# Electroweak Theory and Higgs Mechanism

**1)** The electroweak Lagrangian reads

$$\mathcal{L}_{EW}(x) = -\frac{1}{4}W^a_{\mu\nu}W^{a\mu\nu} - \frac{1}{4}B_{\mu\nu}B^{\mu\nu} + \bar{\Psi}_e i\gamma^\mu D_\mu \Psi_e \,, \tag{1}$$

with $D_\mu = \partial_\mu \mathbb{1} + igW_\mu + ig'B_\mu Y$, $W_\mu = W^a_\mu T^a$, and $\Psi_e = (\psi_{\nu_e}, \psi_{e_L}, \psi_{e_R})$. The photon $A_\mu$ and the $Z^0$-boson are related to $W^3_\mu$ and $B_\mu$ with

$$W^3_\mu = \cos\theta_W \, Z^0_\mu + \sin\theta_W \, A_\mu \,, \qquad B_\mu = -\sin\theta_W \, Z^0_\mu + \cos\theta_W \, A_\mu \,. \tag{2}$$

**a)** Compute the electromagentic current $j^\mu_{em}$, and the neutral current $j^\mu_{nc}$, and relate $g$ and $g'$ to the electric charge $e$ and the coupling constant for $j^\mu_{nc}$.

**2)** The Higgs sector of the Standard model has the Lagrangian

$$\mathcal{L}_H(x) = D_\mu\phi^\dagger D^\mu\phi - h_e\left(\bar{\psi}_{e_R}\phi^\dagger\Psi_{e_L} + h.c.\right) - V(\phi^\dagger\phi) \,, \tag{3}$$

with Higgs potential $V$, and covariant derivative $D_\mu$,

$$V(\phi^\dagger\phi) = \mu^2\phi^\dagger\phi + \lambda(\phi^\dagger\phi)^2 \,, \qquad D_\mu = \partial_\mu\mathbb{1} + igW_\mu + ig'B_\mu Y_H \tag{4}$$

and $W_\mu = W^a_\mu\sigma^a/2$ with Pauli matrices $\sigma^a$, and $Y_H = \frac{1}{2}\mathbb{1}_2$. The Higgs field $\phi$ can be parameterised as

$$\phi(x) = U(x)\begin{pmatrix} 0 \\ \frac{1}{\sqrt{2}}(v+\rho(x)) \end{pmatrix} \,, \tag{5}$$

with vacuum expectation value $v$, and $U(x) \in SU(2)$.

**a)** Show that

$$\phi_0(x) = U(x)(0, \tfrac{1}{\sqrt{2}}v) \tag{6}$$

is a minimum of the Higgs potential $V$, and prove that $\exp\left(i\omega(\sigma^3/2 + Y_H\right)$ leaves $\phi_0(x)$ invariant.

**b)** Show that $U(x)$ can be absorbed in a gauge transformation of gauge fields and fermions, and rewrite the Lagrangian density in terms of the gauge transformed fields, the radial field $\rho(x)$, and $v$.

**3**$^*$) Write down the one loop corrections to the Higgs mass, and estimate their size.