Flavor Mixing and CP Violation

1. CKM Matrix
2. Mixing of neutral mesons
3. CP violation in the B^0 system

1. CKM Matrix

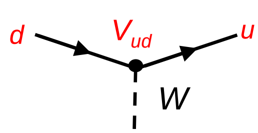
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — **CKM matrix** — **mass eigenstates**

Unitarity

$$V_{CKM} V_{CKM}^{+} = 1$$

Charged currents:



$$J_\mu^+ \propto (\bar{u},\bar{c},\bar{t})\,\gamma_\mu(1-\gamma_5) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (\bar{u},\bar{c},\bar{t})\,\gamma_\mu(1-\gamma_5)\, \mathbf{V_{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak — **mass/ flavor**

1.1 Parameters of CKM matrix

Number of independent parameters: →

18 parameter (9 complex elements)
-5 relative quark phases (unobservable)
-9 unitarity conditions
=4 independent parameters: 3 angles + 1 phase

PDG parametrization

3 Euler angles

$\theta_{23}, \theta_{13}, \theta_{12}$

1 Phase

δ

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

Remark: Unobservable relative quark phases

Phases of left-handed fields in J^{cc} are unobservable: possible redefinition

$$u_L \to e^{i\phi(u)}u_L \qquad c_L \to e^{i\phi(c)}c_L \qquad t_L \to e^{i\phi(t)}t_L$$

$$d_L \to e^{i\phi(d)}d_L \qquad s_L \to e^{i\phi(s)}s_L \qquad b_L \to e^{i\phi(b)}b_L$$

↑ Real numbers

Under phase transformation:

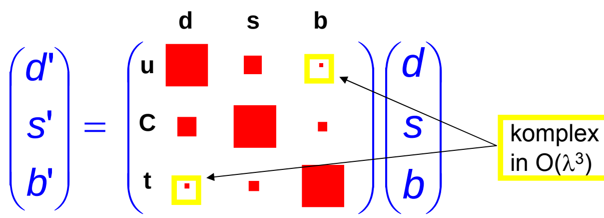
$$V \to \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

$$V\alpha j \to \exp[i(\phi(j) - \phi(\alpha))]V\alpha j$$

5 unobservable phase differences !

Standard Model: Flavor mixing and CP violation

Magnitude of elements



Wolfenstein Parametrization $\lambda,\ A,\ \rho,\ \eta,\ \lambda = 0.22$

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

$V_{ub} = |V_{ub}|e^{-i\gamma}$

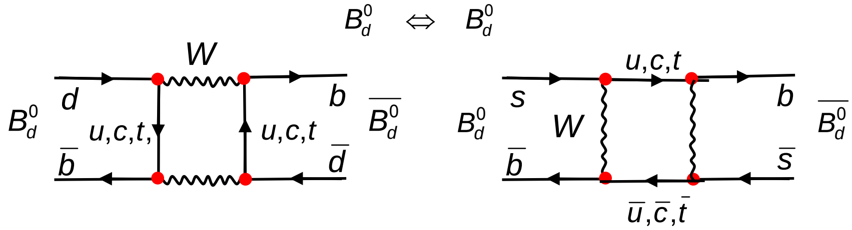
$V_{td} = |V_{td}|e^{-i\beta}$

Reflects hierarchy of elements in O(λ)

2. Mixing of neutral mesons

As result of the quark mixing the Standard Model predicts oscillations of neutral mesons:

$B_d^0 \Leftrightarrow \overline{B_d^0}$



Neutral mesons:

$|P^0\rangle$: $K^0 = |d\bar{s}\rangle$ $D^0 = |\bar{u}c\rangle$ $B_d^0 = |d\bar{b}\rangle$ $B_s^0 = |s\bar{b}\rangle$

$|\overline{P^0}\rangle$: $\overline{K^0} = |\bar{d}s\rangle$ $\overline{D^0} = |\bar{u}c\rangle$ $\overline{B_d^0} = |\bar{d}b\rangle$ $\overline{B_s^0} = |\bar{s}b\rangle$

discovery of mixing: 1960, 2007, 1987, 2006

2.1 Mixing phenomenology

Consider time dependent Schrödinger eq. for 2 component wave function $\begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix}$:

$$i\frac{d}{dt}\begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix} = \mathbf{H}\begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\Gamma\right)\begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix} = \begin{pmatrix} m_{11} - \frac{i}{2}\Gamma_{11} & m_{12}^* - \frac{i}{2}\Gamma_{12}^* \\ m_{12} - \frac{i}{2}\Gamma_{12} & m_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} B^0 \\ \overline{B}^0 \end{pmatrix}$$

Dispersive & absorptive

As the matrix H is not diagonal B^0 and $\overline{B}^0$ are not mass eigenstates.

Diagonalizing H of finds the mass eigenstates:

$$|B_L\rangle = p|B^0\rangle + q|\overline{B}^0\rangle \quad \text{with } m_L, \Gamma_L$$

$$|B_H\rangle = p|B^0\rangle - q|\overline{B}^0\rangle \quad \text{with } m_H, \Gamma_H$$

$$|p|^2 + |q|^2 = 1$$

complex coefficients

Free particle wave funct.

$$|B_{H,L}(t)\rangle = |B_{H,L}(0)\rangle \cdot \underbrace{e^{-im_{H,L}t} \cdot e^{-\frac{1}{2}\Gamma_{H,L}t}}_{b_{H,L}(t)}$$

$$\Delta m = m_H - m_L$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L$$

Time development of B^0 and B^0

$$|B^0\rangle = \frac{1}{2p}\left(|B_L\rangle + |B_H\rangle\right) \qquad |\overline{B}^0\rangle = \frac{1}{2q}\left(|B_L\rangle - |B_H\rangle\right)$$

$$|\psi_{B^0}(t)\rangle = \frac{|B_L(t)\rangle + |B_H(t)\rangle}{2p} = \frac{1}{2p}\left(b_L(t)\cdot\left(p|B^0\rangle + q|\overline{B}^0\rangle\right) + b_H(t)\cdot\left(p|B^0\rangle - q|\overline{B}^0\rangle\right)\right)$$

$$= f_+(t)\cdot|B^0\rangle + \frac{q}{p}f_-(t)\cdot|\overline{B}^0\rangle \qquad f_\pm(t) = \frac{1}{2}\cdot\left[e^{-im_Ht}e^{-\Gamma_H t/2} \pm e^{-im_Lt}e^{-\Gamma_L t/2}\right]$$

$$|\psi_{\overline{B}^0}(t)\rangle = f_+(t)\cdot|\overline{B}^0\rangle + \frac{p}{q}f_-(t)\cdot|B^0\rangle$$

B^0

$$P\left(B^0(0) \to B^0(t)\right) = |f_+(t)|^2$$

$$P\left(B^0(0) \to \overline{B}^0(t)\right) = \left|\frac{q}{p}\right|^2 |f_-(t)|^2$$

$\overline{B}^0$

$$P\left(\overline{B}^0(0) \to \overline{B}^0(t)\right) = |f_+(t)|^2$$

$$P\left(\overline{B}^0(t) \to B^0(t)\right) = \left|\frac{p}{q}\right|^2 |f_-(t)|^2$$

Oscillation frequency

$$\underbrace{P(B^0 \to B^0) = P(\overline{B^0} \to \overline{B^0})}_{\text{CPT}} = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{4}\left|\frac{q}{p}\right|^2\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

$$P(\overline{B^0} \to B^0) = \frac{1}{4}\left|\frac{p}{q}\right|^2\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

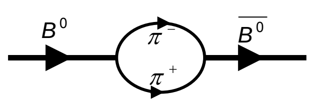
CP- violation in mixing: $P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0) \Rightarrow \left|\frac{q}{p}\right| \neq 1$

2.2 Standard Model prediction for B^0 mixing

Mixing mechanisms:

- **Mixing through decay:**

„long distant, on-shell states“

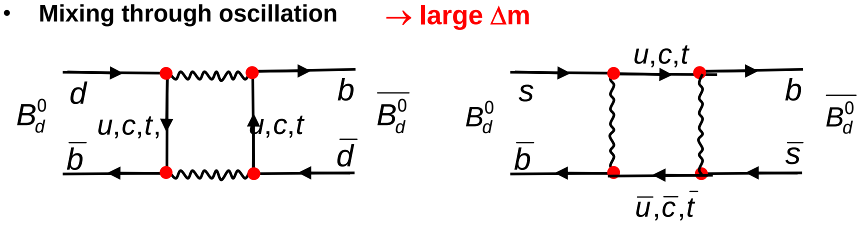


$\to$ **large $\Delta\Gamma$**

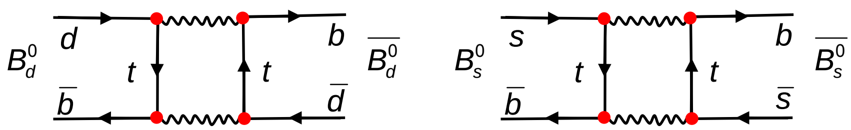
For B mesons there are many possible hadronic decays $\to \Gamma$ is large in addition decays like $B \to \pi\pi$ are suppressed

$$\Rightarrow y = \frac{\Delta\Gamma}{2\Gamma} \text{ is small} = \begin{cases} \approx 0 \text{ for } B_d^0 \\ \approx O(0.1) \text{ for } B_s^0 \end{cases} \Rightarrow \text{don't expect mixing via decay}$$

- **Mixing through oscillation** $\to$ **large Δm**



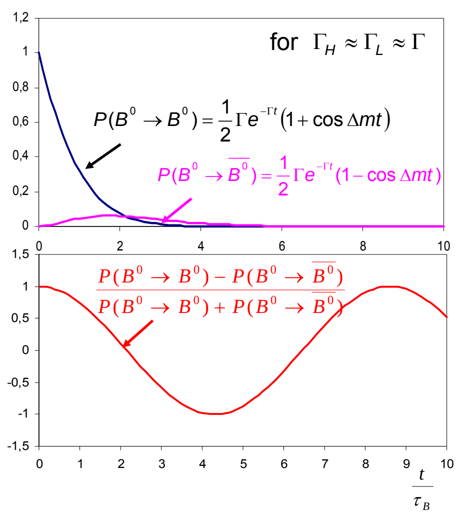
- **Standard Model result** → Significant contribution only from top loop



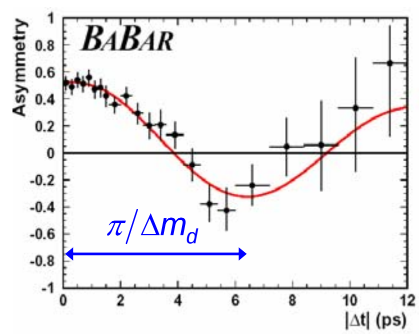
$$\Delta m \sim m_t^2 |V_{tb} V_{td}|^2 \sim m_t^2 \cdot O(\lambda^6) \qquad \Delta m \sim m_t^2 |V_{tb} V_{ts}|^2 \sim m_t^2 \cdot O(\lambda^4)$$

Large $\Delta m_{s,d}$: $\Delta m_s \sim 1/\lambda^2 \, \Delta m_d$ → B_s osc. is about 35 times faster than B_d osc.

2.3 Experimental observation of B meson mixing



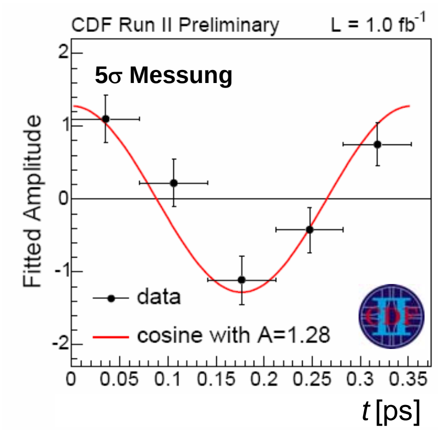
$$A = \frac{unmixed - mixed}{unmixed + mixed}$$



$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004 \text{ ps}^{-1} \approx \frac{0.774}{\tau_B}$$

Observation:

Spring 2006



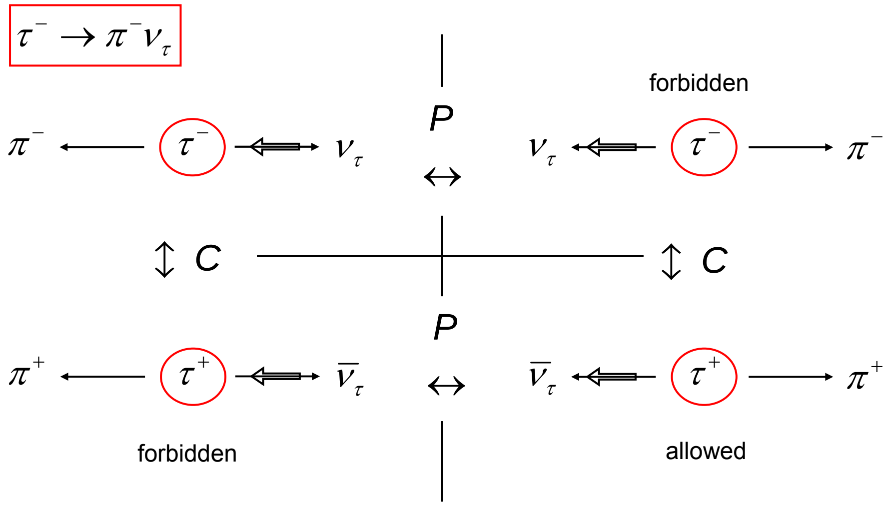
$$\Delta m_s = 17.77 \pm 0.10\,(\text{stat.}) \pm 0.07\,(\text{syst.})\ \text{ps}^{-1} = \frac{26}{\tau}$$

(CDF Collaboration, September 2006)

35 times faster than B^0

3. CP violation in the B^0 system

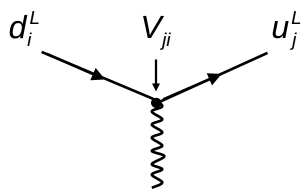
$\tau^- \rightarrow \pi^- \nu_\tau$

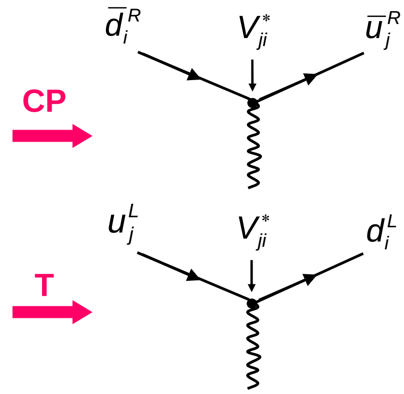


- C and P violated in weak decays
- CP conserved in weak interaction ? $\rightarrow$ No !

Standard Model: Flavor mixing and CP violation

3.1 CP Violation in Standard Model: complex CKM elements





CP (T) violation $\Leftrightarrow$ $V_{ji} \neq V_{ji}^*$

i.e. Complex elements

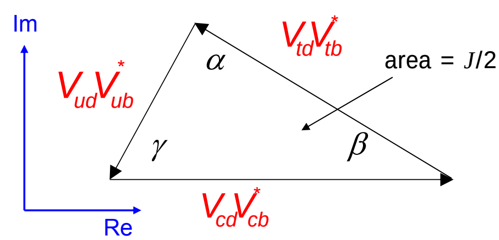
Remark: For 2 quark generations the mixing is described by the **real 2x2** Cabbibo matrix $\rightarrow$ **no CP violation** !!. To explain **CPV** in the SM Kobayashi and Maskawa have predicted a **third quark generation**.

Moreover, as can be shown, CPV requires that all u-type and all d-type quarks have different masses.

3.2 Unitarity Triangle

Unitary CKM matrix: $\mathbf{VV^\dagger = 1}$ $\rightarrow$ 6 “triangle” relations in complex plane:

$$V = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$



$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$
$$V_{td}V_{ud}^* + V_{ts}V_{us}^* + V_{tb}V_{ub}^* = 0$$

Important for $\mathbf{B_d}$ and $\mathbf{B_s}$ decays

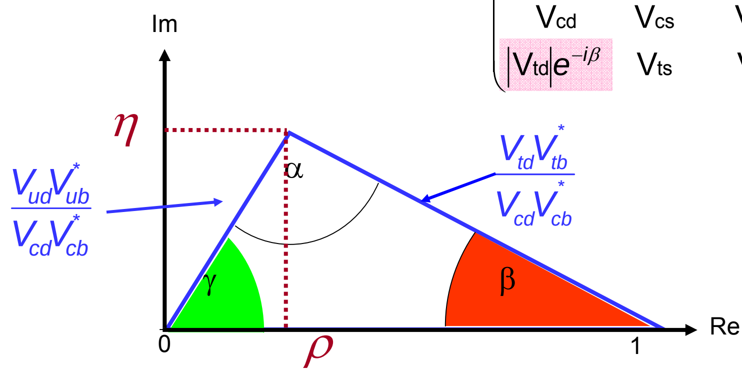
Strength of CPV: Characterized by Jarlskog invariant $J = \mathrm{Im}\left(V_{ij}\, V_{kl}\, V_{il}^* V_{kj}^*\right)$

In SM: $J = \mathrm{Im}[V_{us}V_{cb}V_{ub}^*V_{cs}^*] = A^2\lambda^6\eta\left(1-\lambda^2/2\right) + O(\lambda^{10}) \sim 10^{-5}$

Standard Model: Flavor mixing and CP violation

Rescaled unitarity condition $V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$

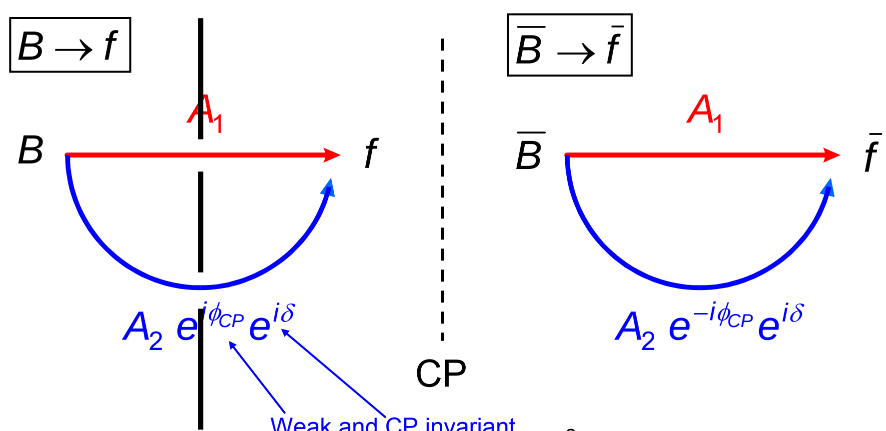
$$\begin{pmatrix} V_{ud} & V_{us} & |V_{ub}|e^{-i\gamma} \\ V_{cd} & V_{cs} & V_{cb} \\ |V_{td}|e^{-i\beta} & V_{ts} & V_{tb} \end{pmatrix}$$



$$\alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^*}{V_{ud}V_{ub}^*}\right] \qquad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^*}{V_{td}V_{tb}^*}\right] \qquad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}\right]$$

3.3 Observation of CP Violation

→ Phase measurement
→ Interference experiment



Weak and CP invariant phase difference

$$|A|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_{CP} + \delta)$$

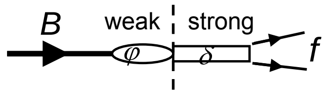
$$|\bar{A}|^2 = A_1^2 + A_2^2 + 2A_1A_2\cos(\phi_{CP} - \delta)$$

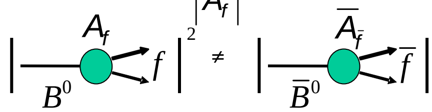
Need two phase differences between A_1 and A_2: Weak difference which changes sign under CP and another phase difference (strong) which is unchanged.

"3 Ways" of CP violation in meson decays

a) Direct CP violation

$$\left|\frac{\bar{A}_{\bar{f}}}{A_f}\right| \neq 1$$



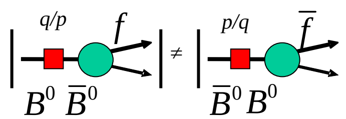


$$A(B \to f) = |A|\, e^{i\varphi}\, e^{i\delta}$$

$$P(\bar{B} \to \bar{f}) \neq P(B \to f)$$

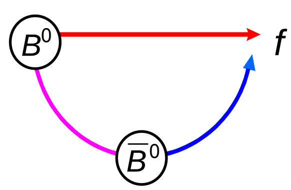
b) CP violation in mixing

$$\left|\frac{q}{p}\right| \neq 1$$



$$P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0)$$

c) CP violation through interference of mixed and unmixed amplitudes



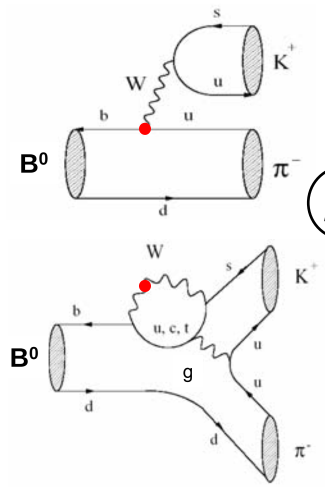
$$\Gamma(B^0_{t=0} \to f)(t) \neq \Gamma(\bar{B}^0_{t=0} \to f)(t)$$

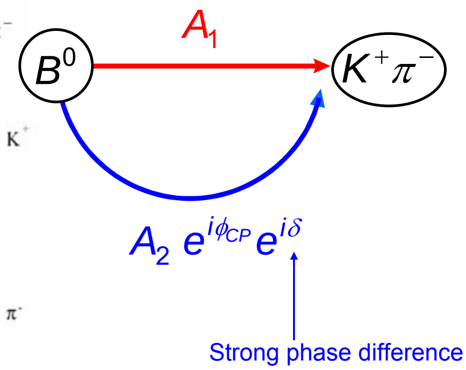
Asymmetrie modulated by $\sim \sin \Delta m t$

Combinations of the 3 ways are possible!

Standard Model: Flavor mixing and CP violation

ad a) Direct CP violation (B system)





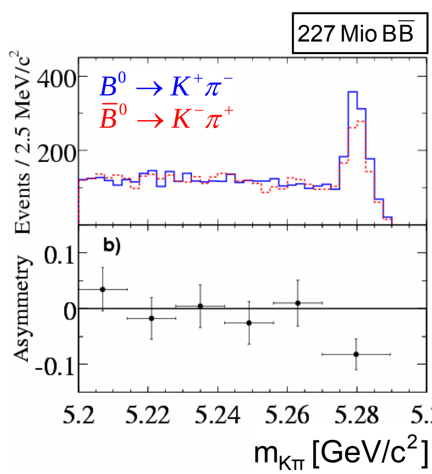
CP Asymmetrie $\quad |\overline{A}|^2 - |A|^2 = 4|A_1||A_2|\sin\varphi\sin\delta$

$N(B^0/\overline{B}^0 \to K^\pm\pi^\mp) = 1606 \pm 51$

$$A_{CP} = \frac{N(\overline{B}^0 \to K^+\pi^-) - N(B^0 \to K^-\pi^+)}{N(\overline{B}^0 \to K^+\pi^-) + N(B^0 \to K^-\pi^+)}$$

$$A_{CP} = -0.133 \pm 0.030 \pm 0.009$$

4.2σ



PRL93(2004) 131801.

b) CP (T) violation in mixing

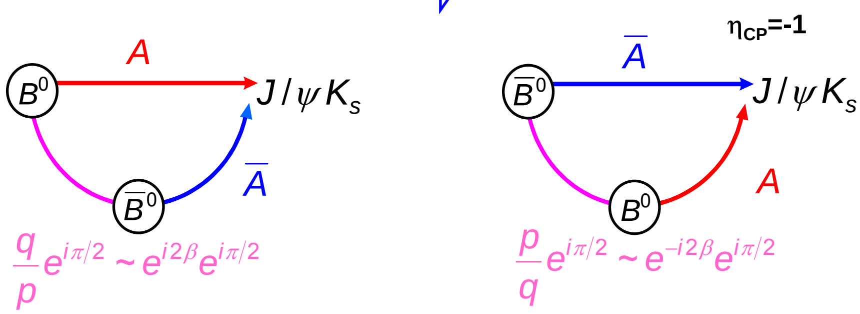
T violation

$$\left|\frac{q}{p}\right| \neq 1 \qquad P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0)$$

Skipped.

c) CP violation in interference between mixing and decay

$B^0 \to J/\psi K_s$ —CP→ $\overline{B}^0 \to J/\psi K_s$



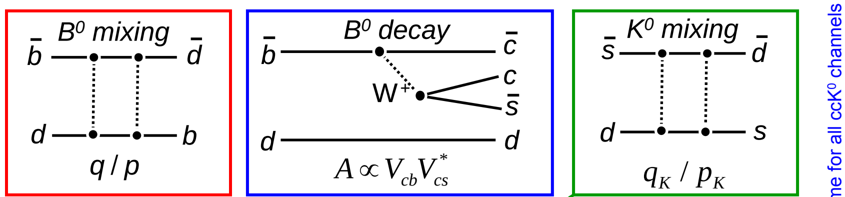
$$\left|B^0 \to J/\psi K_s\right\rangle = A\left(f_+(t) + \lambda_{CP} f_-(t)\right) \qquad \left|\overline{B}^0 \to J/\psi K_s\right\rangle = \overline{A}\left(f_+(t) + \frac{1}{\lambda_{CP}} f_-(t)\right)$$

$$\lambda_{CP} \equiv \frac{q}{p} \cdot \frac{\overline{A}}{A}$$

Standard Model: Flavor mixing and CP violation

SM prediction of λ_{CP} for $B^0 \to J/\psi K_S$

$\eta_{CP} = -1$



$\frac{q}{p} \sim e^{2i\beta}$

$$\lambda_{CP} = \frac{q}{p}\frac{\bar{A}}{A} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{V_{cb} V_{cs}^*}{V_{cb}^* V_{cs}}\frac{V_{cs} V_{cd}^*}{V_{cs}^* V_{cd}} = -\frac{V_{tb}^* V_{td}}{V_{tb} V_{td}^*}\frac{V_{cb} V_{cd}^*}{V_{cb}^* V_{cd}} = -e^{-2i\beta}$$

Beside V_{td} all other CKM elements are real

$$V_{td} \approx |V_{td}| e^{-i\beta} \Rightarrow \begin{array}{c} |\lambda_{CP}| = 1 \\ \mathrm{Im}(\lambda_{CP}) = \sin(2\beta) \end{array}$$

no direct CPV, no CPV in mixing

Calculation of the time-dependent CP asymmetry

$$\Gamma(B^0 \to f_{CP})(t) \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{\left(1+|\lambda_{CP}|^2\right)} \times \left[\frac{1+|\lambda_{CP}|^2}{2} - \mathrm{Im}(\lambda_{CP})\sin(\Delta m_d t) + \frac{1-|\lambda_{CP}|^2}{2}\cos(\Delta m_d t)\right]$$

$$\neq$$

$$\Gamma(\bar{B}^0 \to f_{CP})(t) \propto \frac{e^{-|\Delta t|/\tau_{B^0}}}{\left(1+|\lambda_{CP}|^2\right)} \times \left[\frac{1+|\lambda_{CP}|^2}{2} + \mathrm{Im}(\lambda_{CP})\sin(\Delta m_d t) - \frac{1-|\lambda_{CP}|^2}{2}\cos(\Delta m_d t)\right]$$

$$A_{CP}(t) = \frac{\Gamma(\bar{B}^0(t) \to f_{CP}) - \Gamma(B^0(t) \to f_{CP})}{\Gamma(B^0(t) \to f_{CP}) + \Gamma(\bar{B}^0(t) \to f_{CP})} = [S_f \sin(\Delta m_d t) - C_f \cos(\Delta m_d t)]$$

negligible

Time resolved

$$S_f = \frac{2\,\mathrm{Im}\,\lambda_{CP}}{1+|\lambda_{CP}|^2} \qquad C_f = \frac{1-|\lambda_{CP}|^2}{1+|\lambda_{CP}|^2}$$

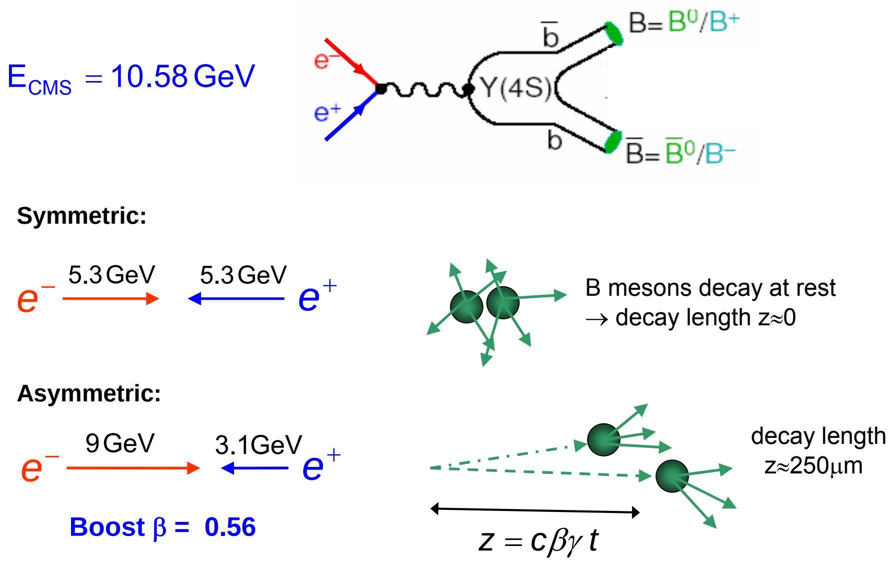
Interference
= $\sin 2\beta$ for $B^0 \to J/\psi K_S$

indicates direct CP violation
if $|q/p| \neq 1$

To measure CP violation in B_d system:

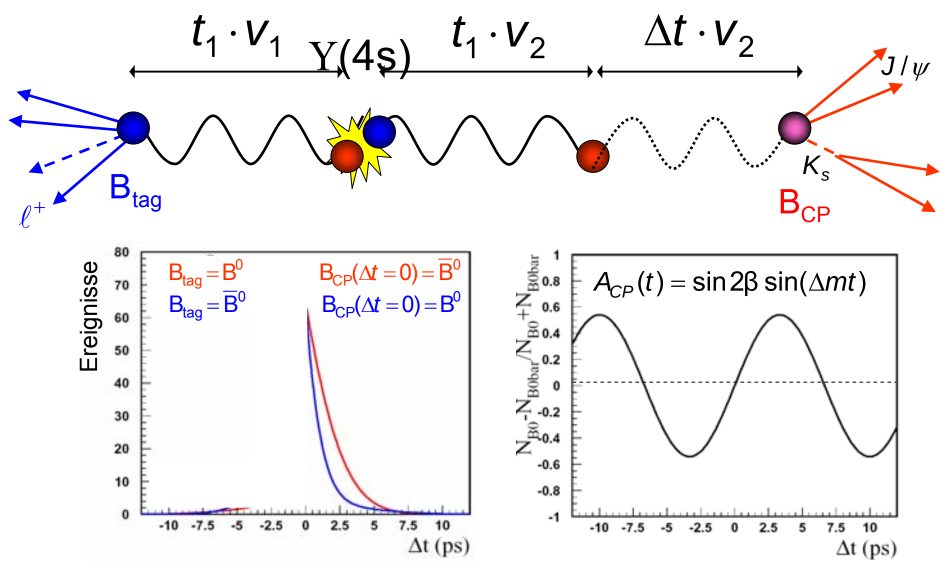
- Need many B (several 100 × 10^9)
- Need to know the flavor of the B at t=0
- Need to reconstruct the decay length to measure t

3.4 Measurement of sin2β: Asymmetric $e^+ e^-$ B factory

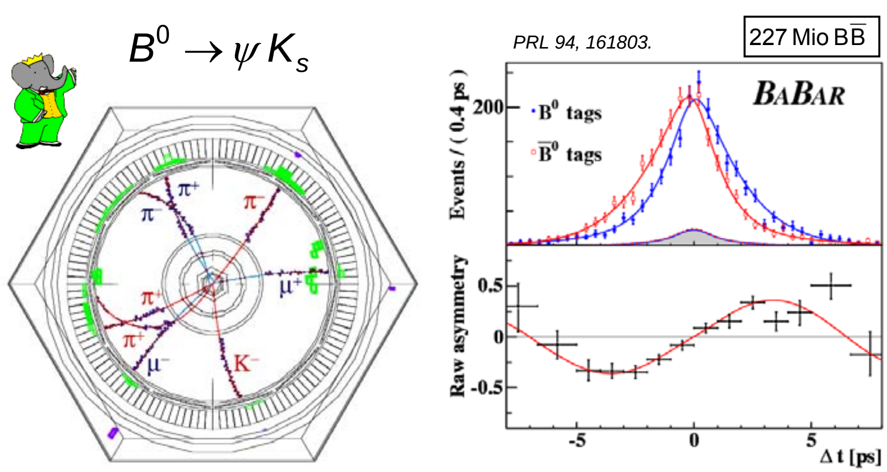


Standard Model: Flavor mixing and CP violation

Measurement of sin2β: Coherent oscillation



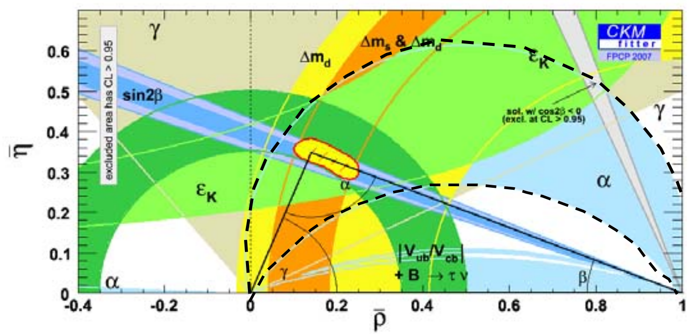
Measurement of sin2β: Golden decay channel



$A_{CP}(t) = \sin 2\beta \sin(\Delta m t)$

$\sin 2\beta = \ 0.722 \pm 0.040 \pm 0.023$

3.5 Experimental status of the Unitarity Triangle



Standard Model CKM mechanism confirmed

1. Large CP Violation in B decays
2. Large direct CP violation observed
3. CPV parameter related to magnitude of non-CP observables

A triple triumph