\left[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu \cos\theta\right]$$

asymmetric in cosθ

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W}\left[(g_V^{e2} + g_A^{e2})(g_V^{\mu 2} + g_A^{\mu 2})(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta\right]$$

At the Z-pole $\sqrt{s} \approx M_Z$ → Z contribution is dominant
→ interference vanishes

$$\sigma_{tot} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16\sin^4\theta_w \cos^4\theta_w} \cdot \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$

Forward-backward asymmetry

$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$

$$A_{FB} = 3 \cdot \frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \cdot \frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2}$$

Cross section at the Z-pole $\sqrt{s} \approx M_Z$: Breit-Wigner Resonance

$$\sigma_{tot} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16\sin^4\theta_w \cos^4\theta_w} \cdot \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z\Gamma_Z)^2}$$

$$\sigma_Z\left(\sqrt{s} = M_Z\right) = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

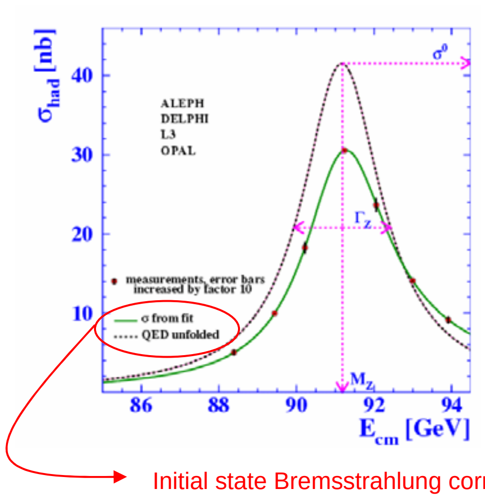
With partial and total widths:

$$\Gamma_f = \frac{\alpha M_Z}{12\sin^2\theta_w \cos^2\theta_w} \cdot \left[(g_V^f)^2 + (g_A^f)^2\right]$$

$$\Gamma_Z = \sum_i \Gamma_i$$

← Cross sections and widths can be calculated within the Standard Model if all parameters are known

Measurement of the Z lineshape



Z Resonance curve:

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

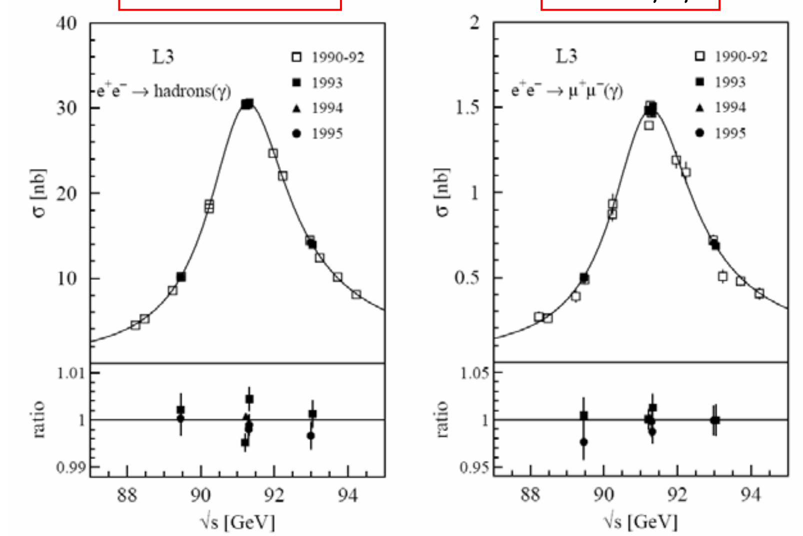
Peak: $$\sigma_0 = \frac{12\pi}{M_Z^2}\frac{\Gamma_e\Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_\mu$
- Width $\rightarrow \Gamma_Z$

Initial state Bremsstrahlung corrections

$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^{1} G(z)\sigma_{ff}^0(zs)\,dz \qquad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

$e^+e^- \rightarrow hadrons$ $\qquad$ $e^+e^- \rightarrow \mu^+\mu^-$



Resonance looks the same, independent of final state: Propagator the same

Z line shape parameters (LEP average)

$M_Z = 91.1876 \pm 0.0021$ GeV ± 23 ppm (*)

$\Gamma_Z = 2.4952 \pm 0.0023$ GeV
$\Gamma_{had} = 1.7458 \pm 0.0027$ GeV
$\Gamma_e = 0.08392 \pm 0.00012$ GeV
$\Gamma_\mu = 0.08399 \pm 0.00018$ GeV
$\Gamma_\tau = 0.08408 \pm 0.00022$ GeV

± 0.09 %

3 leptons are treated independently

test of lepton universality

$\Gamma_Z = 2.4952 \pm 0.0023$ GeV
$\Gamma_{had} = 1.7444 \pm 0.0022$ GeV
$\Gamma_e = 0.083985 \pm 0.000086$ GeV

Assuming lepton universality: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

*) error of the LEP energy determination: ± 1.7 MeV (19 ppm)

http://lepewwg.web.cern.ch/ (Summer 2005)

Number of light neutrino generations

In the Standard Model:

$$\Gamma_Z = \Gamma_Z + 3 \cdot \Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible}: \Gamma_{inv}}$$

$$\left\{ \begin{array}{l} e^+e^- \to Z \to \nu_e \bar{\nu}_e \\ e^+e^- \to Z \to \nu_\mu \bar{\nu}_\mu \\ e^+e^- \to Z \to \nu_\tau \bar{\nu}_\tau \end{array} \right.$$

$$\Gamma_{inv} = 0.4990 \pm 0.0015 \text{ GeV}$$

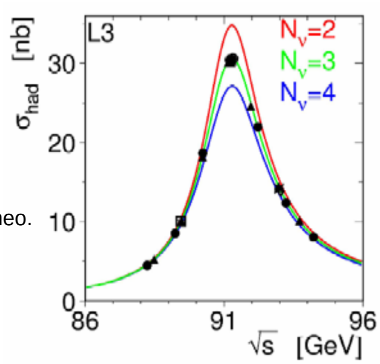
To determine the number of light neutrino generations:

$$N_\nu = \left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp} \cdot \left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}$$

5.9431±0.0163   =1/(1.991±0.001) (small theo. uncertainties from m_{top} M_H)

$N_\nu = 2.9840 \pm 0.0082$

No room for new physics: Z→new



Standard Model: Experimental Tests of Electroweak Interaction

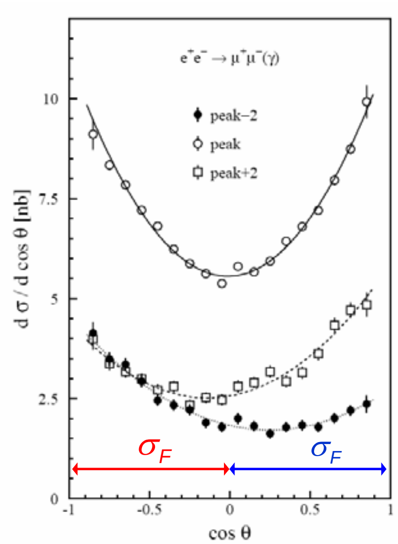
Leptonic forward-backward asymmetry

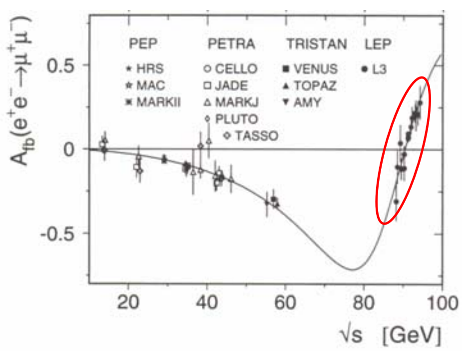
$e^+ e^- \to Z \to \mu^+ \mu^-$

$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with $A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$





Lepton couplings

Forward-backward asymmetry:

- Away from the resonance A_{FB} large
  → interference term dominates

$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2}$$

- At the Z pole: Interference = 0

$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

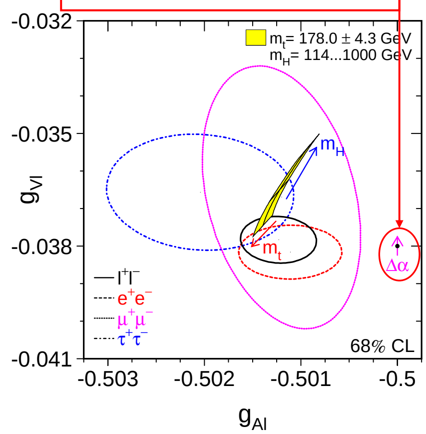
→ very small because g_V^l small in SM

Pole cross section:

$$\sigma_Z^0 \sim \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right]$$

Asymmetries together with cross sections allow the determination of the fermion couplings g_A and g_V (elliptical regions)

Lowest order SM prediction:
$g_V = T_3 - 2q\sin^2\theta_W \quad g_A = T_3$



Confirms lepton universality

Higher order corrections seen

Higher order corrections

Including radiative corrections

Lowest order SM predictions

$$\rho = \frac{m_W^2}{m_Z^2 \cos^2\theta_W} = 1 \quad \Longrightarrow \quad \bar{\rho} = 1 + \Delta\rho$$

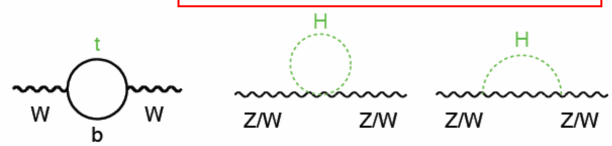
$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \quad \Longrightarrow \quad \sin^2\theta_{\text{eff}} = (1 + \Delta\kappa)\sin^2\theta_W$$

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F} \quad \Longrightarrow \quad m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F}(1 + \Delta r)$$

$$\alpha(0) \quad \Longrightarrow \quad \alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

with : $\Delta\alpha = \Delta\alpha_{\text{lept}} + \Delta\alpha_{\text{top}} + \Delta\alpha_{\text{had}}^{(5)}$

$\Delta\rho, \Delta\kappa, \Delta r = f\,(m_t^2, \log(m_H), \ldots)$



$\sin^2\theta_w$ → $\sin^2\theta_{eff}$

g_A, g_V → $\bar{g}_A, \bar{g}_V$

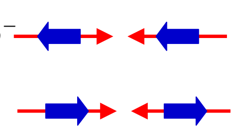
Coupling to LH and RH fermions

Z boson coupling to LH and RH fermions different:

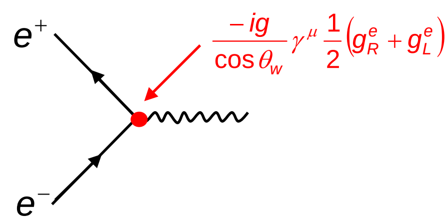
$$\left|g_L = \frac{1}{2}(g_V + g_A)\right| > \left|g_R = \frac{1}{2}(g_V - g_A)\right|$$

➡ Coupling to LH leptons stronger

<u>Experimental configuration:</u>

e^-  e^+ $\Rightarrow g_L$

$\Rightarrow g_R$



Left-Right Asymmetry at SLC

Measure cross section σ_L (σ_R) for LH (RH) initial state electrons:

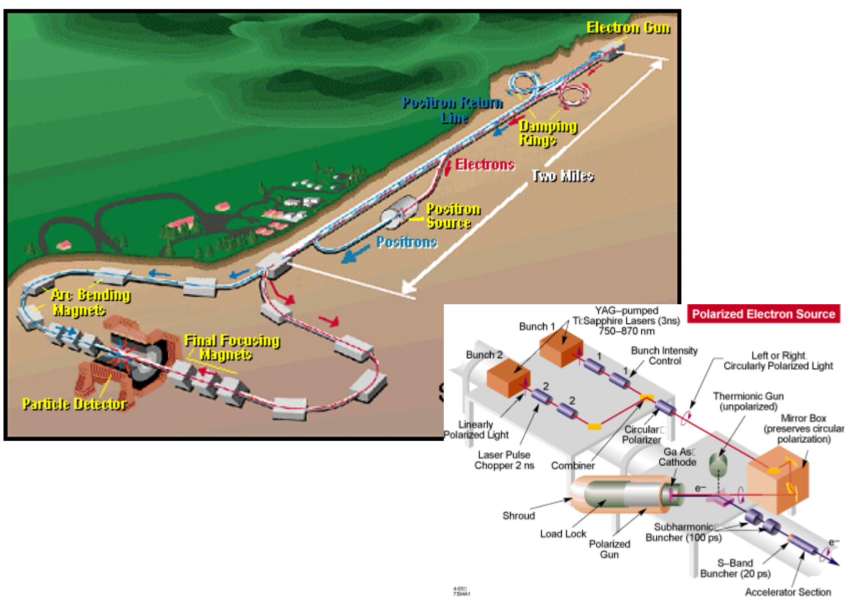
Polarization of electron beam: P~70 – 80%

$$A_{LR} = \frac{1}{P}\frac{\sigma_L^f - \sigma_R^f}{\sigma_L^f + \sigma_R^f} = \frac{1}{P}\frac{2g_V^e g_A^e}{\left(g_V^e\right)^2 + \left(g_A^e\right)^2}$$

$$= \frac{2(1-4\sin^2\theta_w)}{1+(1-4\sin^2\theta_w)^2}$$

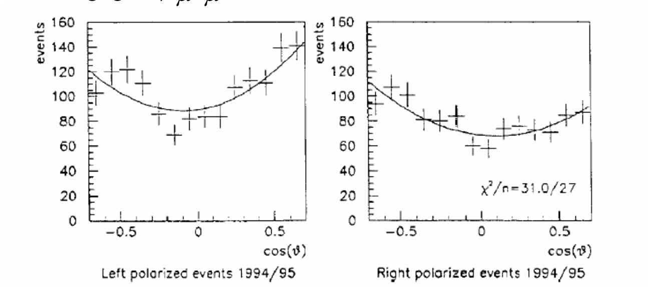
Powerful determination of $\sin^2\theta_w$. Requires longitudinal polarization of colliding beams: only possible in case of Linear Collider: SLC

SLAC Linear Collider



Leptonic final states:

$e^+e^- \to \mu^+\mu^-$ 1994/1995



All data: $A_{LR} = 0.1513 \pm 0.0021$

$\sin^2\theta_W = 0.23098 \pm 0.00026$