2.3 Number of light neutrino generations

In the Standard Model:

$$\Gamma_Z = \Gamma_{had} + 3 \cdot \Gamma_\ell + \underbrace{N_\nu \cdot \Gamma_\nu}_{\text{invisible }: \Gamma_{inv}} \rightarrow \begin{cases} e^+e^- \to Z \to \nu_e \bar{\nu}_e \\ e^+e^- \to Z \to \nu_\mu \bar{\nu}_\mu \\ e^+e^- \to Z \to \nu_\tau \bar{\nu}_\tau \end{cases}$$

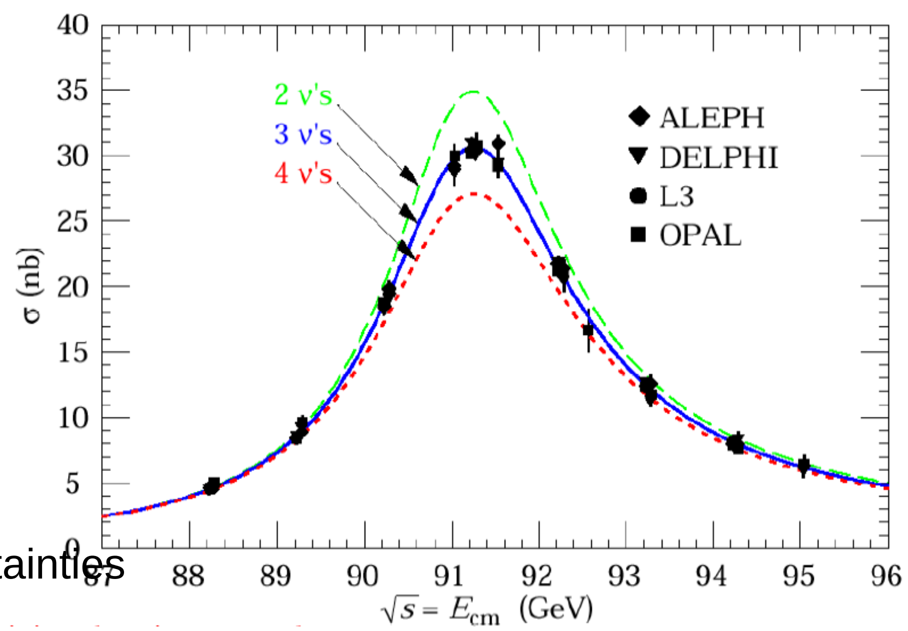
$$\Gamma_{inv} = 0.4990 \pm 0.0015 \text{ GeV}$$

To determine the number of light neutrino generations:

$$N_\nu = \frac{\Gamma_{inv}}{\Gamma_{\nu,SM}} = \underbrace{\left(\frac{\Gamma_{inv}}{\Gamma_\ell}\right)_{exp}}_{5.9431 \pm 0.0163} \cdot \underbrace{\left(\frac{\Gamma_\ell}{\Gamma_\nu}\right)_{SM}}_{=1/1.991 \pm 0.001}$$

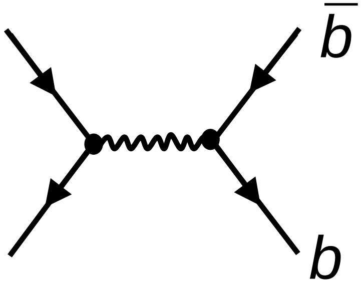
(small theo. uncertainties from m_{top} M_H)

$$N_\nu = 2.9840 \pm 0.0082$$

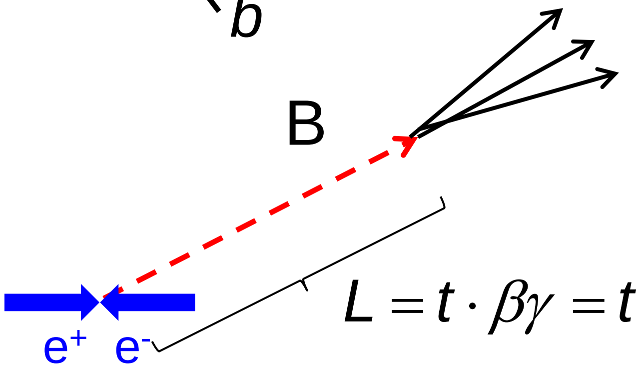


No room for new physics: Z→new

Heavy Quark production



$$R_b = \frac{\Gamma_b}{\Gamma_{had}}$$

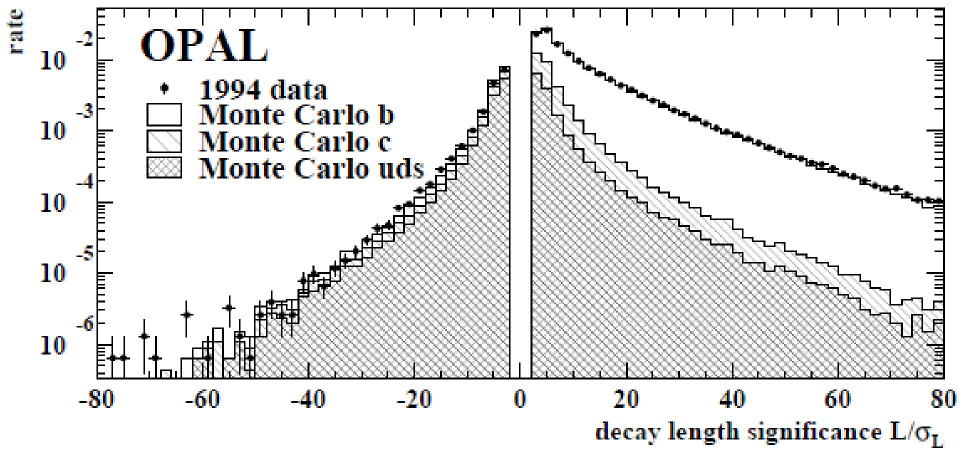


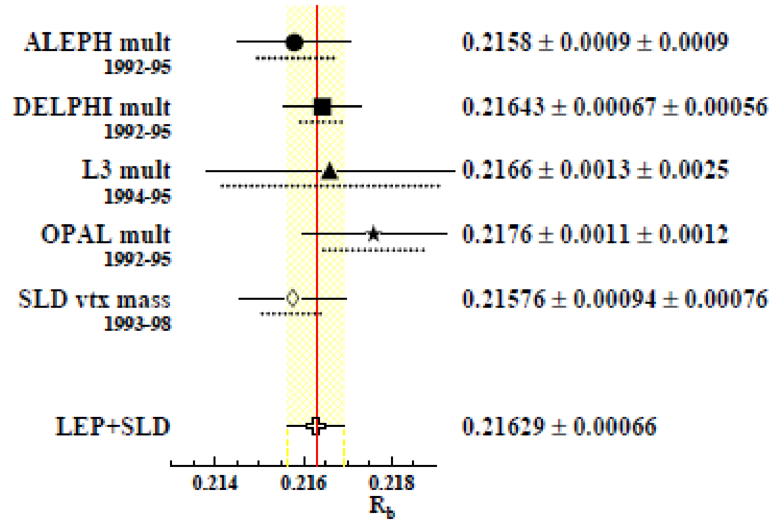
$$L = t \cdot \beta\gamma = t \cdot \frac{p_B}{m_B}$$

Identification of b-Quark events:

b-quarks hadronize to b-hadrons (B's, Λ_b) with typical lifetime of ~ 1 ps $\rightarrow$ decay length

Use displaced "2nd" B decay vertex as signature.





Significance = L / error

2.4 Lepton couplings to the Z boson

In the following ignore the difference between chirality and helicity: good approximation as leptons are produced with energies >> mass.

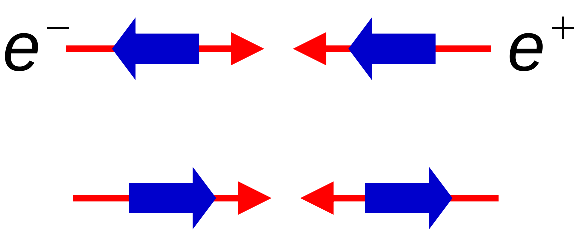
Z boson couples differently to LH and RH leptons:

$$\left| g_L = \frac{1}{2}(g_V + g_A) \right| \quad > \quad \left| g_R = \frac{1}{2}(g_V - g_A) \right|$$

Coupling to LH leptons stronger

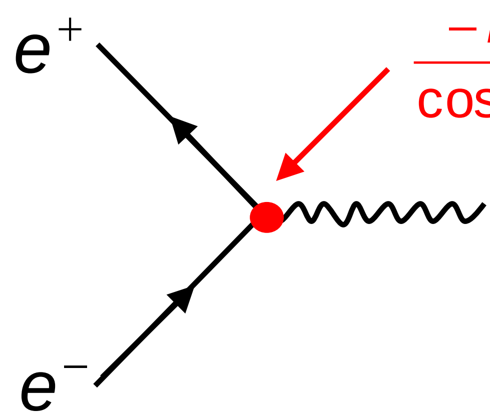
Z produced in e+e- collisions is polarized.

Experimental configuration:



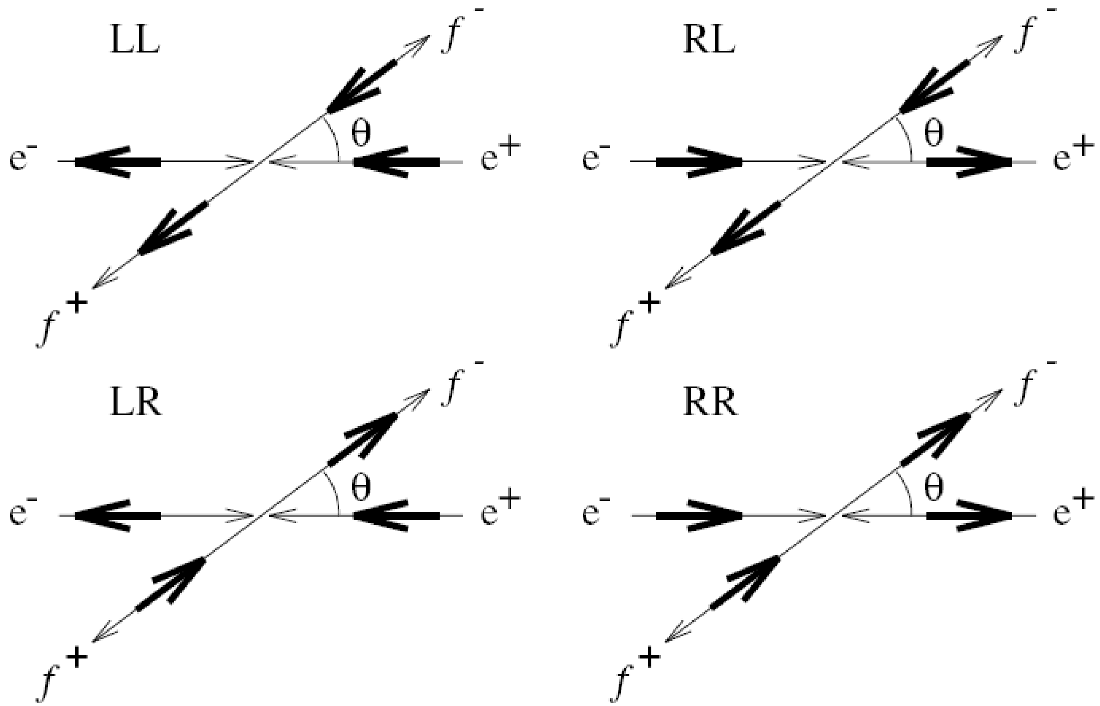
$\Rightarrow \quad g_L$

$\Rightarrow \quad g_R$

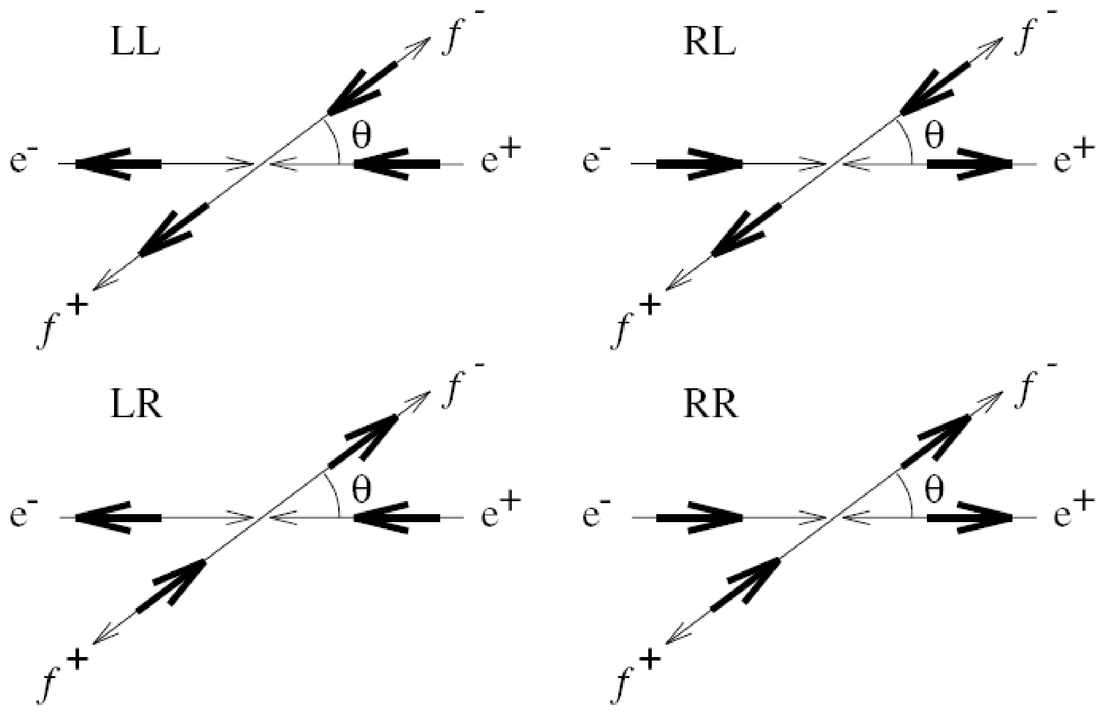


$$\frac{-ig}{\cos\theta_W}\gamma^\mu \frac{1}{2}\left(g_R^e + g_L^e\right)$$

Instead of measuring the spin averaged transition amplitudes try to decompose the different "helicity" components to the cross section:



Chirality e	Chirality f	amplitude	
L	L	$\mathcal{M}_{\mathrm{LL}} \propto g_L^f g_L^{\mathrm{e}} d_{11}^1(\theta)$	$\propto g_L^f g_L^{\mathrm{e}} (1 + \cos\theta)$
R	L	$\mathcal{M}_{\mathrm{RL}} \propto g_L^f g_R^{\mathrm{e}} d_{11}^1(\theta + \pi)$	$\propto g_L^f g_R^{\mathrm{e}} (1 - \cos\theta)$
L	R	$\mathcal{M}_{\mathrm{LR}} \propto g_R^f g_L^{\mathrm{e}} d_{11}^1(\theta + \pi)$	$\propto g_R^f g_L^{\mathrm{e}} (1 - \cos\theta)$
R	R	$\mathcal{M}_{\mathrm{RR}} \propto g_R^f g_R^{\mathrm{e}} d_{11}^1(\theta)$	$\propto g_R^f g_R^{\mathrm{e}} (1 + \cos\theta)$



Observables:

$\sigma_F = \sigma_{LL} + \sigma_{RR}$ $\sigma_B = \sigma_{RL} + \sigma_{LR}$

$$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

Forward-backward asym. (final)

$\sigma_L = \sigma_{LL} + \sigma_{LR}$ $\sigma_R = \sigma_{RL} + \sigma_{RR}$

$$A_{LR} = \frac{\sigma_L - \sigma_R}{\sigma_L + \sigma_R}$$

Left right asym. (initial)

$\sigma_- = \sigma_{LL} + \sigma_{RL}$ $\sigma_+ = \sigma_{RR} + \sigma_{LR}$

$$\mathcal{P}_f = \frac{\sigma_+ - \sigma_-}{\sigma_+ + \sigma_-}$$

fermion polarization (final)

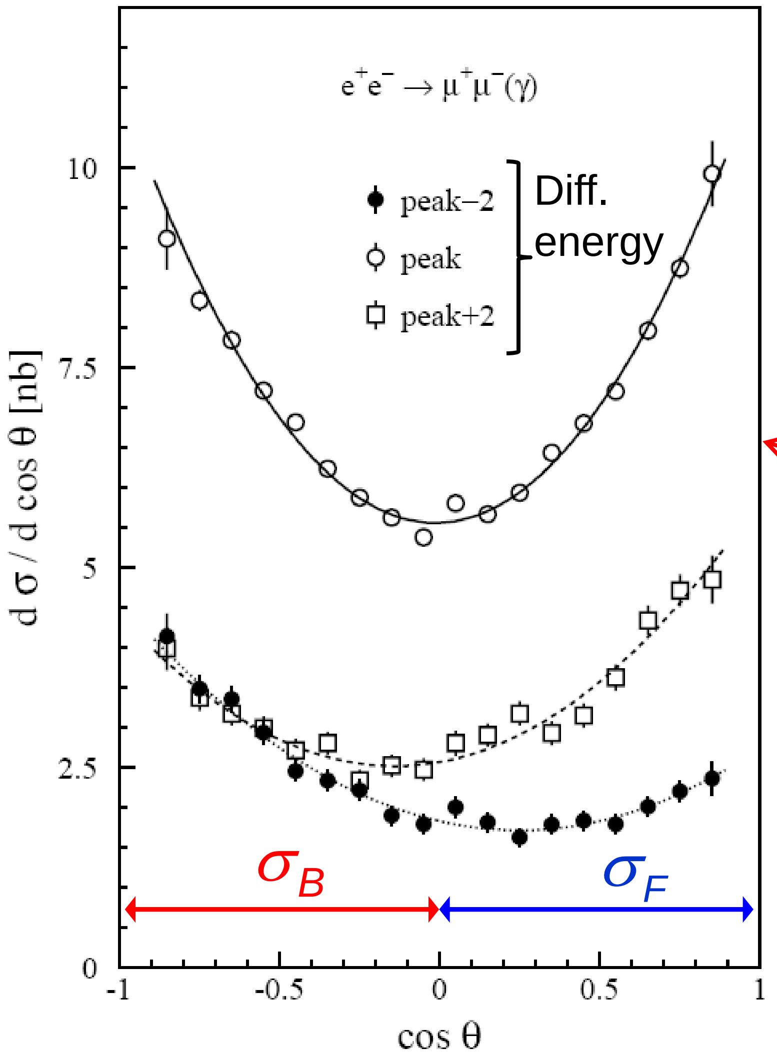
2.5 Forward-backward asymmetry and fermion couplings to Z

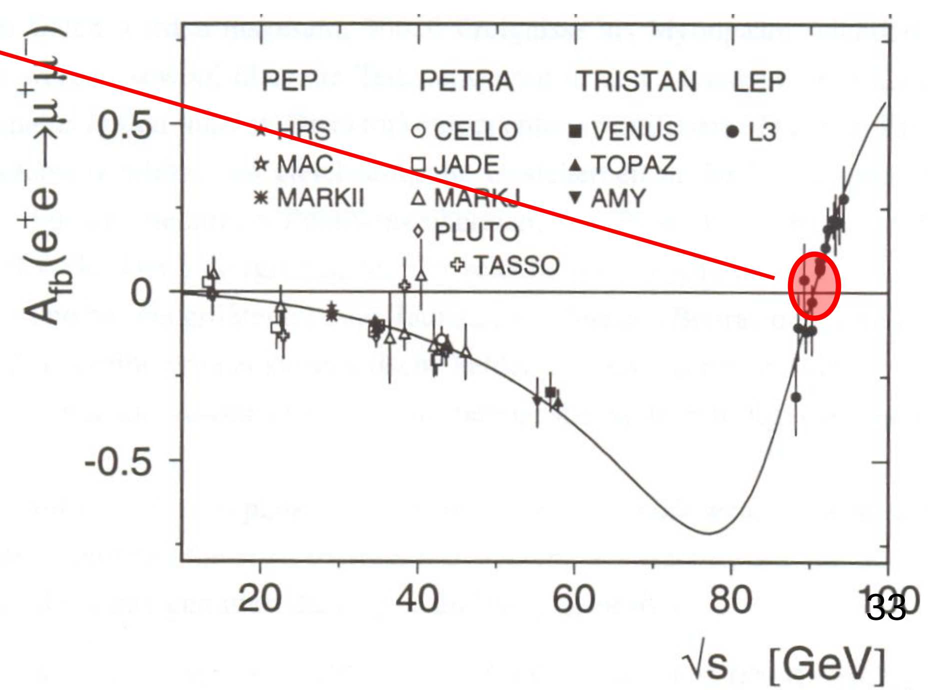
$$e^+ e^- \to Z \to \mu^+ \mu^-$$

$$\frac{d\sigma}{d\cos\theta} \sim (1+\cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

with $$A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$$

$$\sigma_{F(B)} = \int_{0\,(-1)}^{1\,(0)} \frac{d\sigma}{d\cos\theta} d\cos\theta$$





Angular distribution: (see above)

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4\sin^2\theta_W \cos^2\theta_W}\left[2g_V^e g_V^\mu(1+\cos^2\theta) + 4g_A^e g_A^\mu \cos\theta\right]$$

$$F_Z(\cos\theta) = \frac{1}{16\sin^4\theta_W \cos^4\theta_W}\left[({g_V^e}^2 + {g_A^e}^2)({g_V^\mu}^2 + {g_A^\mu}^2)(1+\cos^2\theta) + 8g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta\right]$$

Forward-backward asymmetry A_{FB}

- Away from the resonance large → interference term dominates

$$A_{FB} \sim g_A^e g_A^f \cdot \frac{s(s-M_Z^2)}{(s-M_Z^2)^2 + M_Z^2\Gamma_Z^2} \qquad \to \text{large}$$

- At the Z pole: Interference = 0 (see energy dependence of interference term)

$$A_{FB} = 3\cdot\frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2}\cdot\frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2}$$

→ very small because g_V^l small in SM

[Asymmetrie at the Z pole]

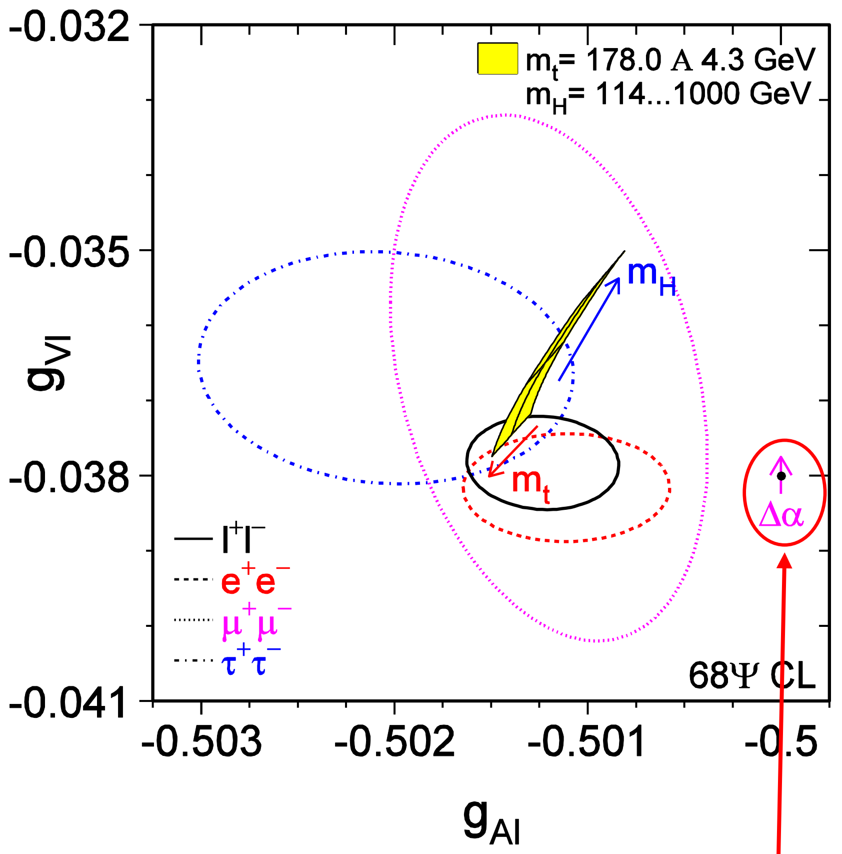
$$A_{FB} \sim g_A^e g_V^e g_A^f g_V^f$$

[Cross section at the Z pole]

$$\sigma_Z \sim \left[(g_V^e)^2 + (g_A^e)^2\right]\left[(g_V^\mu)^2 + (g_A^\mu)^2\right]$$

Lepton asymmetries together with lepton pair cross sections allow the determination of the lepton couplings g_A and g_V.

Good agreement between the 3 lepton species confirms "lepton universality"



Lowest order SM prediction:

$$g_V = T_3 - 2q\sin^2\theta_W \quad g_A = T_3$$

Deviation from lowest order SM prediction is an effect of higher-order electroweak corrections.

2.6 Polarization of final state leptons: tau pol.

$$\mathcal{P}_f(\cos\theta) = \frac{\dfrac{d\sigma_+}{d\cos\theta} - \dfrac{\sigma_-}{d\cos\theta}}{\dfrac{d\sigma_+}{d\cos\theta} + \dfrac{\sigma_-}{d\cos\theta}}$$

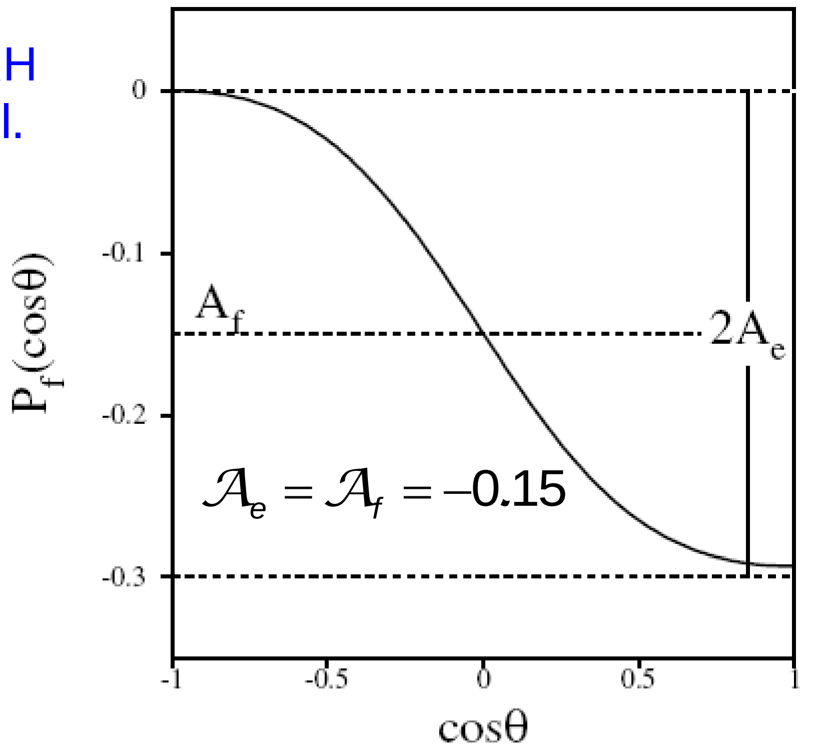
$\sigma_{L(R)}$= LH/RH fermion pol.

$$\mathcal{P}_f(\cos\theta) = \frac{\mathcal{A}_f(1+\cos^2\theta) + 2\mathcal{A}_e\cos\theta}{(1+\cos^2\theta) + 8/3\, A_{FB}\cos\theta}$$

with

$$\mathcal{A}_i = \frac{2g_V^i g_A^i}{(g_V^i)^2 + (g_A^i)^2}$$

$$\mathcal{P}_\ell \approx -2\frac{2g_V^\ell}{g_A^\ell} = -2(1-4\sin^2\theta_w)$$



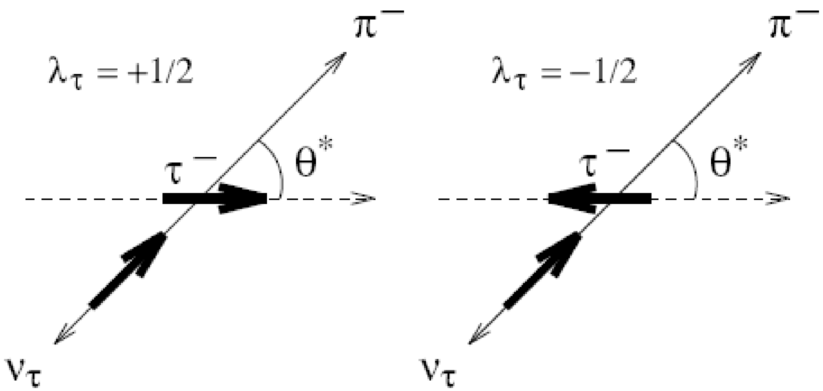
($\cos\theta$ is the fermion scattering angle)

Lepton polarization measures directly $\sin^2\theta_W$. The only lepton for which polarization can be measured at LEP is the tau!

Experimental Method to measure tau polarization:

$$\tau^- \rightarrow \pi^- \nu_\tau$$

Spin ½ →Spin ½ + Spin 0

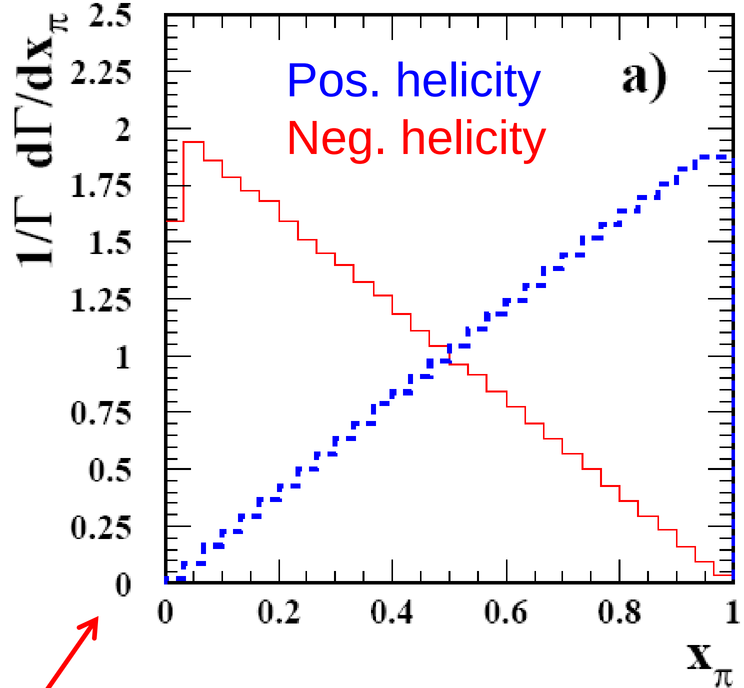


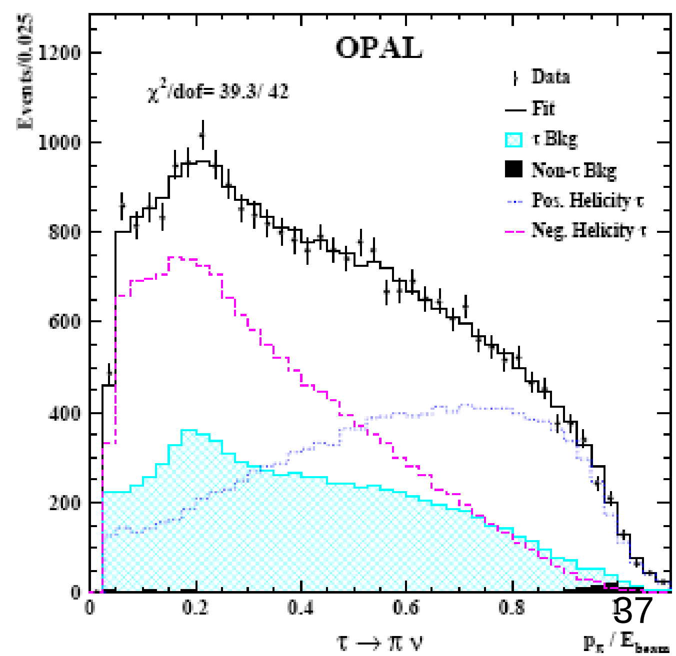
$$\frac{1}{\Gamma}\frac{d\Gamma}{d\cos\theta^*} = \frac{1}{2}(1 + \mathcal{P}_\tau \cos\theta^*)$$

Boost into lab frame

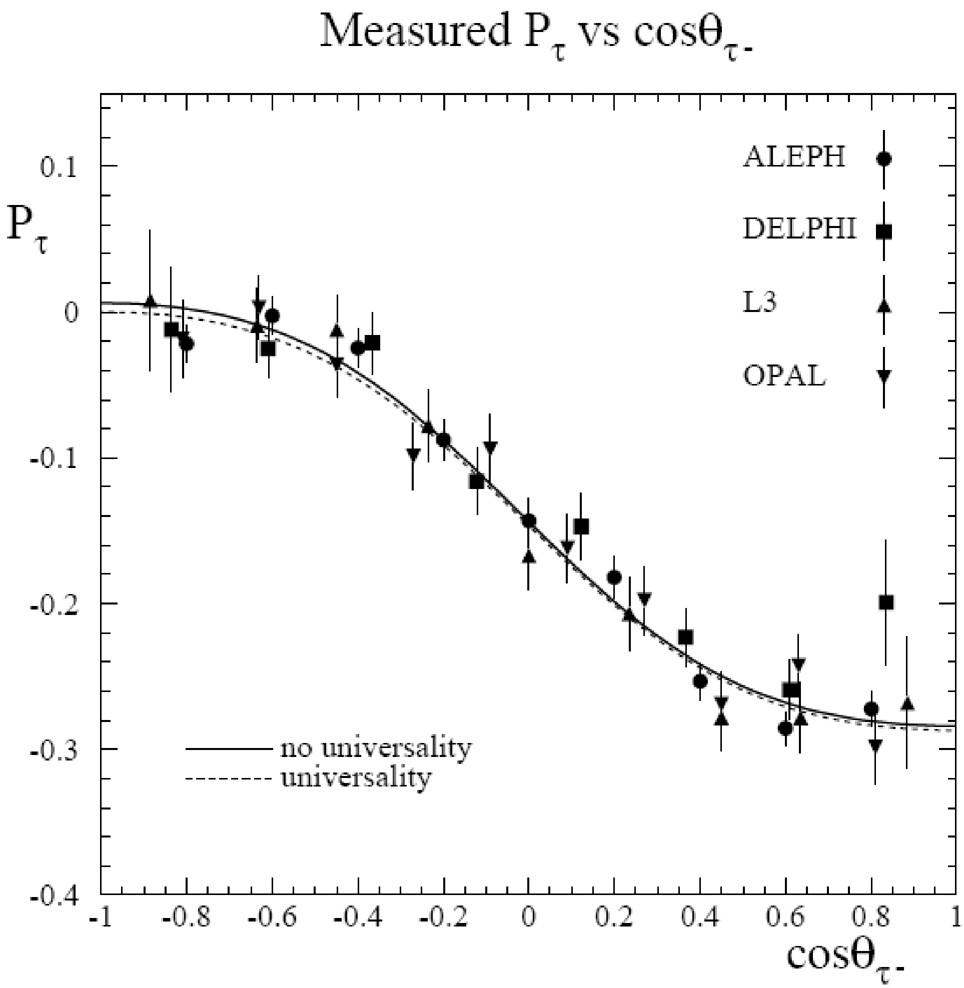
$$\frac{1}{\Gamma}\frac{d\Gamma}{dx_\pi} = 1 + \mathcal{P}_\tau(2x_\pi - 1) \qquad x_\pi = E_\pi / E_\tau$$

Fit of the two theoretical distribution to data yields the polarization: ~ 0.15





Measured Tau Polarization



$$\mathcal{A}_\tau = 0.1439 \pm 0.0043$$

$$\mathcal{A}_e = 0.1498 \pm 0.0049$$

$$\mathcal{A}_\ell = 0.1465 \pm 0.0033$$

$$\sin^2 \theta_w^{eff} = 0.23159 \pm 0.00041$$

hep-ex/0509008

2.7 Left-Right Asymmetry at SLC

Measure cross section σ_L (σ_R) for LH (RH) initial state electrons:

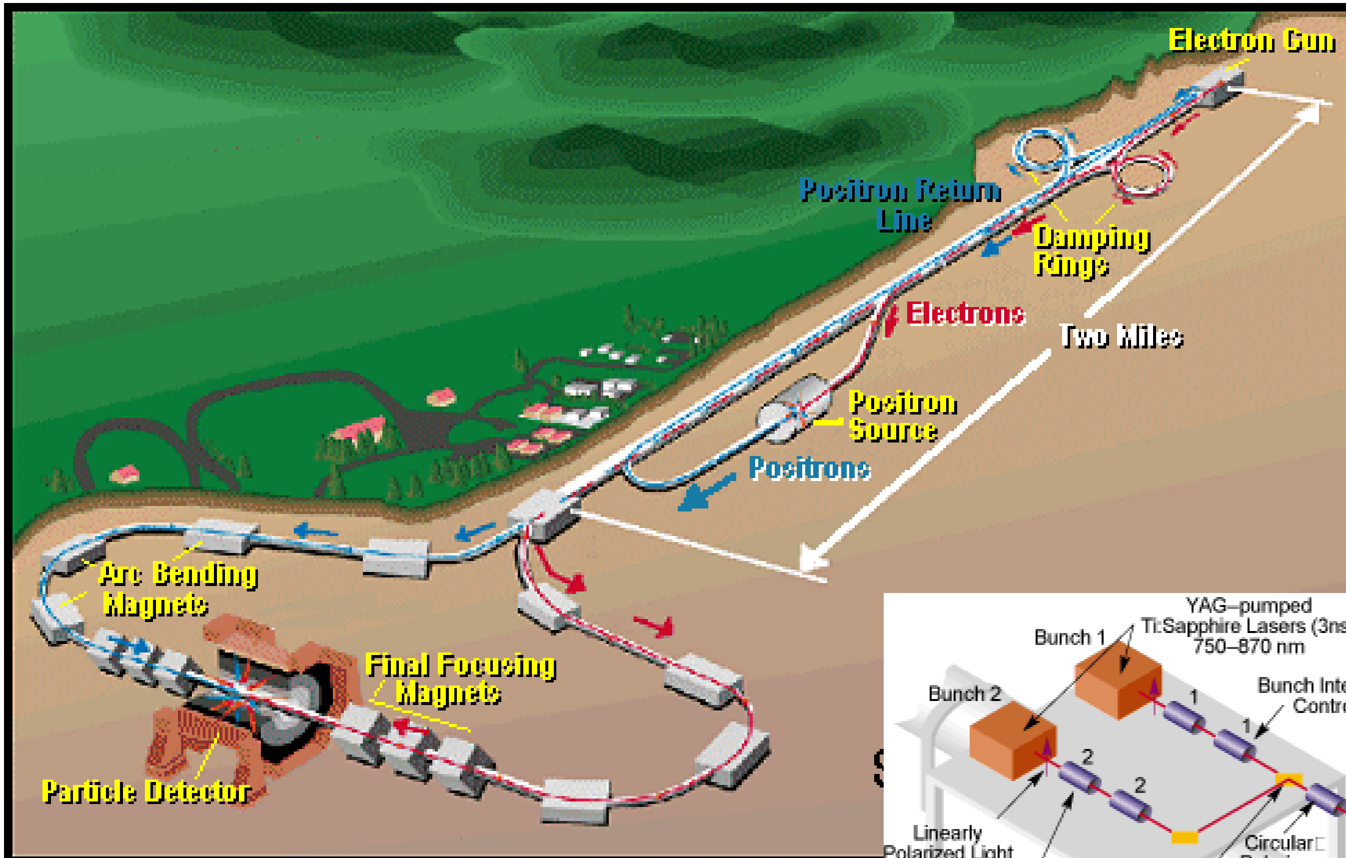
Polarization of electron beam: P~70 – 80%

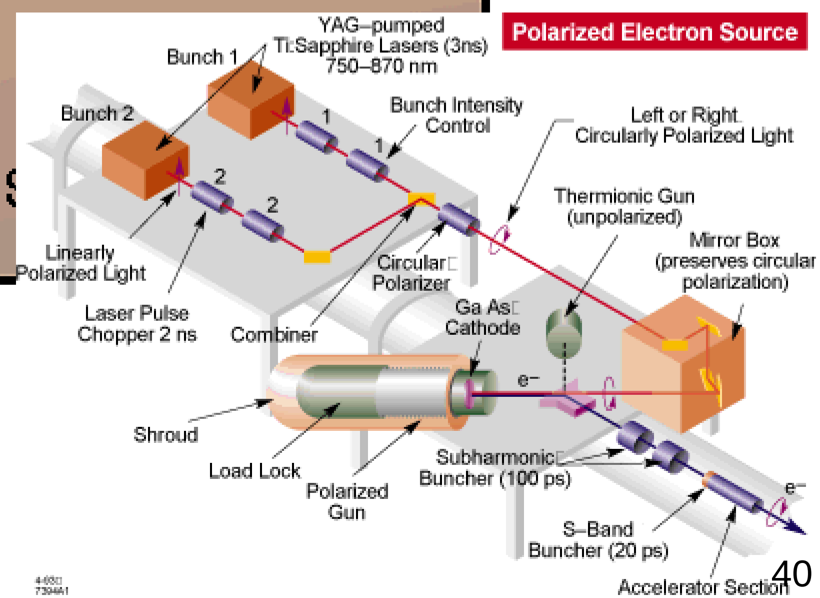
$$A_{LR} = \frac{1}{\mathcal{P}_e} \frac{\sigma_L^f - \sigma_R^f}{\sigma_L^f + \sigma_R^f}$$

$$A_{LR} = \frac{2 g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} = \frac{2(1 - 4\sin^2\theta_w)}{1 + (1 - 4\sin^2\theta_w)^2}$$

Powerful determination of $\sin^2\theta_w$.
Requires longitudinal polarization of colliding beams

SLAC Linear Collider





Typical beam polarization of 70%.

Precise determination of beam polarization using a Compton Polarimeter

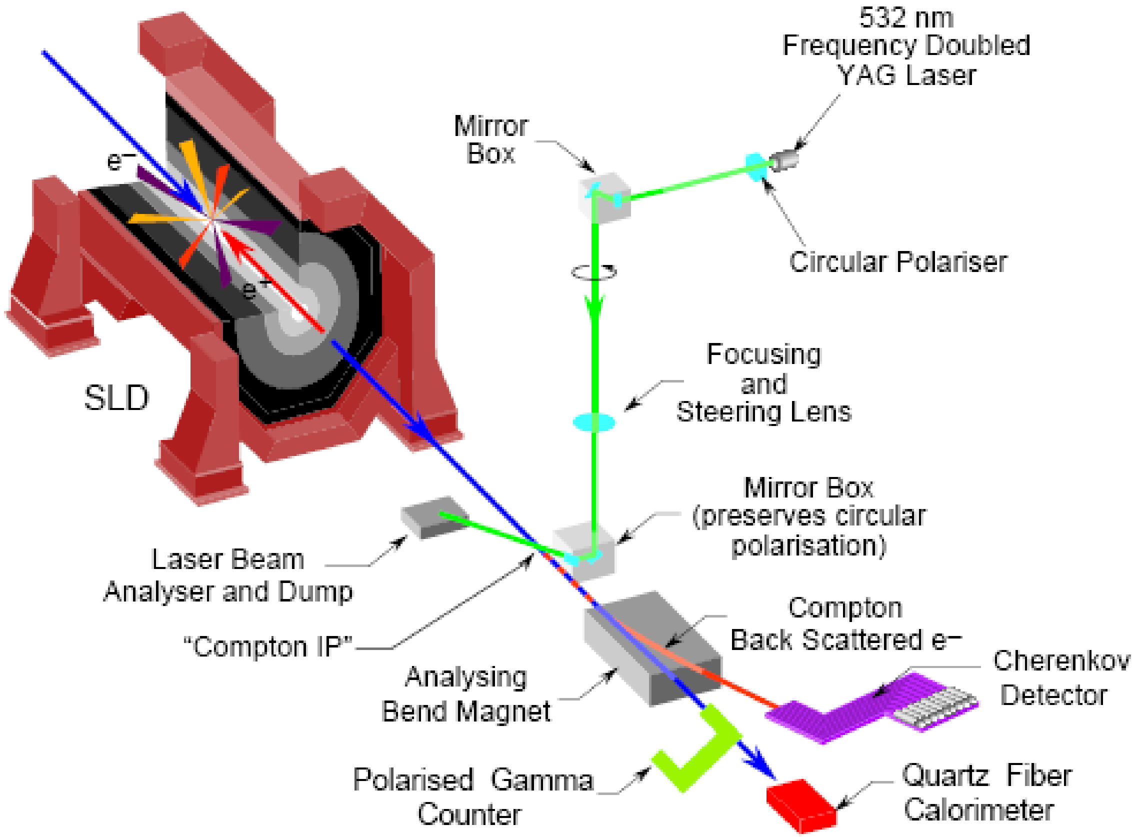
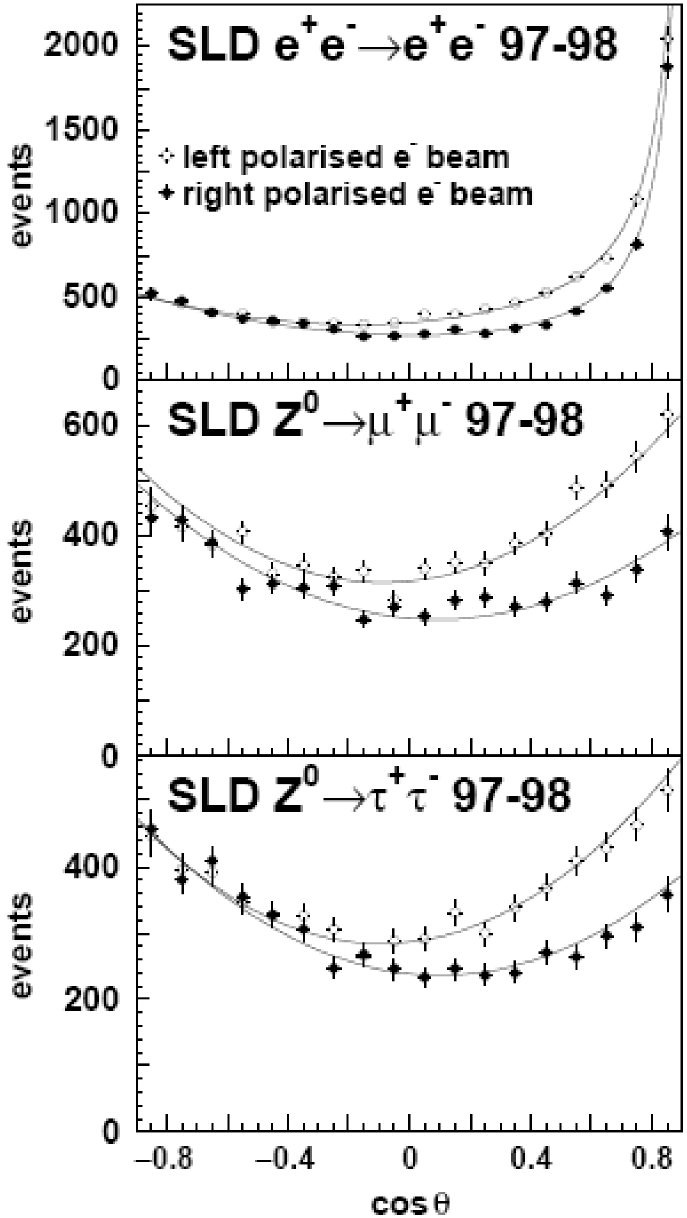


Figure 3.1: A conceptual diagram of the SLD Compton Polarimeter. The laser beam, consisting of 532 nm wavelength 8 ns pulses produced at 17 Hz and a peak power of typically 25 MW, were circularly polarised and transported into collision with the electron beam at a crossing angle of 10 mrad approximately 30 meters from the IP. Following the laser/electron-beam collision, the electrons and Compton-scattered photons, which are strongly boosted along the electron beam direction, continue downstream until analysing bend magnets deflect the Compton-scattered electrons into a transversely-segmented Cherenkov detector. The photons continue undeflected and are detected by a gamma counter (PGC) and a calorimeter (QFC) which are used to cross-check the polarimeter calibration.

Leptonic final states:



SLD

Asymmetry clearly seen for LH and RH cross section.

SLD

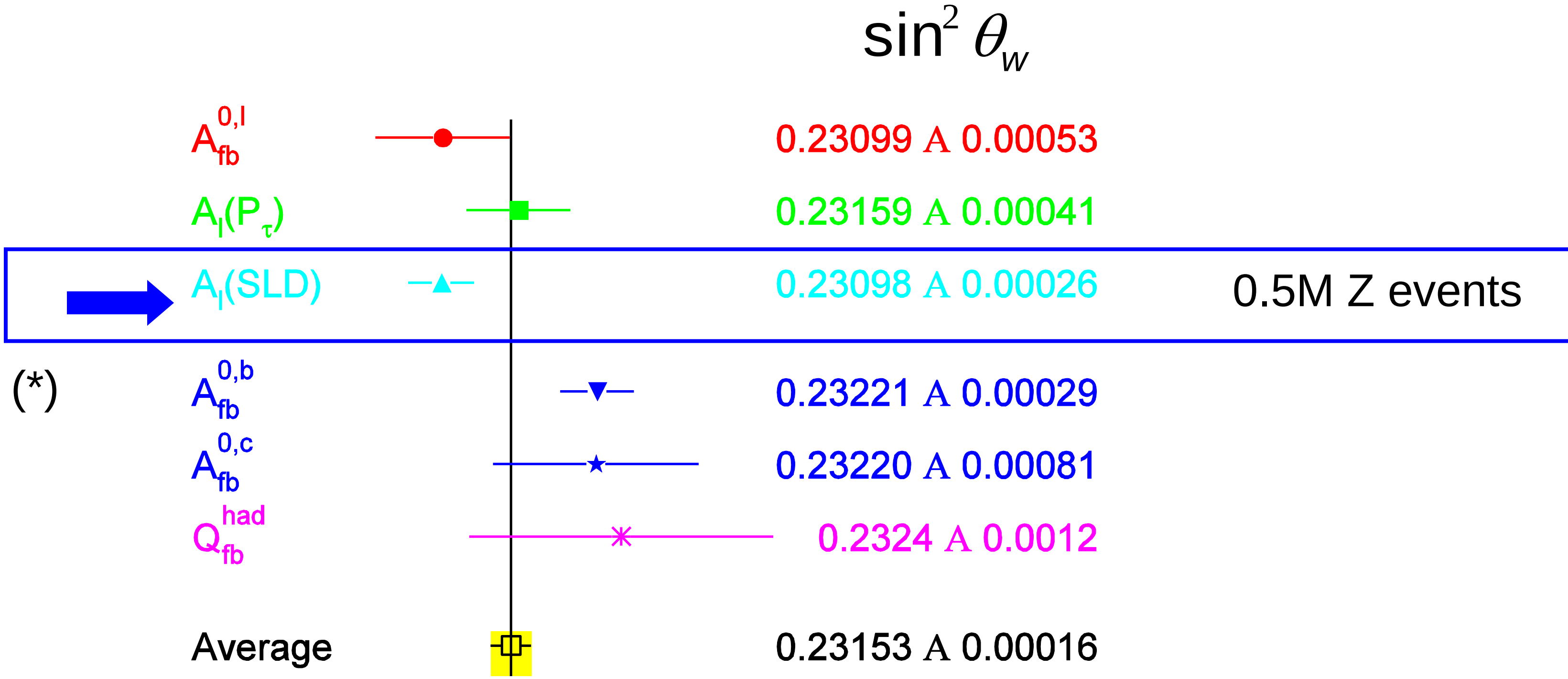
All data:

$$A_{LR} = 0.1513 \pm 0.0021$$

$$\sin^2 \theta_W = 0.23098 \pm 0.00026$$

With 0.5×10^6 Z-decays

SLD versus 4×4.5 ×10^6 Z-decays at LEP

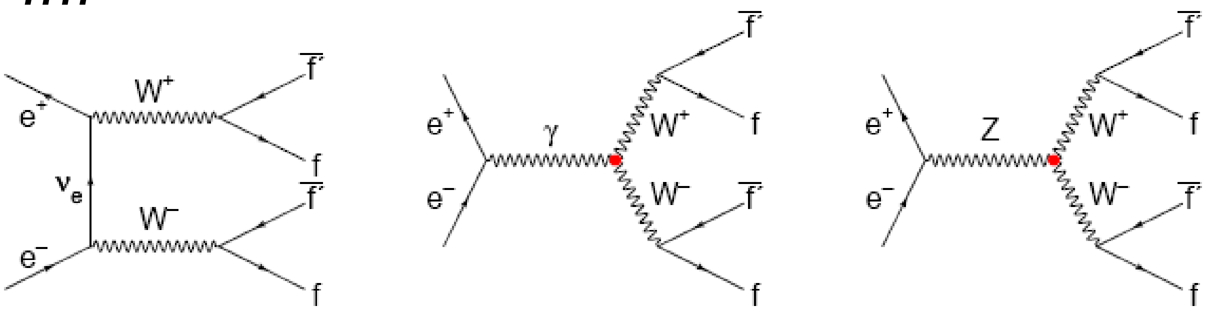


(*) similar to R_b one can also determine the forward-backward asymmetry for bb-events.

3. Precision tests of the W sector (LEP2 and Tevatron)

~10K WW events / experiment

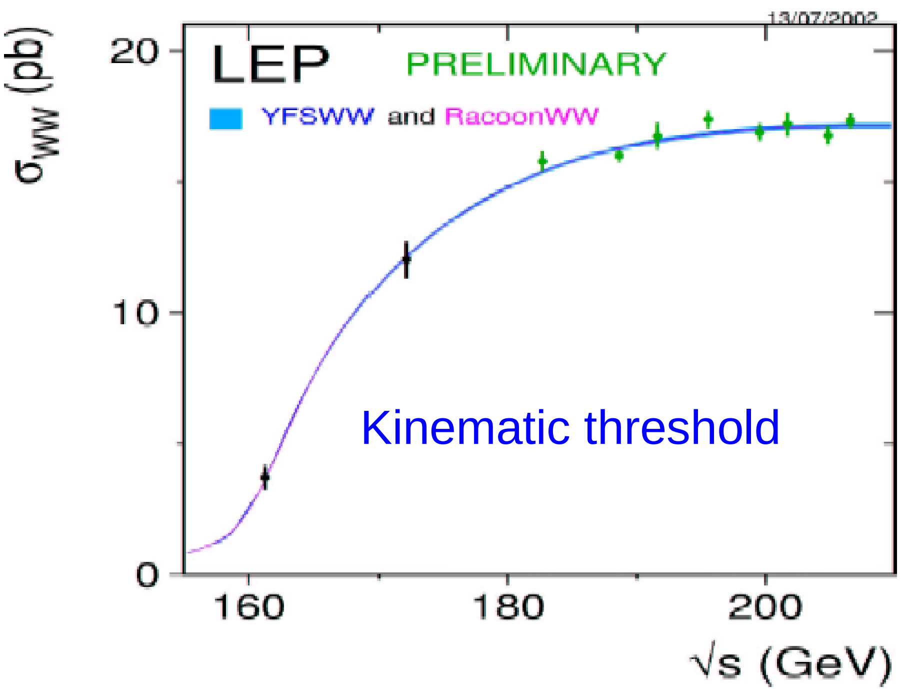
$$e^+e^- \to WW \to f\bar{f}f\bar{f}$$



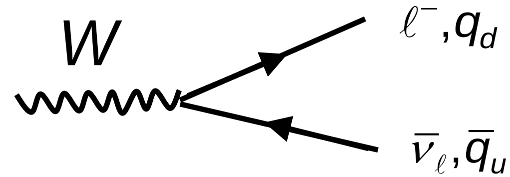
Threshold behavior of the cross section (kinematics, phase space) for ee→WW production:

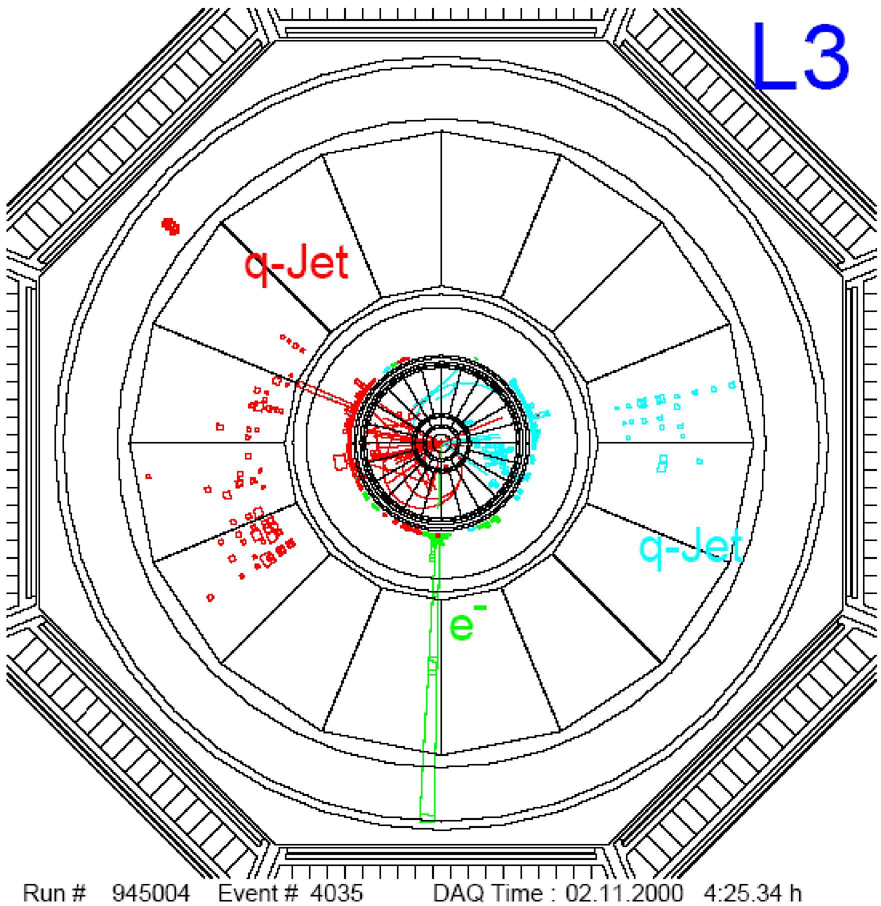
Phase space factor = f(M_W, $\sqrt{s}$):

→ Allows determination of M_W

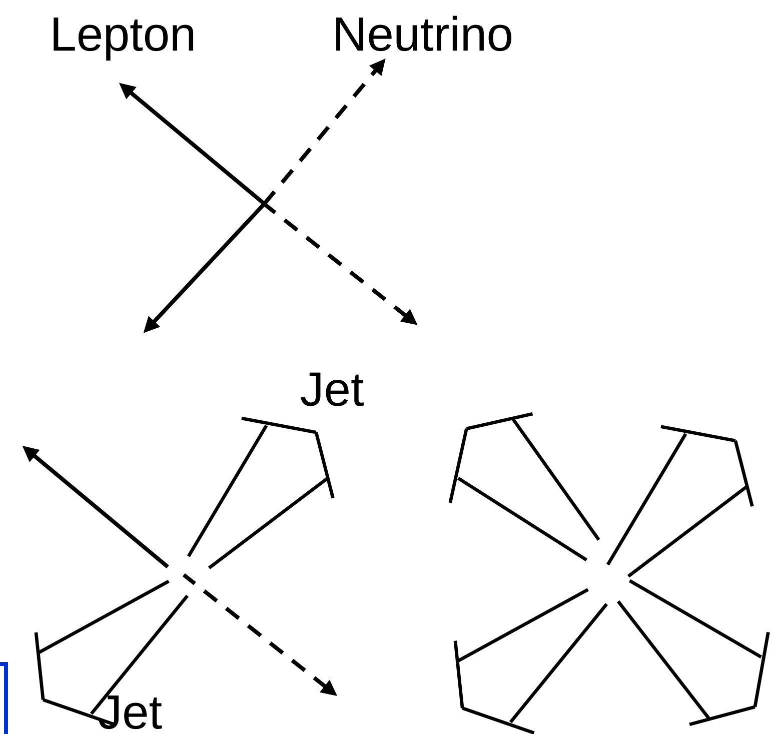


W decays





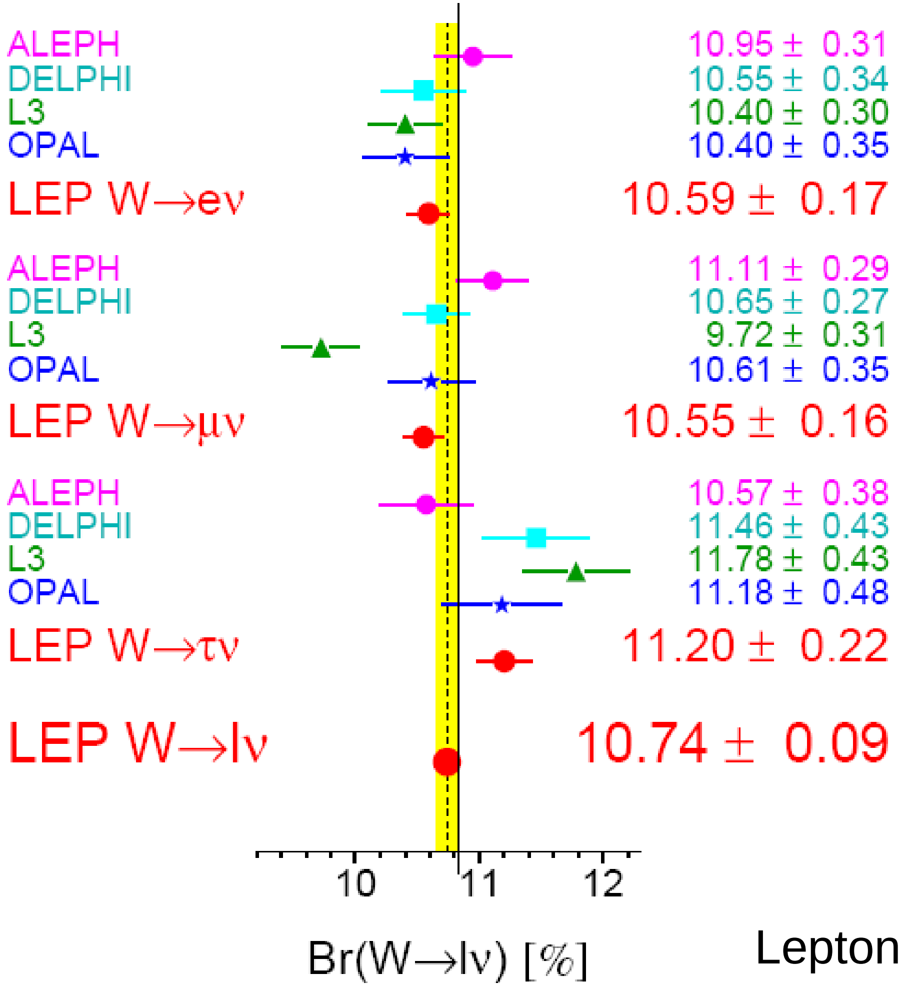
$$WW \rightarrow \begin{cases} qq\ell\nu & 44\% \\ qqqq & 45\% \\ \ell\nu\ell\nu & 11\% \end{cases}$$



Easiest signature for a mass measurement:

$W_1 \rightarrow l\nu$ $W_2 \rightarrow$ JetJet: use JetJet invariant mass

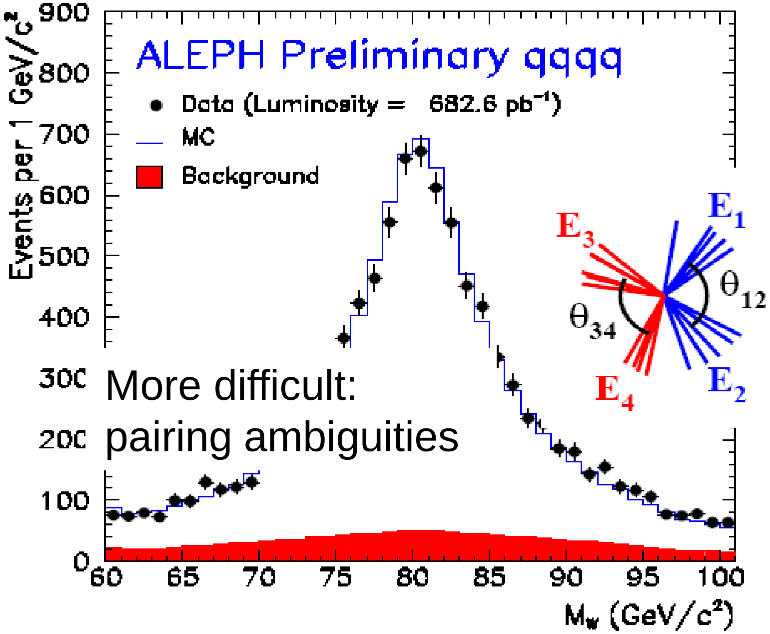
W leptonic branching fractions



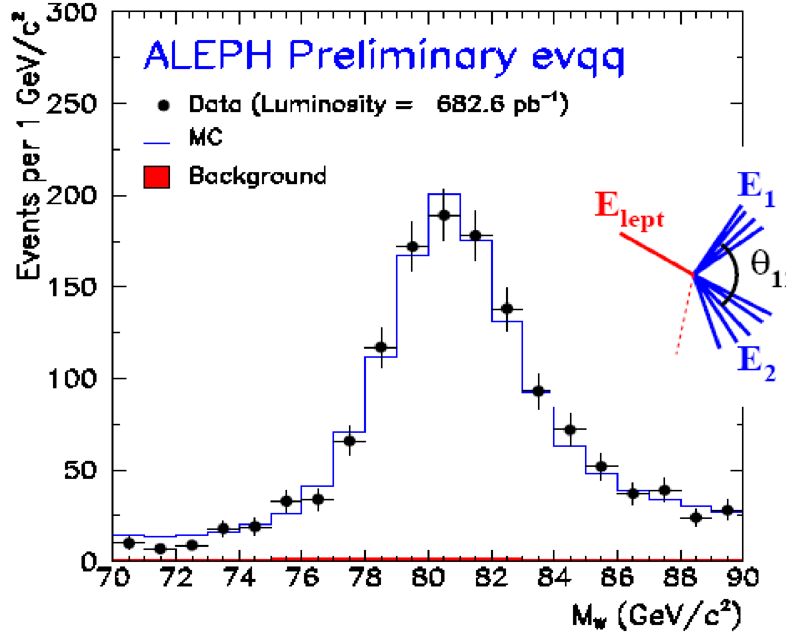
$Br(W \to q\bar{q}) = (67.77 \pm 0.28)\%$

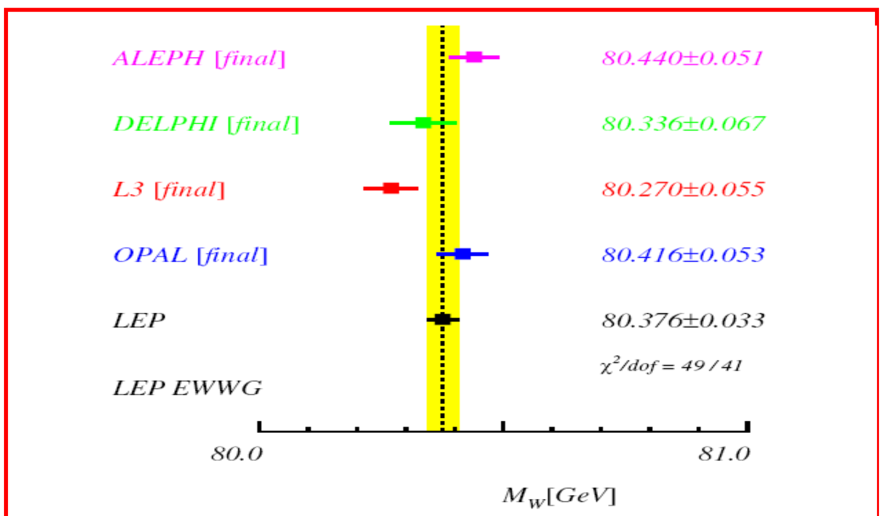
Lepton universality tested to 2%

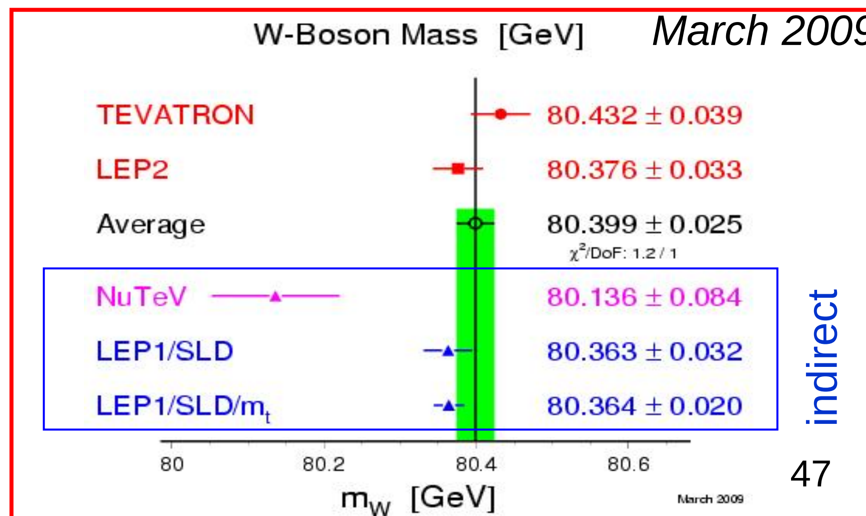
Invariant W mass recontruction



More difficult:
pairing ambiguities

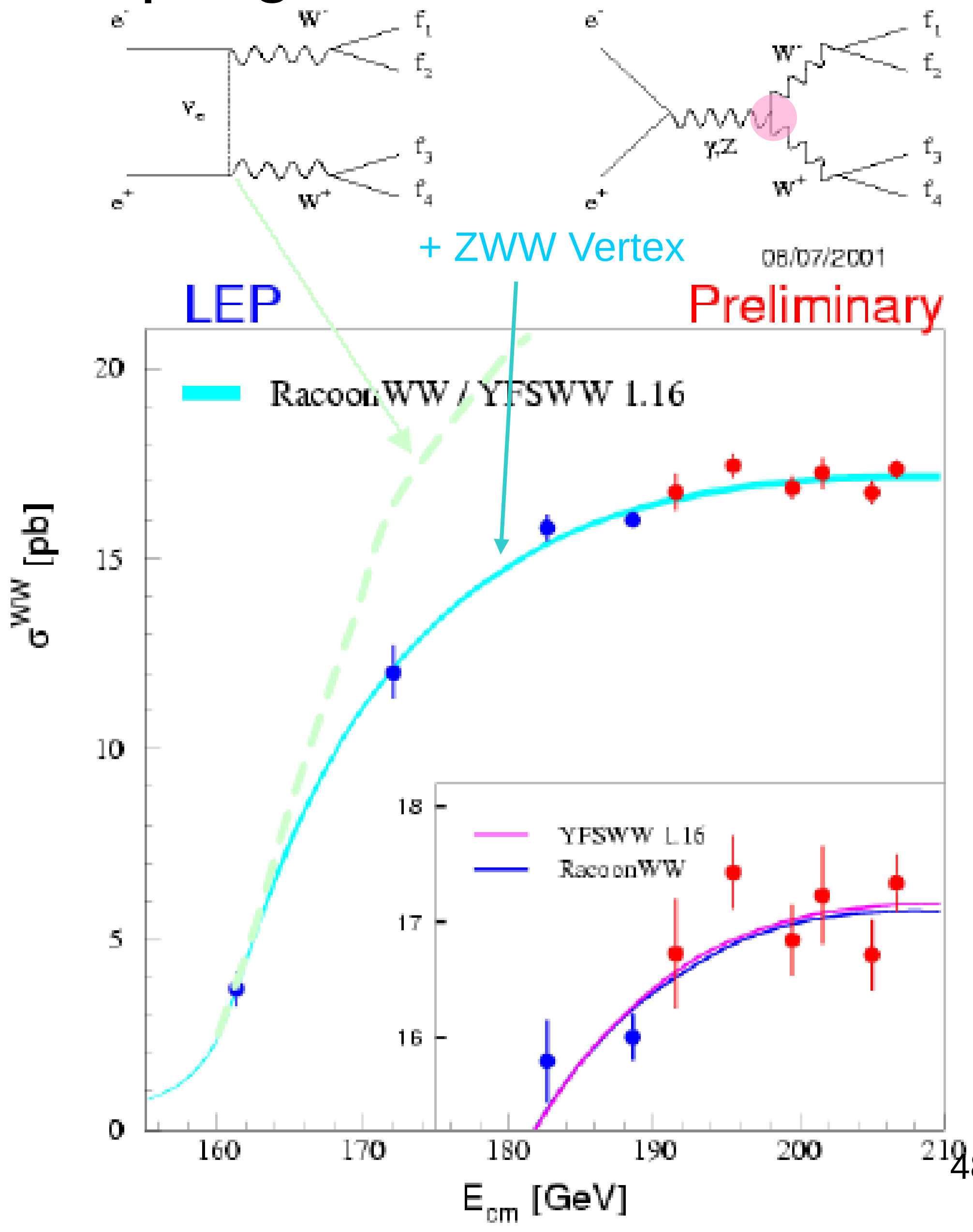




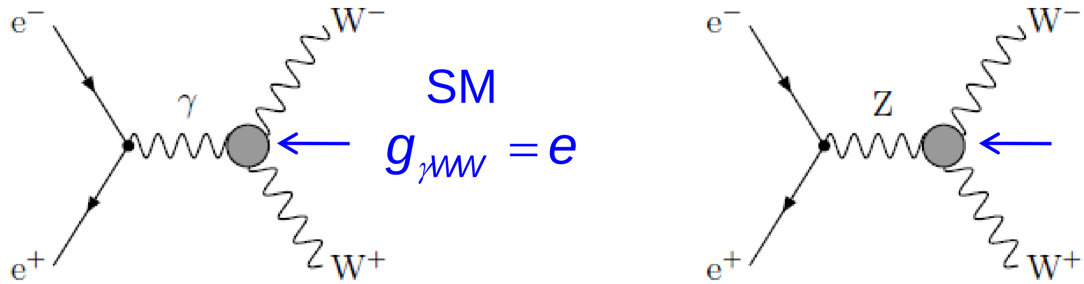


Effect of triple gauge coupling

Data confirms the existence of the γ/ZWW triple gauge boson vertex



Test of trilinear gauge boson coupling in WW production



SM $g_{\gamma WW} = e$

$g_{ZWW} = e \frac{\cos\theta_w}{\sin\theta_w}$

Triple gauge coupling an important result of the non-abelian gauge structure.

Most general Lagrangian for VWW:

$$\begin{aligned} i\mathcal{L}_{\text{eff}}^{\text{VWW}}/g_{\text{VWW}} &= \boxed{g_1^V} V^\mu \left(W_{\mu\nu}^- W^{+\nu} - W_{\mu\nu}^+ W^{-\nu}\right) \\ &+ \boxed{\kappa_V} W_\mu^+ W_\nu^- V^{\mu\nu} + \frac{\lambda_V}{m_W^2} V^{\mu\nu} W_\nu^{+\rho} W_{\rho\mu}^- \\ &+ i g_5^V \varepsilon_{\mu\nu\rho\sigma} \left((\partial^\rho W^{-\mu}) W^{+\nu} - W^{-\mu}(\partial^\rho W^{+\nu})\right) V^\sigma \\ &+ i g_4^V W_\mu^+ W_\nu^- \left(\partial^\mu V^\nu + \partial^\nu V^\mu\right) \\ &- \frac{\tilde{\kappa}_V}{2} W_\mu^- W_\nu^+ \varepsilon^{\mu\nu\rho\sigma} V_{\rho\sigma} - \frac{\tilde{\lambda}_V}{2m_W^2} W_{\rho\mu}^- W_\nu^{+\mu} \varepsilon^{\nu\rho\alpha\beta} V_{\alpha\beta}. \end{aligned}$$

$\square$ =1, all others 0

$\Delta\kappa, \Delta g_1 \neq 0$ Deviation from SM

Interpretation for γWW

$q_W = \pm g_V^\gamma$ charge

$\mu_W = \frac{e}{2M_W}(1 + \kappa_\gamma + \lambda_\gamma)$

Dipol moment

W-polarization in $e^+ e^- \to W^+ W^-$

W polarization
- Transversely: +, -
- Longitudinally: 0

TGC Parametrisation		
$\Delta\lambda$	$(\lambda\lambda')$	$A^V_{\lambda\lambda'}$
1	$(+,0)$	$\gamma(g_1^V + \kappa_V + \lambda_V - ig_4^V + \beta g_5^V + \frac{i}{\beta}(\tilde{\kappa}_V - \tilde{\lambda}_V))$
1	$(0,-)$	$\gamma(g_1^V + \kappa_V + \lambda_V + ig_4^V + \beta g_5^V - \frac{i}{\beta}(\tilde{\kappa}_V - \tilde{\lambda}_V))$
0	$(+,+)$	$g_1^V + 2\gamma^2\lambda_V + \frac{i}{\beta}(\tilde{\kappa}_V - \tilde{\lambda}_V)$
0	$(0,0)$	$g_1^V + 2\gamma^2\kappa_V$
0	$(-,-)$	$g_1^V + 2\gamma^2\lambda_V - \frac{i}{\beta}(\tilde{\kappa}_V - \tilde{\lambda}_V)$
-1	$(0,+)$	$\gamma(g_1^V + \kappa_V + \lambda_V + ig_4^V - \beta g_5^V - \frac{i}{\beta}(\tilde{\kappa}_V - \tilde{\lambda}_V))$
-1	$(-,0)$	$\gamma(g_1^V + \kappa_V + \lambda_V - ig_4^V - \beta g_5^V - \frac{i}{\beta}(\tilde{\kappa}_V - \tilde{\lambda}_V))$

Angular distribution of the corresponding helicity amplitude given by rotation matrices (d-functions):

$$d^{J_0}_{\sigma,\Delta\lambda}$$

Electron/positron helicity: $+\sigma/2$, $-\sigma/2$, $J_0 = \max(|\sigma|, |\Delta\lambda|)$

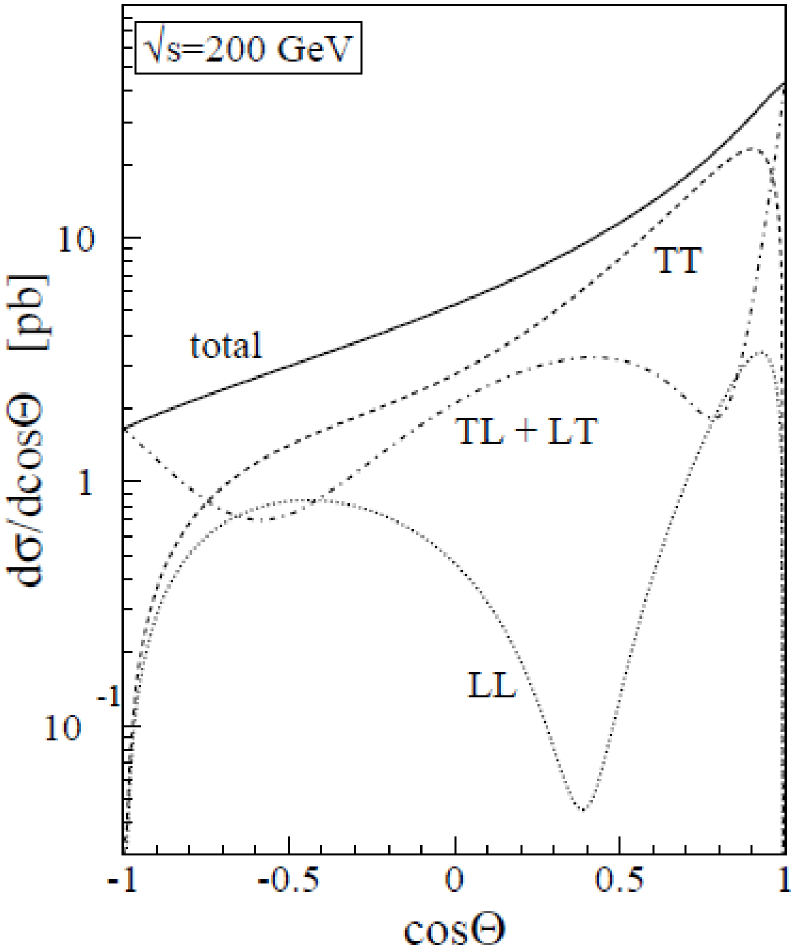


Figure 1.2: *The differential cross section for the process* $e^+e^- \rightarrow W^+W^-$ *at* 200 GeV *as function of the cosine of the* W^- *production angle. The separate contributions from different helicity combinations of the produced W bosons are also given, with* $TT = (-,+) + (+,-) + (-,-) + (+,+)$, $TL{+}LT = (-,0) + (+,0) + (0,-) + (0,+)$ *and* $LL = (0,0)$.

M.E.T. Dierckxsens, Thesis, Nijmegen, 2004

Triple Gauge couplings:

Assuming electromagnetic gauge invariance as well as C and P conservation, the number of independent TGCs reduces to five. Common set: { g_1^Z, κ_Z, κ_γ, λ_Z, λ_γ }

Parameters used by the LEP experiments are: g_1^Z, κ_γ, λ_γ

With additional gauge constraints

$$\kappa_Z = g_1^Z - (\kappa_\gamma - 1)\tan^2\theta_W$$
$$\lambda_Z = \lambda_\gamma ,$$

From a fit to the angular distribution of the WW:

Parameter	68% C.L.
g_1^Z	$0.984^{+0.022}_{-0.019}$
κ_γ	$0.973^{+0.044}_{-0.045}$
λ_γ	$-0.028^{+0.020}_{-0.021}$

g_1^Z, κ_γ: =1 in SM

λ_γ: =0 in SM

Standard Model structure of VWW triple boson coupling confirmed.

4. Higher order corrections and the Higgs mass

$$\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \qquad \sin\theta_W = \frac{e}{g}$$

Lowest order SM predictions

$$\rho = \frac{m_W^2}{m_Z^2\cos^2\theta_W} = 1 \quad \Longrightarrow \quad \bar{\rho} = 1 + \Delta\rho$$

$$\sin^2\theta_W = 1 - \frac{m_W^2}{m_Z^2} \quad \Longrightarrow \quad \sin^2\theta_{eff} = (1 + \Delta\kappa)\sin^2\theta_W$$

$$m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F} \quad \Longrightarrow \quad m_W^2 = \frac{\pi\alpha}{\sqrt{2}\sin^2\theta_W G_F}(1 + \Delta r)$$

$$\alpha(0) \quad \Longrightarrow \quad \alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta\alpha}$$

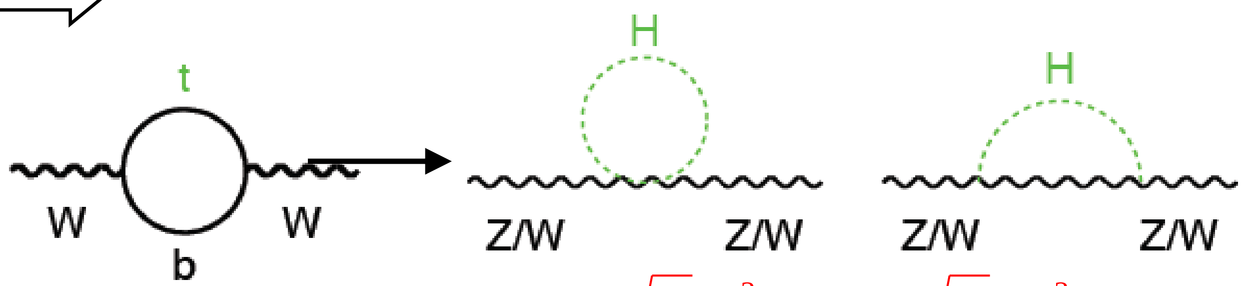
Including radiative corrections

with : $\Delta\alpha = \Delta\alpha_{lept} + \Delta\alpha_{top} + \Delta\alpha_{had}^{(5)}$

$\sin^2\theta_W$

g_A, g_V

$\Delta\rho, \Delta\kappa, \Delta r = f\left(m_t^2, \log(m_H), \ldots\right)$



$\sin^2\theta_{eff}$

$\bar{g}_A, \bar{g}_V$

$$\bar{g}_A = \sqrt{\bar{\rho}}\, T^3 \qquad \bar{g}_V = \sqrt{\bar{\rho}}\,(T^3 - 2Q\sin^2\theta_{eff})$$

Top mass prediction from radiative corrections

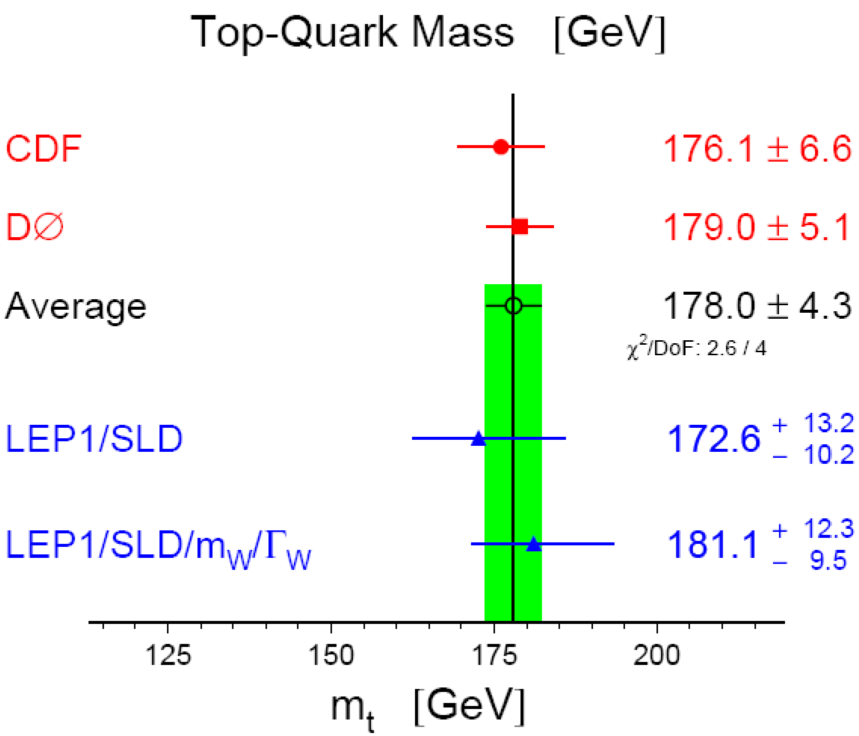
e.g.: $$\Delta r(m_t, M_H) = -\frac{3\alpha\cos^2\theta_w}{16\pi\sin^4\theta_w}\frac{m_t^2}{M_W^2} - \frac{11\alpha}{48\pi\sin^2\theta_w}\ln\frac{M_H^2}{M_W^2} + \dots$$

The measurement of the radiative corrections:

$$\sin^2\theta_{eff} \equiv \frac{1}{4}(1-\bar{g}_V/\bar{g}_A)$$

$$\sin^2\theta_{eff} = (1+\Delta\kappa)\sin^2\theta_w$$

Allows the indirect determination of the unknown parameters m_t and M_H.



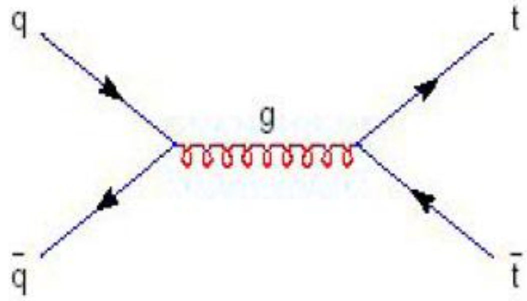
Direct measurement of m_t

Good agreement between the indirect prediction of m_t and the value obtained in direct measurements confirm the radiative corrections of the SM

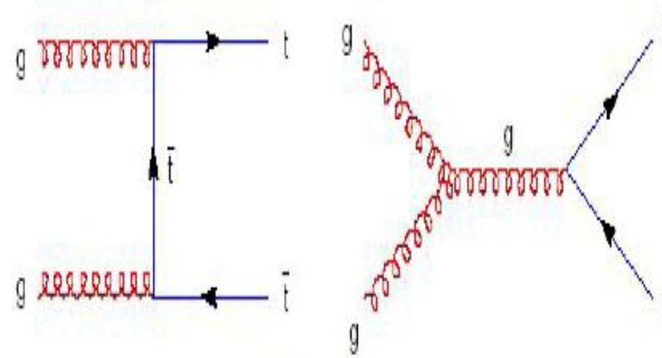
Prediction of m_t by LEP before the discovery of the top at TEVATRON.

Observation of the top quark at TEVATRON (1995)

$p\bar{p}$ @ 2 TeV

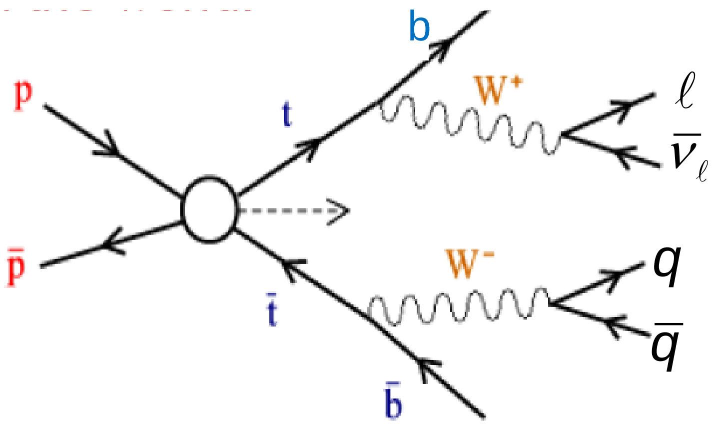


$q\bar{q}$ annihilation (85%)



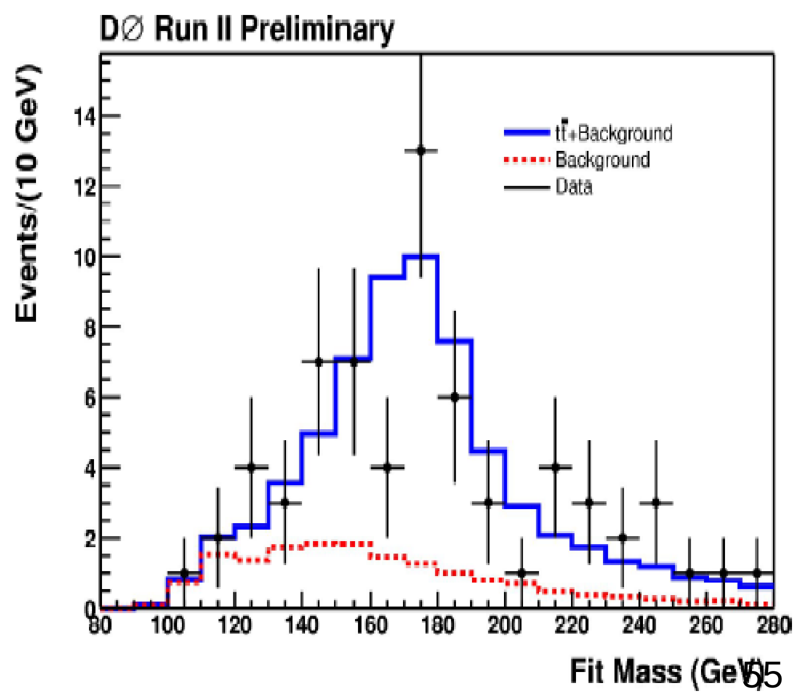
gluon fusion (15%)

Top decay (decays before hadronization)

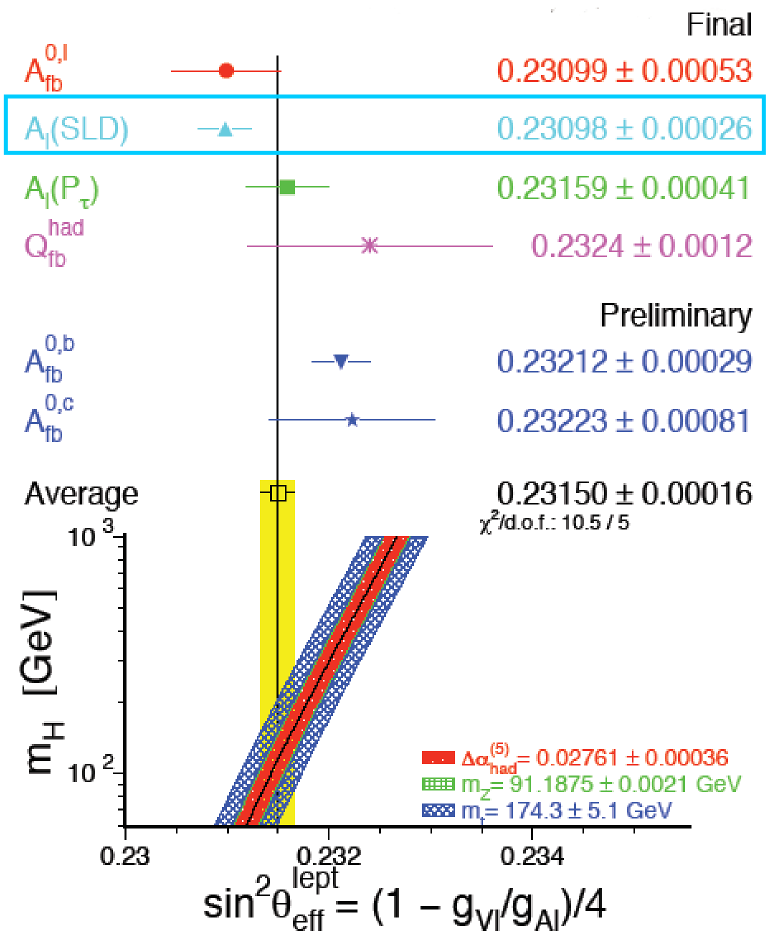


Channel used for mass reconstruction:

$$m_t = m_{inv}(b - jet, W \rightarrow jet + jet)$$



Higgs mass prediction from radiative corrections

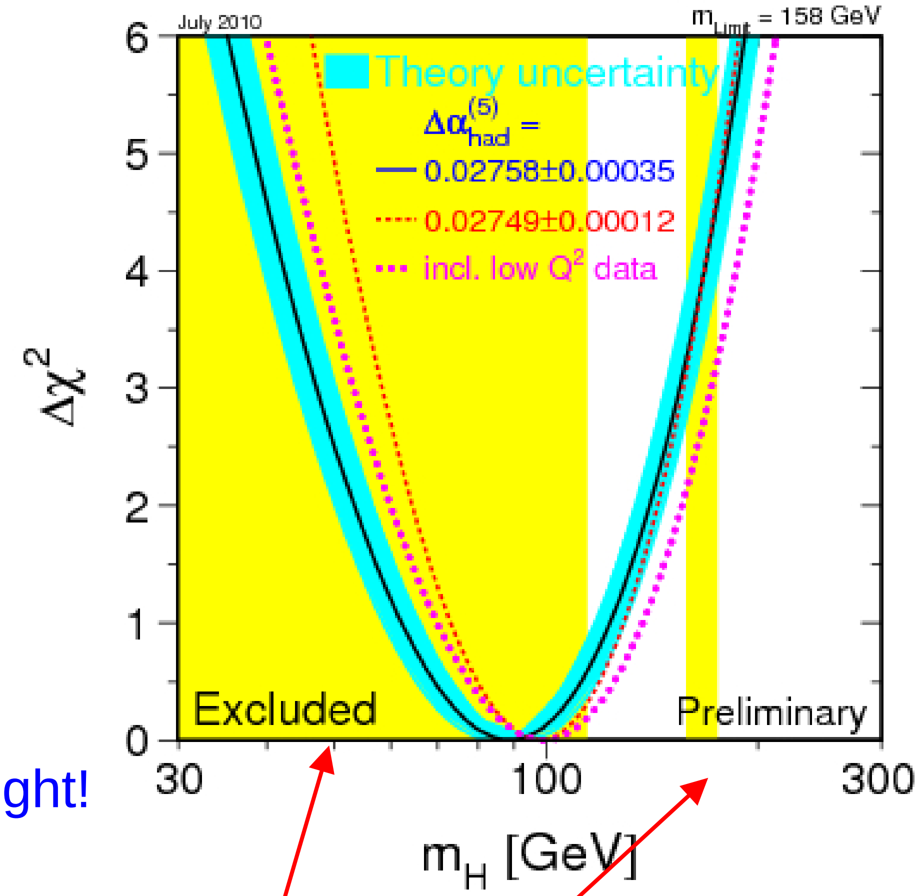


Theoretical prediction of $\sin^2\theta_{eff}$ as function of the Higgs mass.

Take the top mass from direct measurements and use the radiative corrections to determine the Higgs mass.

$$\Delta r(m_t, M_H) = -\frac{3\alpha\cos^2\theta_W}{16\pi\sin^4\theta_W}\frac{m_t^2}{M_W^2} - \frac{11\alpha}{48\pi\sin^2\theta_W}\boxed{\ln\frac{M_H^2}{M_W^2}} + \cdots$$

http://lepewwg.web.cern.ch/LEPEWWG/



Fits to electro-weak data:

$m_H = 89\ ^{+35}_{-26}$ GeV

m_H < 158 GeV (95% CL)

Assumption for fit:

- SM including Higgs
- No confirmation of Higgs mechanism

If existing, Higgs seems to be light!

- Direct searches at LEP: m_H > 114.4 GeV @ 95 % CL
- Direct searches at Tevatron

Unfortunately this mass region is the most difficult to explore!