Weak interaction

1. Phenomenology of weak decays
2. Parity violation and neutrino helicity
3. V-A theory
4. Neutral currents

The weak interaction **was** and **is a topic** with a lot of **surprises:**

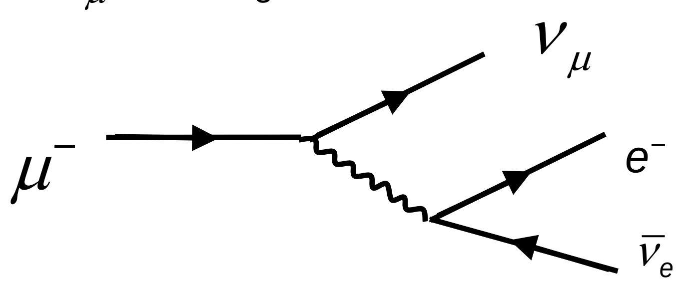
Past: Flavor violation, P and CP violation.
Today: Weak decays used as probes for new physics

1. Phenomenology of weak decays

All particles (except photons and gluons) participate in the weak interaction. At small q^2 weak interaction can be shadowed by strong and electro-magnetic effects.

- Observation of weak effects only possible if strong/electro-magnetic processes are forbidden by conservation laws.
- Today's picture for charge current interaction is the **exchange** of **massive W-bosons** coupling only to **left-handed fermion currents**

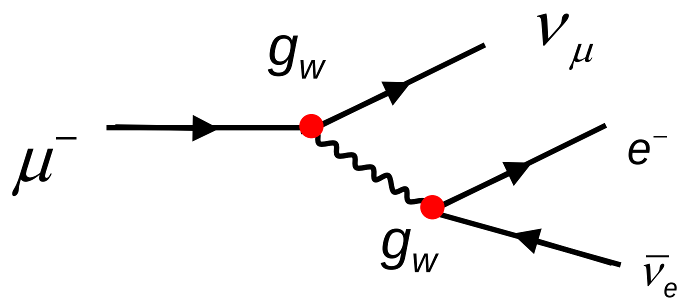
$$\mu^- \to \bar{\nu}_\mu + e^- \bar{\nu}_e$$



$$\tau = 2.6 \cdot 10^{-6}\,\mathrm{s}$$

Electromagnetic decay $\mu^- \to e^- \gamma$ forbidden by lepton number conservation

Application of Feynman-rules for massive W boson and LH coupling:



$$\mathcal{M} = (-ig_w)^2\left(\bar{u}_\nu\gamma^\mu\underbrace{(1-\gamma^5)}_{\mathbf{P}_L}u_\mu\right)\left(\frac{-g_{\mu\nu}+\dfrac{q_\mu q_\nu}{M_W^2}}{q^2-M_W^2}\right)\left(\bar{u}_e\gamma^\nu\underbrace{(1-\gamma^5)}_{\mathbf{P}_L}v_\nu\right)$$

“weakness” result of $(1/M_w)^2$ suppression

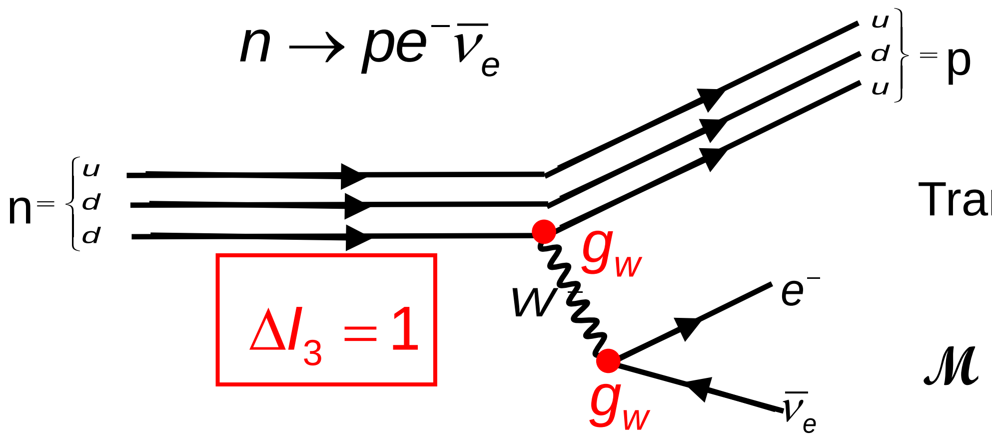
Calculation is straight forward ... (everything known!)

Today's picture of the β-decay

- Nucleons are composed of quarks, which are the fundamental fermions. Fundamental forces couple to the the quarks .

Using the the "quark level" decay one can describe weak hadron decays (treating the quarks which are not weakly interacting as spectators)

$$n \rightarrow p e^- \bar{\nu}_e$$



Currents:
V-A structure

Transition matrix:

$$\mathcal{M} = (-ig_w)^2 J^{\mu}_{quark} \left(\frac{-g^{\mu\nu} + \frac{q_\mu q_\nu}{M_W^2}}{q^2 - M_W^2} \right) J^{\nu}_{lepton}$$

Strong isospin I_3 not conserved. All other quark flavor numbers also violated.

Motivation of massive boson exchange:

- Long range electromagnetic force mediated by massless photon:

Potential: $$V_{Coulomb}(r) = \frac{e^2}{4\pi r} \longrightarrow \text{infinite range}$$

- Replace massless photon by massive W boson for weak interaction:

$$\longrightarrow \quad V_{Yukawa}(r) = \frac{g^2}{4\pi r} \cdot e^{-M_w r}$$

Yukawa potential:
Screened Coulomb potential

Can be verified by using Fourier transform of propagtor (Greens Funktion of Klein-Gordon Eq.)

$$V_{Yukawa}(\vec{r}) = \frac{g^2}{(2\pi)^3} \int e^{i\vec{k}\cdot\vec{r}} \frac{4\pi}{q^2 - M_W^2} d\vec{k}$$

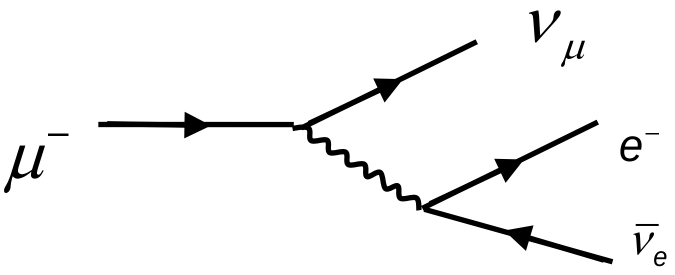
For $M_w \to \infty$:
or $M_w >> |q|$

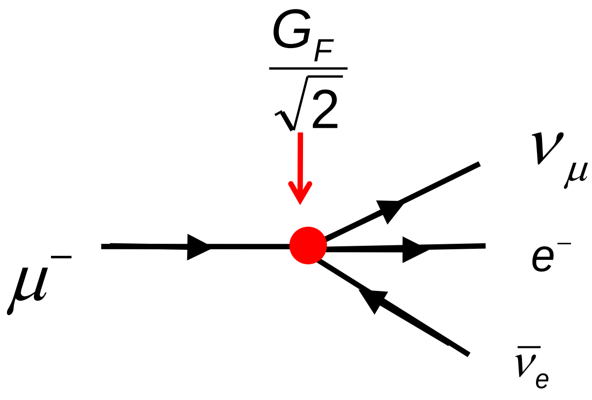
$$V_{Yukawa}(r) \quad \to \quad \frac{g^2}{4\pi r} \cdot \delta(\vec{r})$$

point-coupling

Non local current – current coupling

Point-like 4-fermion interaction





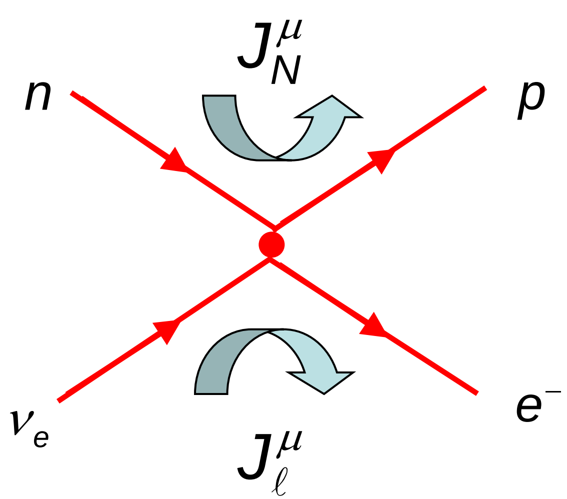
Fermi coupling constant,
dimension = $(1/M)^2$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} \left(\bar{u}_{\nu,L} \gamma^\mu u_{\mu,L} \right) \left(\bar{u}_{e,L} \gamma^\mu (1-\gamma^5) v_{\nu,L} \right)$$

- 4-fermion theory is an **effective theory** valid for small q^2. Gives reliable results for most low energy problems.
- Conceptual problems in the high-energy limit (see later)
- Introduced by Fermi in 1933 to explain nuclear β decay.

Fermi's treatment of nuclear β-Decay: $n \to p\, e^- \, \bar{\nu}_e$

Fermi's explanation (1933/34) of the nuclear β-decay:



Two fermionic vector currents coupled by a **weak coupling const.** at single point (4-fermion interact.)

Apply "Feynman Rules"

$$\boxed{M = \frac{G_F}{\sqrt{2}} \cdot J_{N,\mu} \cdot J_e^{\mu^+} = \frac{G_F}{\sqrt{2}} \cdot \left(\bar{u}_p \gamma_\mu u_n\right) \cdot \left(\bar{u}_e \gamma_\mu v_\nu\right)}$$

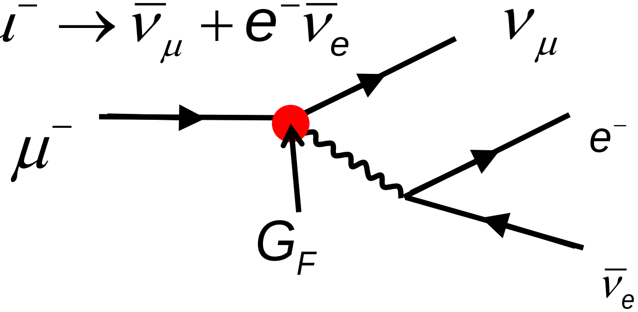
Weak coupling constant G_F is a very small number ~10^{-5} GeV^{-2}.
Explains the "weakness" of the force.

Fermi's ansatz was inspired by the structure of the electromagnetic interaction and the fact that there is essentially no energy dependence observed.

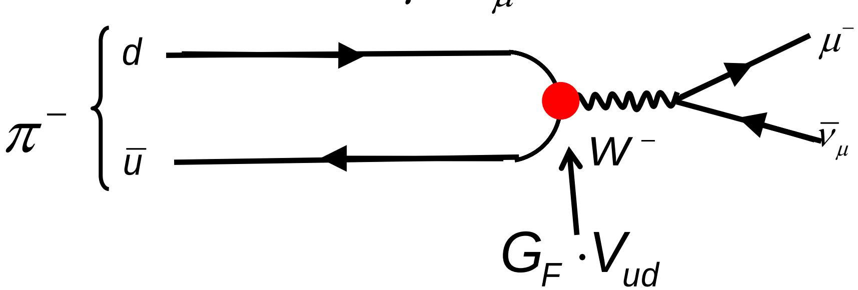
Problem: Ansatz cannot explain parity violation (was no a problem in 1933)

Universality of weak coupling constant:

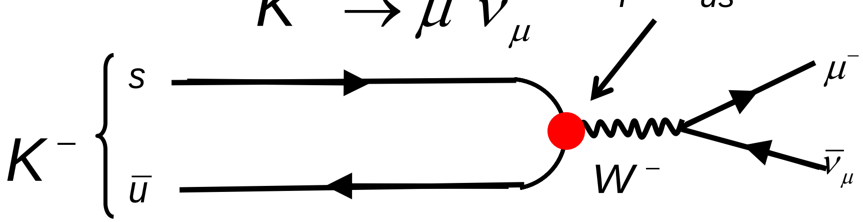
$\mu^- \to \bar{\nu}_\mu + e^- \bar{\nu}_e$

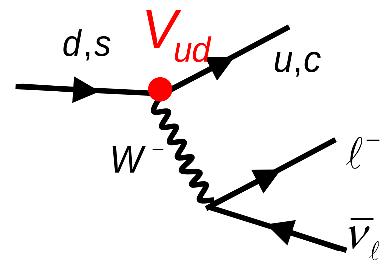


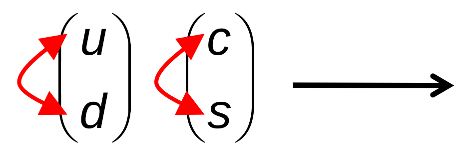
$\pi^- \to \mu^- \bar{\nu}_\mu$



$K^- \to \mu^- \bar{\nu}_\mu$

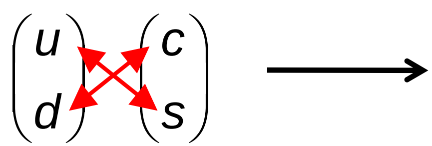






$G_F \cdot V_{ud}$

$G_F \cdot V_{cs}$



$G_F \cdot V_{us}$

$G_F \cdot V_{cd}$

If one considers the quark mixing the weak coupling constant G_F is universal.

2. Parity violation

$$x'^{\mu} = \Lambda_{P\ \nu}^{\ \mu}\, x^{\nu}$$

Parity transformations (P) = space inversion

$$\Lambda_P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

P transformation properties: $P:$

$$\vec{r} \rightarrow -\vec{r}$$

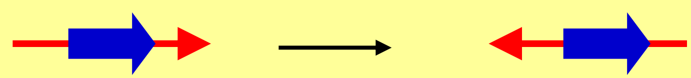
$$t \rightarrow t$$

$$\vec{p} \rightarrow -\vec{p}$$

$$\vec{\ell} = \vec{r} \times \vec{p} \rightarrow \vec{\ell}$$ Axial/pseudo vector

e.g.: Helicity operator

$$H = \frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \xrightarrow{P} -\frac{\vec{s} \cdot \vec{p}}{|\vec{p}|} \quad \text{(pseudo-scalar)}$$



H=+1/2 **P** H=-1/2

Experimentally:

$\Leftrightarrow$ mirroring at plane + rotation around axis perpendicular to plane

$\Rightarrow$ To test parity it is sufficient to study the process in the "mirrored system": physics invariant under rotation

2.1 Historical θ/τ puzzle (1956)

Until 1956 parity conservation as well as T and C symmetry was a “dogma”:
$\rightarrow$ very little experimental tests done

In 1956 Lee and Yang proposed parity violation in weak processes.

Starting point: Observation of two particles $\underbrace{\boldsymbol{\theta^+} \text{ and } \boldsymbol{\tau^+}}^{\text{Historical names}}$ with exactly equal mass, charge and strangeness **but** with different parity:

$$\theta^+ \rightarrow \pi^+\pi^0 \quad w/ \quad P(\theta^+) = P(\pi)^2(-1)^\ell \rightarrow J^P(\theta^+) = 0^+ \qquad P(\pi)=-1$$

$$\tau^+ \rightarrow \pi^+\pi^+\pi^- \qquad P(\tau^+) = P(\pi)^3(-1)^{2\ell} \rightarrow J^P(\tau^+) = 0^-, 2^-$$

Lee + Yang: θ^+ and τ^+ same particle, but decay violates parity

$\Rightarrow$ today, particle is called K^+:

$$K^+(0^-) \rightarrow \pi^+\pi^0 \qquad \text{P is violated}$$

$$K^+(0^-) \rightarrow \pi^+\pi^+\pi^- \qquad \text{P is conserved}$$

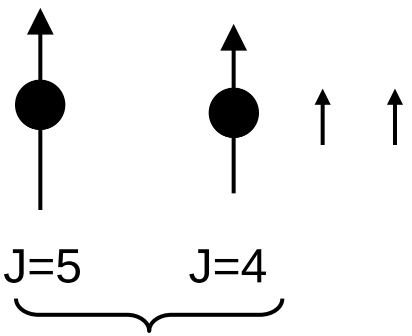
To search for possible P violation, a number of experimental tests of parity conservation in weak decays has been proposed:

1957 Observation of P violation in nuclear β decays by Chien-Shiung Wu et al.

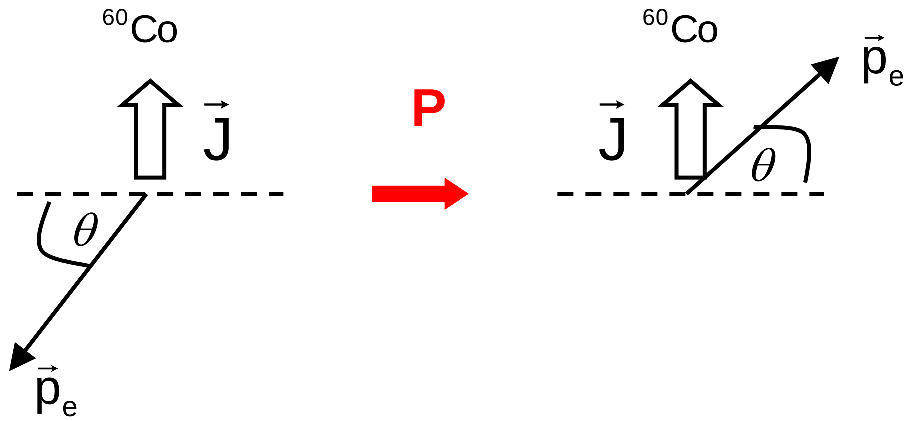
2.2 Observation of parity violation, C.S. Wu et al. 1957

Idea: Measurement of the angular distribution of the emitted e^- in the decay of polarized ^{60}Co nuclei

$$^{60}\mathrm{Co} \rightarrow {}^{60}\mathrm{Ni}^* + e^- + \bar{\nu}_e$$



Gamov-Teller Transition ΔJ=1



Observable: $\vec{J}\cdot\vec{p}_e \rightarrow -\vec{J}\cdot\vec{p}_e$

If P is conserved, the angular distribution must be symmetric in θ (symmetric to dashed line): transition rates for $\vec{J}\cdot\vec{p}_e$ and $-\vec{J}\cdot\vec{p}_e$ are identical.

Experiment: Invert Co polarization and compare the rates at the same position θ.

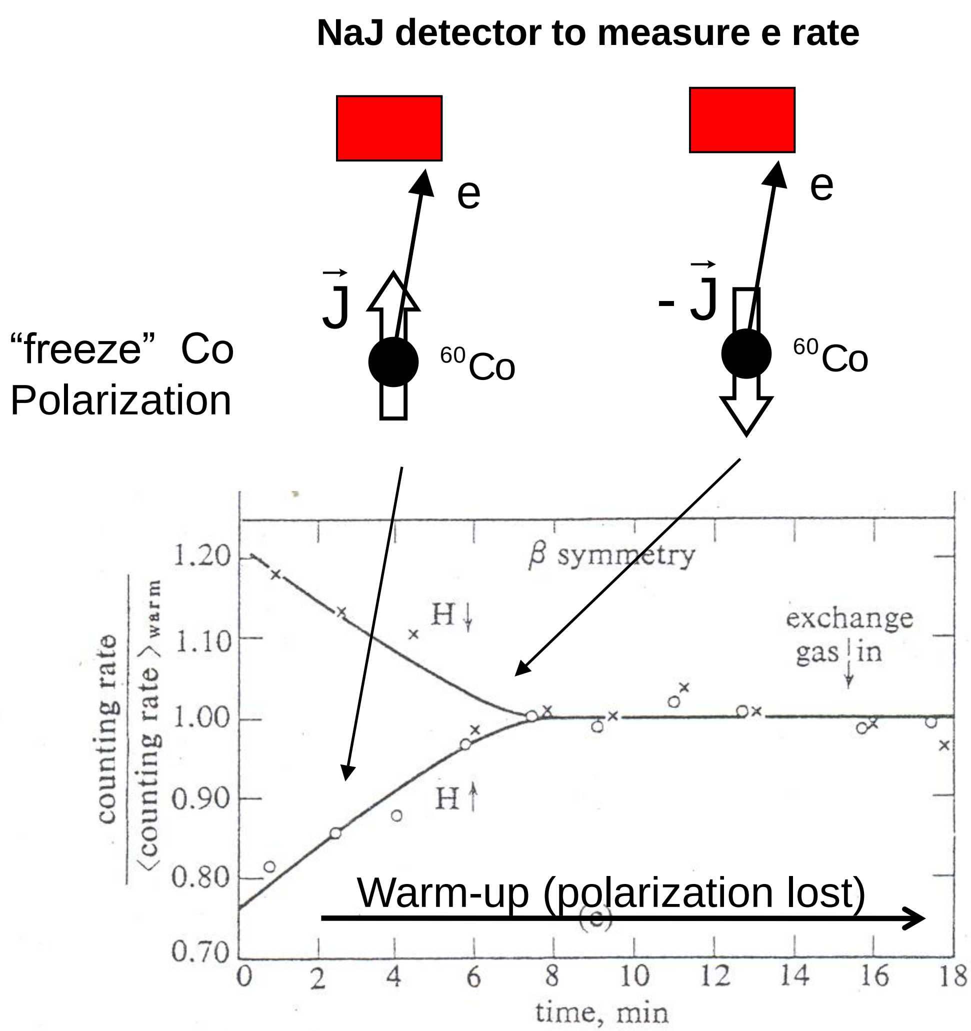


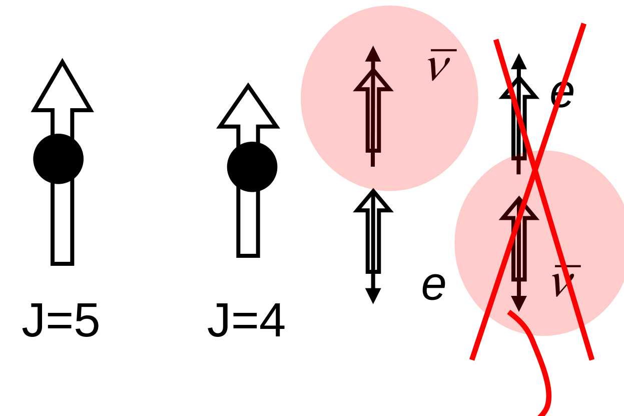
Figure 9-12 Gamma anisotropy (as determined from the two NaI counters) and beta asymmetry for the polarizing field pointing up and down as a function of time. The times for disappearance of the beta and gamma asymmetry coincide; this is the warm-up time. The warm-up time for the sample is approximately 6 min and the counting rates for the warm unpolarized sample are independent of the field direction. [From C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *Phys. Rev.*, **105**, 1413 (1957).]

Result:

Electron rate opposite to Co polarization is higher than along the ^{60}Co polarization:

parity violation

Qualitative explanation:



Today we know:

Consequence of existence of only left-handed (LH) neutrinos (RH anti-neutrinos)

Electron polarization in β decays

$$H_{e^-} = -\frac{1}{2}\frac{v}{c}$$

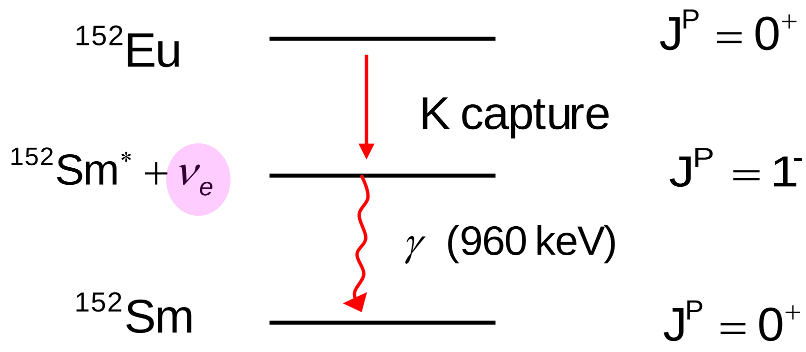
2.3 Determination of the neutrino helicity

Goldhaber et al., 1958

Indirect measurement of the neutrino helicity in a K capture reaction:

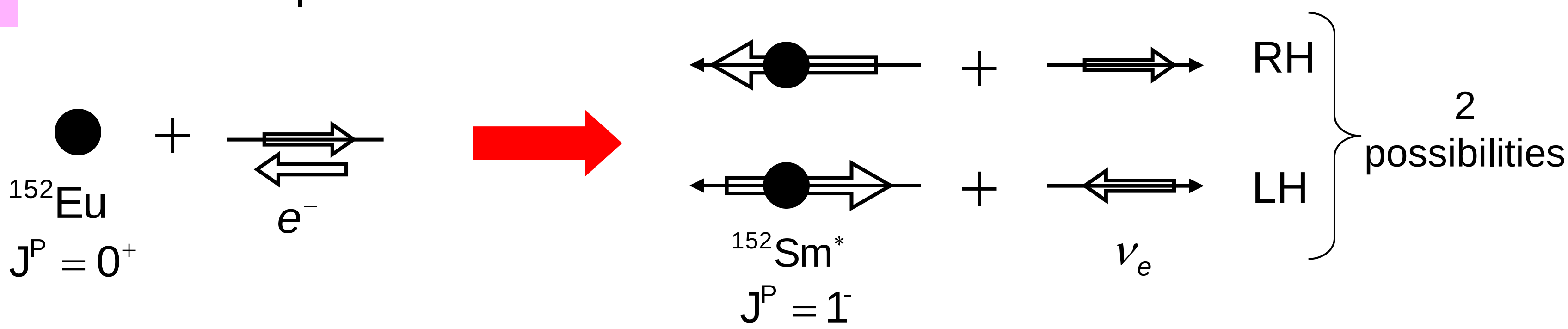
$$^{152}\text{Eu} + e^- \rightarrow \ ^{152}\text{Sm}^* + \nu_e$$

$$E_\nu \approx 950\,\text{KeV}$$



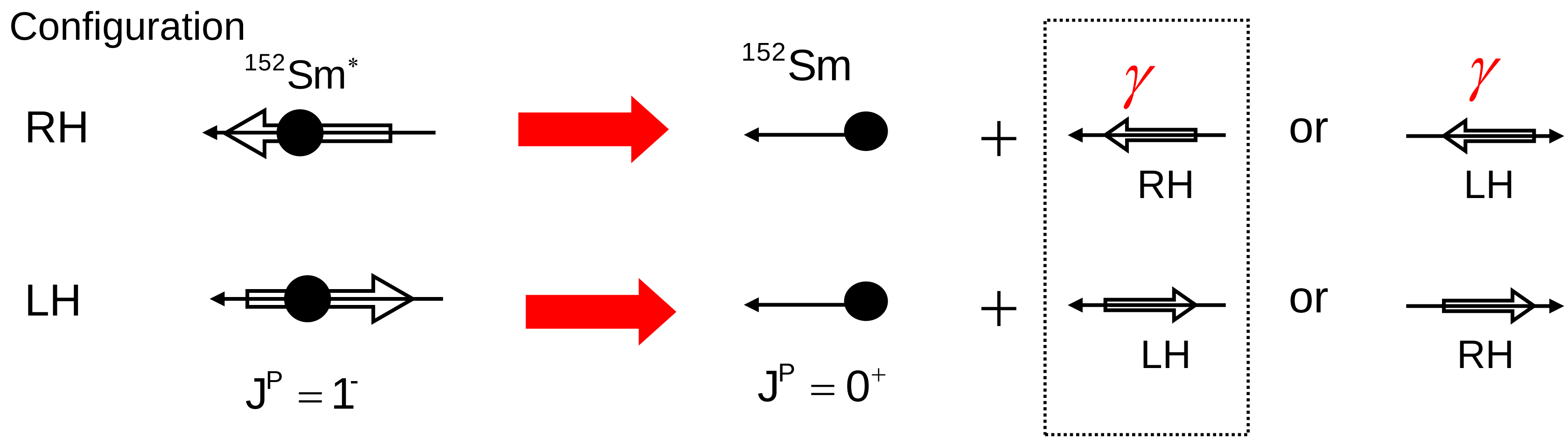
<u>Idea of the experiment:</u>

1. Electron capture and ν emission



Sm undergoes a small **recoil (p_{recoil} =950 KeV)**. Because of angular momentum conservation Spin J=1 of Sm* is opposite to neutrino spin. Important: **neutrino helicity is transferred to the Sm nucleous.**

2. γ emission: ${}^{152}\mathrm{Sm}^*(J^P = 1^-) \rightarrow {}^{152}\mathrm{Sm}(J^P = 0^+) + \gamma$



Photons along the Sm recoil direction carry the polarization of the Sm* nucleus

- How to select photons along the recoil direction ? $\Rightarrow$ 3
- How to determine the polarization of these photons ? $\Rightarrow$ 4

3. Resonant photon scattering: $\gamma + {}^{152}\text{Sm} \rightarrow {}^{152}\text{Sm}^* \rightarrow {}^{152}\text{Sm} + \gamma$

<u>Resonant scattering:</u>

To compensate the nuclear recoil, the photon energy must be slightly larger than 960 keV.

This is the case for photons which have been emitted in the direction of the Eu→Sm recoil (Doppler-effect).

Resonant scattering only possible for “forward” emitted photons, which carry the polarization of the Sm* and thus the polarization of the neutrinos.

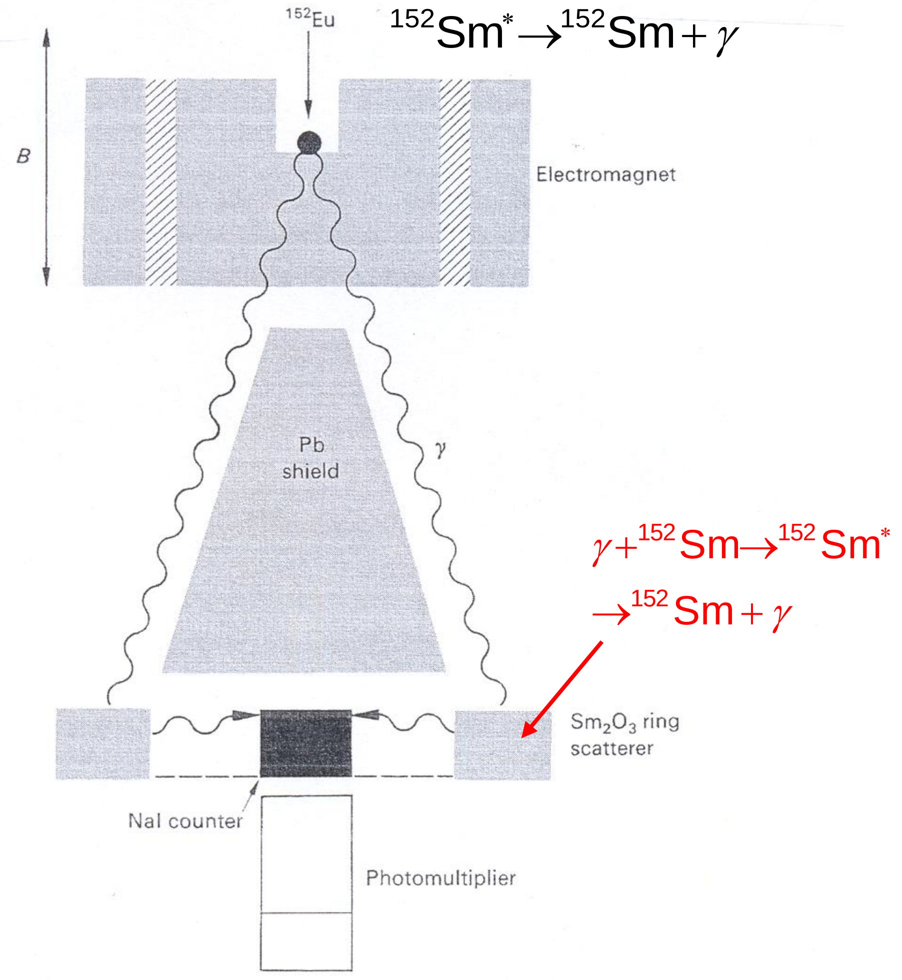
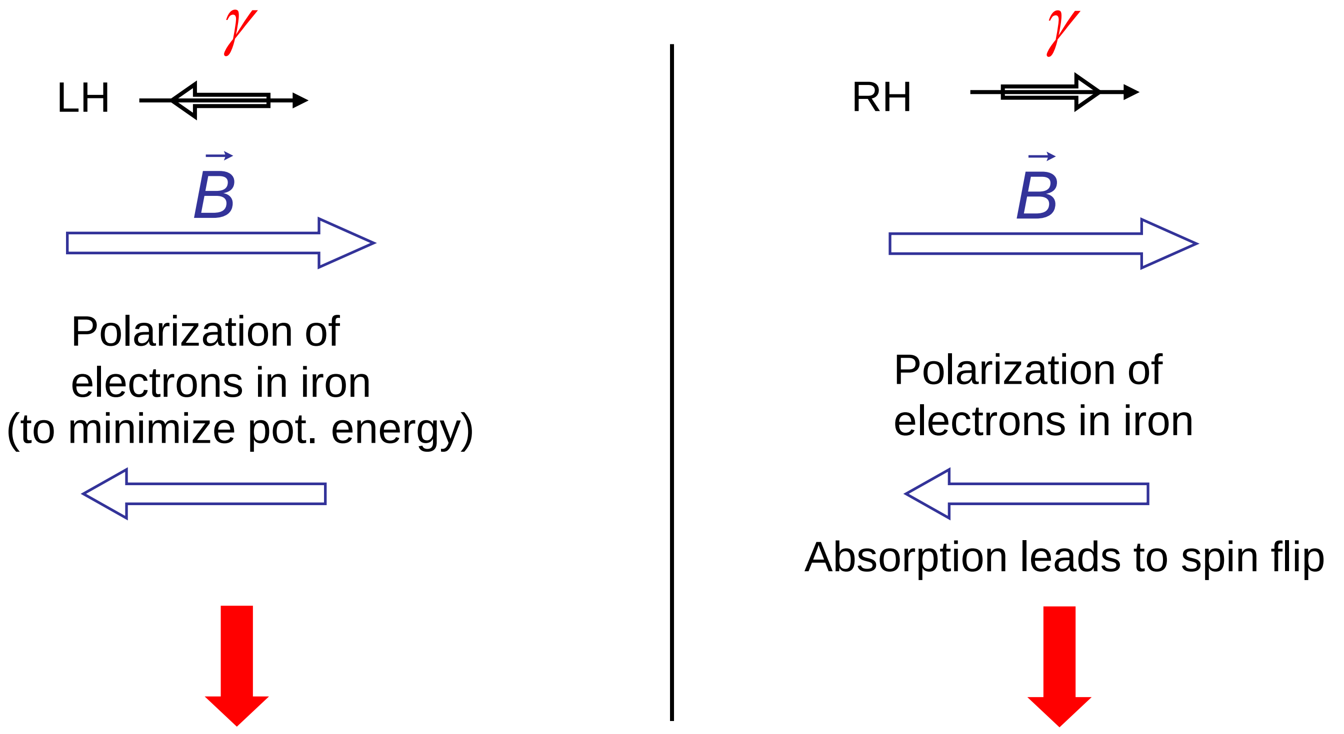


Fig. 7.8. Schematic diagram of the apparatus used by Goldhaber *et al.*, in which γ-rays from the decay of $^{152}\text{Sm}^*$, produced following K-capture in ^{152}Eu, undergo resonance scattering in Sm_2O_3 and are recorded by a sodium iodide scintillator and photomultiplier. The transmission of photons through the iron surrounding the source depends on their helicity and the direction of the magnetic field **B**.

4. Determination of the photon polarization

Exploit that the transmission index through magnetized iron is polarization dependent: <u>Compton scattering in magnetized iron</u>



LH photons cannot be absorbed:

Good transmission

RH photons undergo Compton scattering: Bad transmission

Photons w/ polarization anti-parallel to magnetization undergo less absorption

Experiment

Sm^* emitted photons pass through the magnetized iron. Resonant scattering allows the photon detection by a NaJ scintillation counter. The counting rate difference for the two possible magnetizations measures the polarization of the photons and thus the helicity of the neutrinos.

Results:

$$P_\gamma = -0.66 \pm 0.14$$

$\rightarrow$ photons from Sm^* are left-handed. The measured photon polarization is compatible with a neutrino helicity of H=–1/2.

From a calculation with 100% photon polarization one expects a measurable value P_γ~0.75. Reason is the finite angular acceptance.
$\rightarrow$ Also not exactly forward-going γ's can lead to resonant scattering.

Summary: Lepton polarization in β decays

	e^-	e^+	ν	$\bar{\nu}$
$H=\frac{1}{2}\cdot$	-v/c	+v/c	-1	+1

3. "V-A Theory" for charged current weak interactions

3.1 Lorentz structure of the weak currents

Fermi:

$$M = C_V \, J_{N,\mu} \cdot J_e^{\mu^+} = C_V \left(\bar{u}_p \gamma_\mu u_n\right) \cdot \left(\bar{u}_e \gamma_\mu v_\nu\right)$$

Cannot explain the parity violation in beta decays.
(Treats LH and RH current components the same).

More general ansatz:
(proposed by Gamov & Teller)

$$M = \sum_i C_i \left(\bar{u}_p \Gamma_i u_n\right) \cdot \left(\bar{u}_e \Gamma_i v_\nu\right)$$

$$i = \mathrm{S, P, V, A, T}$$

$$\bar{u}_p \; \Gamma_i \; u_n$$

bilinear Lorentz covariants:

$$\bar{\psi} \; (4 \times 4) \psi$$

$$\bar{u}_p \, \Gamma_i \, u_n = \begin{cases} S: & \bar{u}_p u_n & \text{scalar} \\ P: & \bar{u}_p \gamma^5 u_n & \text{pseudo-scalar} \\ V: & \bar{u}_p \gamma^\mu u_n & \text{vector} \\ A: & \bar{u}_p \gamma^5 \gamma^\mu u_n & \text{pseudo-vector} \\ T: & \bar{u}_p \sigma^{\mu\nu} u_n & \text{tensor} \end{cases}$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu\right)$$

$$\gamma^5 = i\,\gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \quad \Rightarrow \quad \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\left(\gamma^5\right)^2 = 1 \qquad \gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0$$

Remark: Pure P or A couplings do not lead to observable parity violation!

Mixtures like $(1 \pm \gamma^5)$ or $\gamma^\mu (1 \pm \gamma^5)$ do violate parity.

3.2 Chirality

Operators

$$\boxed{\begin{aligned} P_L &\equiv \frac{1}{2}(1-\gamma^5) \\ P_R &\equiv \frac{1}{2}(1+\gamma^5) \end{aligned}}$$

are projection operators:

$$(P_i)^2 = P_i \ , \quad P_L + P_R = 1, \quad P_L P_R = 0$$

(properties of γ^5)

working on the fermion spinors they result in the left / right handed chirality components:

$$\left.\boxed{\begin{aligned} u_L &= \frac{1}{2}(1-\gamma^5)u \\ u_R &= \frac{1}{2}(1+\gamma^5)u \end{aligned}}\ \right\}$$

Not observable!

In contrary to helicity, which is an observable: $\frac{1}{2}\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}$

Dirac spinors → Eigenvectors of helicity operator

Specific representation of Dirac spinors

solution spin$\uparrow$ i.e. helicity $\lambda = +\frac{1}{2}$

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + i p_y}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix}$$

solution spin$\downarrow$ i.e. helicity $\lambda = -\frac{1}{2}$

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ \frac{p_x - i p_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix}$$

$$\frac{1}{2}\frac{\Sigma^k p^k}{|\vec{p}|} u_1 = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} u_1 = \frac{1}{2} u_1$$

$$\frac{1}{2}\frac{\Sigma^k p^k}{|\vec{p}|} u_1 = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} u_1 = -\frac{1}{2} u_2$$

Dirac spinors and chirality projection operators:

Positive helicity:

$$\frac{1-\gamma^5}{2}u_1 = \frac{1}{2}\underbrace{\sqrt{E+m}\cdot\left(1-\frac{p}{E+m}\right)}_{\approx 0 \text{ for E} >> m}\cdot\begin{pmatrix}1\\0\\-1\\0\end{pmatrix} \quad \to 0 \quad \text{for E} >> m$$

Negative helicity:

$$\frac{1-\gamma^5}{2}u_2 = \frac{1}{2}\underbrace{\sqrt{E+m}\cdot\left(1+\frac{p}{E+m}\right)}_{\approx \sqrt{E} \quad \text{for E} >> m}\cdot\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \quad \to u_2 \quad \text{for E} >> m$$

In the **relativistic limit** helicity states are also eigenstates of the chirality operators.

Polarization for particles with finite mass

Left handed spinor component of unpolarized electron:

$$u_1, u_2 \rightarrow u_L, u_R$$

$$u_L = \frac{1-\gamma^5}{2}\underbrace{(u_1+u_2)}_{\text{unpolarized}}$$

Not normalized

Helicity polarization of left handed chirality state u_L :

$$Pol = \frac{P(\lambda=+1/2)-P(\lambda=-1/2)}{P(\lambda=+1/2)+P(\lambda=-1/2)} = \frac{\left|\langle u_1|u_L\rangle\right|^2 - \left|\langle u_2|u_L\rangle\right|^2}{\left|\langle u_1|u_L\rangle\right|^2 + \left|\langle u_2|u_L\rangle\right|^2}$$

$$= \frac{\left(1-p/(E+m)\right)^2 - \left(1+p/(E+m)\right)^2}{\left(1-p/(E+m)\right)^2 + \left(1+p/(E+m)\right)^2} = -\frac{p}{E} = -\frac{v}{c}$$

i.e. the LH spinor component for a particle with finite mass is not fully in the helicity state “spin down” (λ=-½) .

For massive particles of a given chirality there is a finite probability to observe the “wrong” helicity state: $\mathcal{P}$ = [1- (p/E)]/2

Neutrino-Electron Vertex

Only left-handed neutrinos are observed in beta decays: $$u_\nu \rightarrow \left(\frac{1-\gamma^5}{2}\right) u_\nu$$

This leads to the following electron-neutrino vertex
(assuming vector coupling between LH neutrino and e): $$\bar{u}_e \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u_\nu$$

If one further exploits that $P_l = 1/2(1 - \gamma^5)$ is a projection operator one finds:

$$\bar{u}_e \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u_\nu = \bar{u}_e \gamma^\mu \left(\frac{1-\gamma^5}{2}\right)^2 u_\nu = u_e^+ \left(\frac{1-\gamma^5}{2}\right) \gamma^0 \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u_\nu = \overline{(u_e)_L} \gamma^\mu (u_\nu)_L$$

The left-handed neutrino thus couples only to left-handed electrons (vector current).

V-A structure: $$\bar{u}_e \gamma^\mu \left(\frac{1-\gamma^5}{2}\right) u_\nu = \frac{1}{2} \bar{u}_e \left(\gamma^\mu - \gamma^\mu \gamma^5\right) u_\nu$$

V - A (vector – axial-vector)