

Experimental tests of the Standard Model

0. Standard Model in a Nutshell
1. Discovery of W and Z boson
2. Precision tests of the Z sector
3. Precision test of the W sector
4. Radiative corrections and prediction of the Higgs mass
5. Higgs searches

Literature for (2):

Precision electroweak measurement on the Z resonance, Phys. Rept. 427 (2006), hep-ex/0509008.

<http://lepewwg.web.cern.ch/LEPEWWG/1/physrep.pdf>

0. Standard Model in a Nutshell

$$\begin{array}{ccc}
 \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L & \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L & \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \\
 e^-_R & \mu^-_R & \tau^-_R
 \end{array}$$

LH weak isospin doublets

RH singlets



weak isospin: T, T₃

$$W^\pm \quad \tau^\pm = \frac{1}{2}(\tau^1 \pm i \cdot \tau^2)$$

Symmetry:

Additional field W^3 which corresponds to the 3rd isospin operator τ^3 .

W^3 only couples to the particles of the weak isospin doublet!

In addition we have two more fields:

- Photon γ which couples to the LH and RH fermions with same strength.
- Z boson which couples to LH and RH fermions with different couplings g_L and g_R

How can we associate the observed fields to W^3 ?

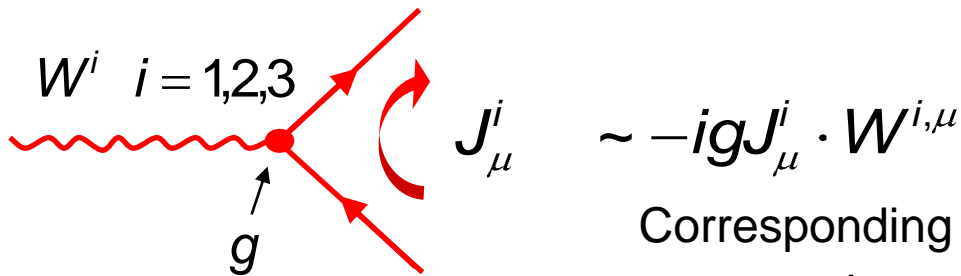
⇒ Additional gauge field B

Gauge field B couples to hyper-charge: $Y = 2 [Q - T_3]$

couples to LH and RH fermions

T.Plehn

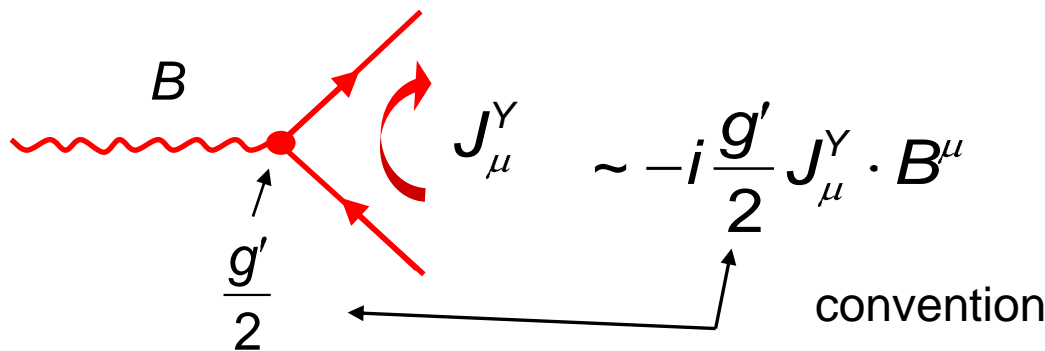
$$q \equiv \frac{y\mathbb{1} + \tau_3}{2}$$



$$\sim -igJ_\mu^i \cdot W^{i,\mu}$$

Corresponding to J^\pm and J^3 there are fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}} (W_\mu^1 \mp iW_\mu^2) \quad \text{and} \quad W_\mu^3$$



$$\sim -i \frac{g'}{2} J_\mu^Y \cdot B^\mu$$

g, g' are coupling constants.

While the charged boson fields W^\pm correspond to the observed W bosons, the neutral fields B and W^3 only correspond to linear combinations of the observed photon and Z boson:

$$\begin{aligned}
 A_\mu &= B_\mu \cos \theta_W + W_\mu^3 \sin \theta_W && \leftarrow \text{massless photon} \\
 Z_\mu &= -B_\mu \sin \theta_W + W_\mu^3 \cos \theta_W && \leftarrow \text{massive Z boson} \\
 B_\mu &= A_\mu \cos \theta_W - Z_\mu \sin \theta_W \\
 W_\mu^3 &= A_\mu \sin \theta_W + Z_\mu \cos \theta_W
 \end{aligned}$$

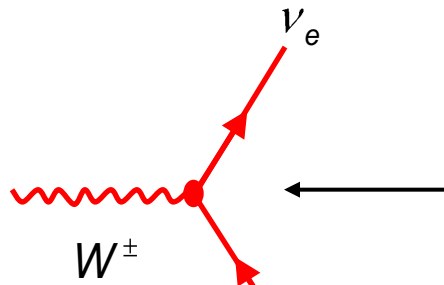
$$g_Z = \frac{g}{\cos \theta_W} \quad g' = g \frac{\sin \theta_W}{\cos \theta_W}$$

The **weak mixing angle** θ_w (Weinberg angle) follows from coupling constants:

$$g = \frac{e}{\sin \theta_w} \quad g' = \frac{e}{\cos \theta_w}$$

Coupling to the photon field $\sim e$

Feynman rules

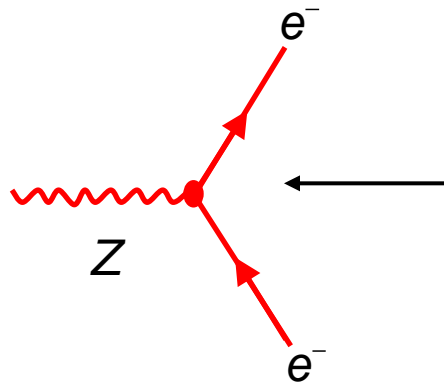


Vertex factors

$$-i \frac{g}{\sqrt{2}} \gamma_\mu \frac{1}{2} (1 - \gamma^5)$$

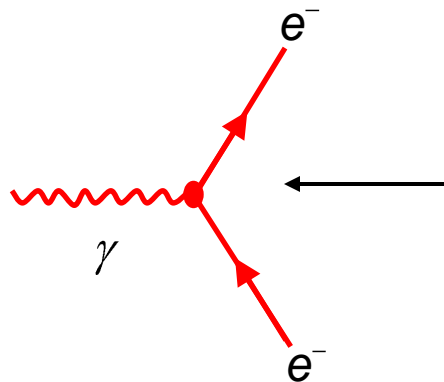
Propagator
(unitary gauge)

$$\frac{g_{\mu\nu} - q_\mu q_\nu / M_W^2}{q^2 - M_W^2}$$



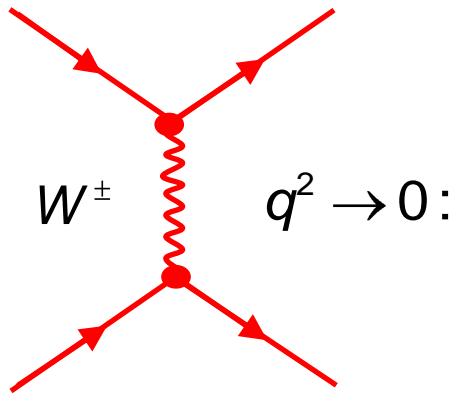
$$-i \frac{g}{\cos \theta_W} \gamma_\mu \frac{1}{2} (g_V - g_A \gamma^5)$$

$$\frac{g_{\mu\nu} - q_\mu q_\nu / M_Z^2}{q^2 - M_Z^2}$$

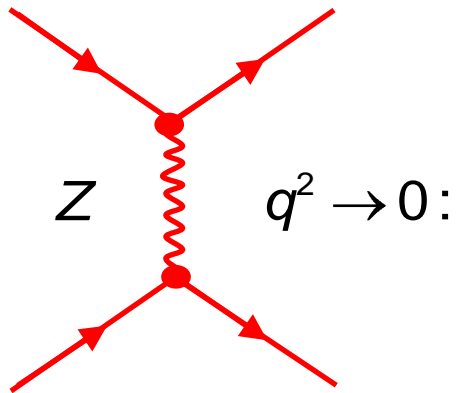
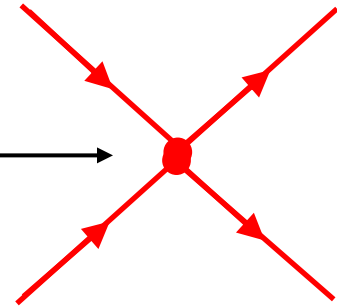


$$-ie\gamma_\mu$$

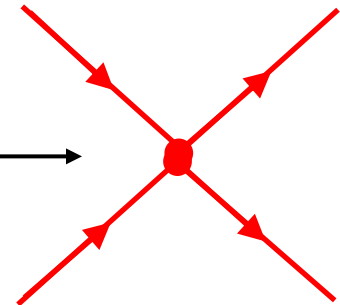
$$\frac{1}{q^2}$$



$$\frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$$



$$\frac{g^2}{8 \cos^2 \theta_W M_Z^2} = \frac{G_{NC}}{\sqrt{2}}$$



Assuming universality

$$\frac{G_F}{\sqrt{2}} \equiv \frac{G_{NC}}{\sqrt{2}}$$

follows

$$\cos^2 \theta_W = \frac{M_W^2}{M_Z^2}$$

$$\Delta\rho=0$$

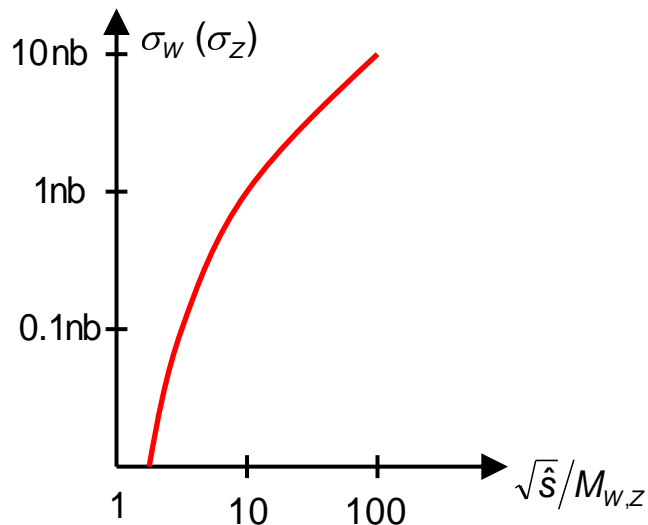
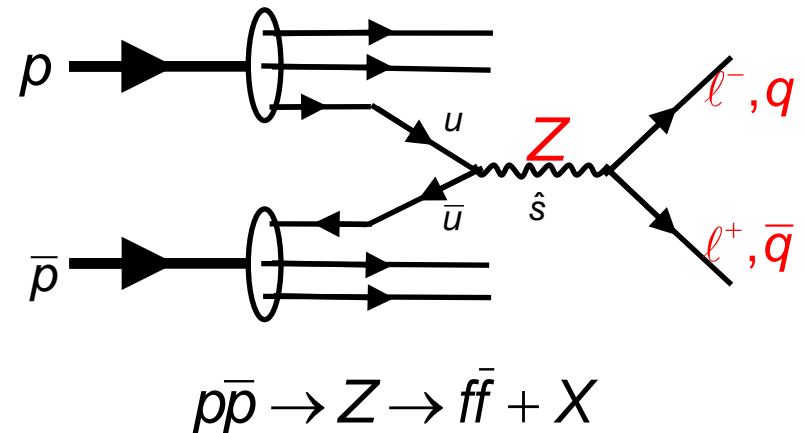
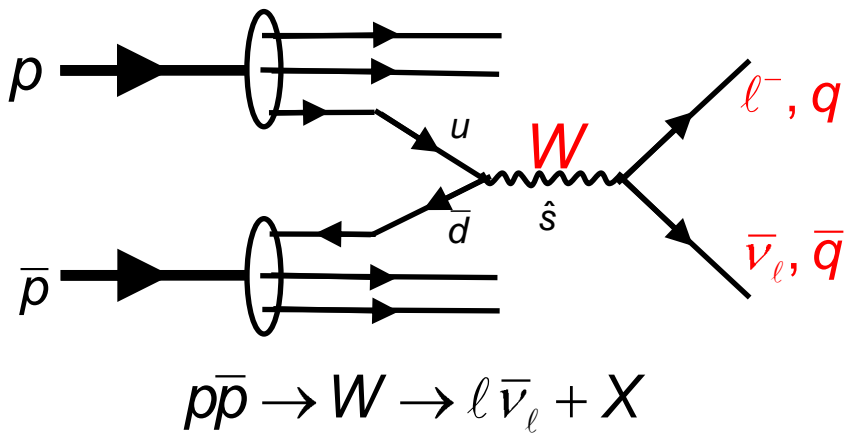
$$\rho = 1 - \Delta\rho = 1$$

$$\rho \cos^2 \theta_W = \frac{M_W^2}{M_Z^2}$$

1. Discovery of the W and Z boson

1983 at CERN Sp \bar{p} S accelerator,
 $\sqrt{s} \approx 540$ GeV, UA-1/2 experiments

1.1 Boson production in $p\bar{p}$ interactions



Similar to Drell-Yan: (photon instead of W)

$$\hat{s} = x_q x_{\bar{q}} s \quad \text{mit} \quad \langle x_q \rangle \approx 0.12$$

$$\hat{s} = \langle x_q \rangle^2 s \approx 0.014s = (65 \text{ GeV})^2$$

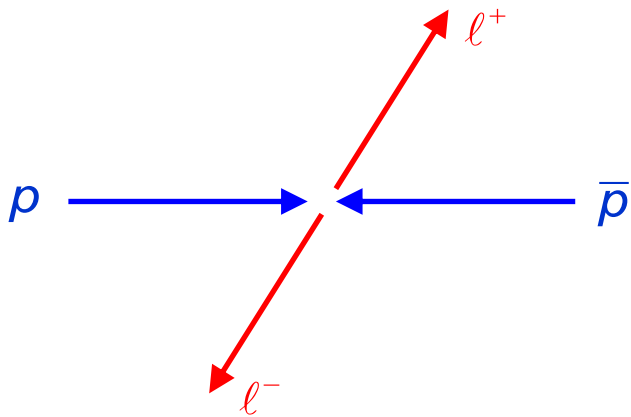
→ Cross section is small !

1.2 Event signature: $p\bar{p} \rightarrow Z \rightarrow f\bar{f} + X$

$$p + \bar{p} \rightarrow Z^0 + X$$

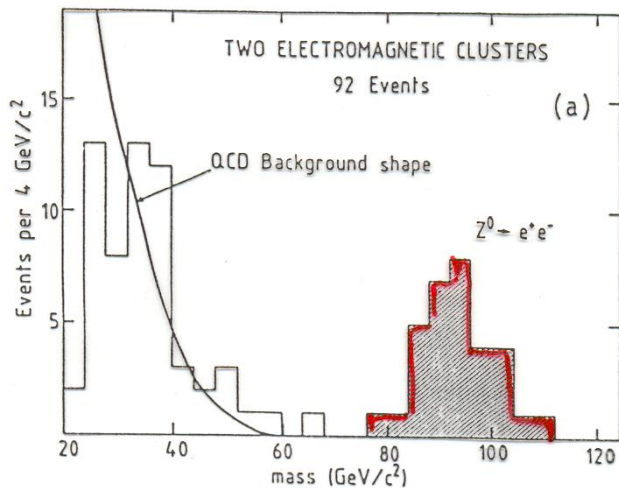
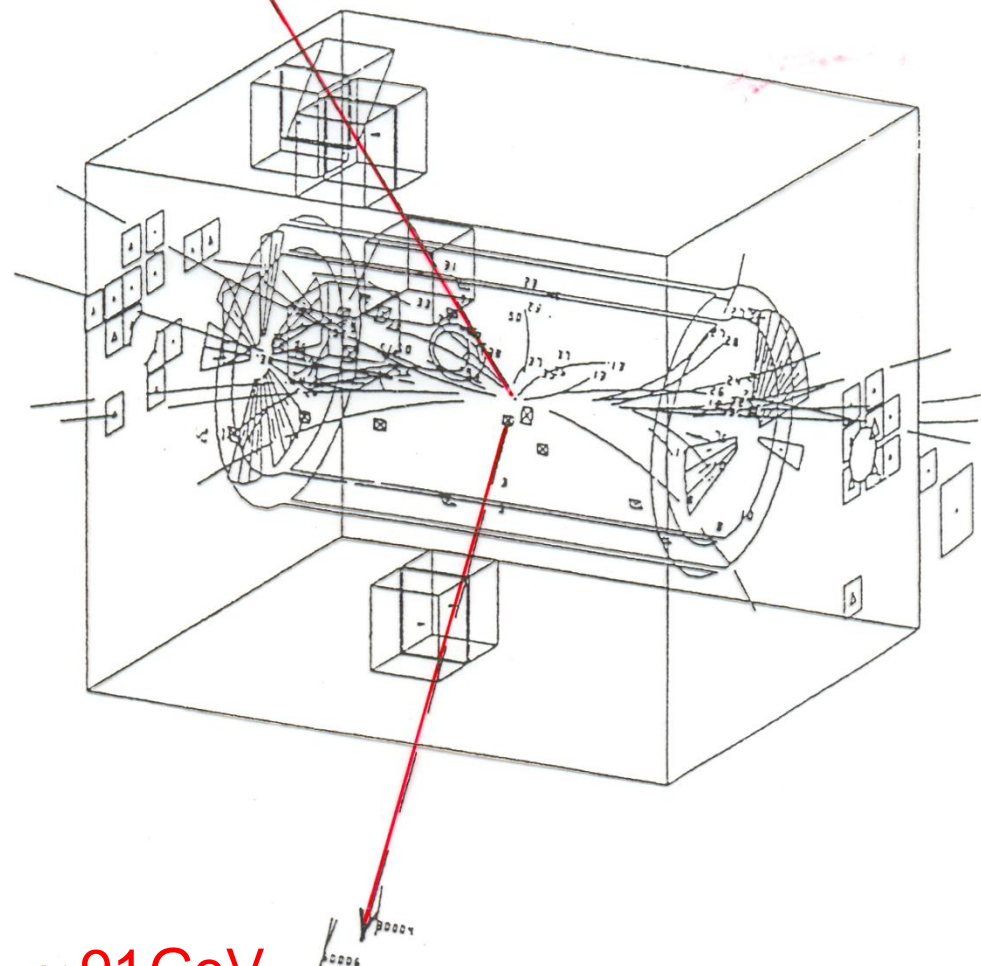
$$\downarrow$$

$$f^+ f^-$$



High-energy lepton pair:

$$m_{\ell\ell}^2 = (p_{\ell^+} + p_{\ell^-})^2 = M_Z^2$$

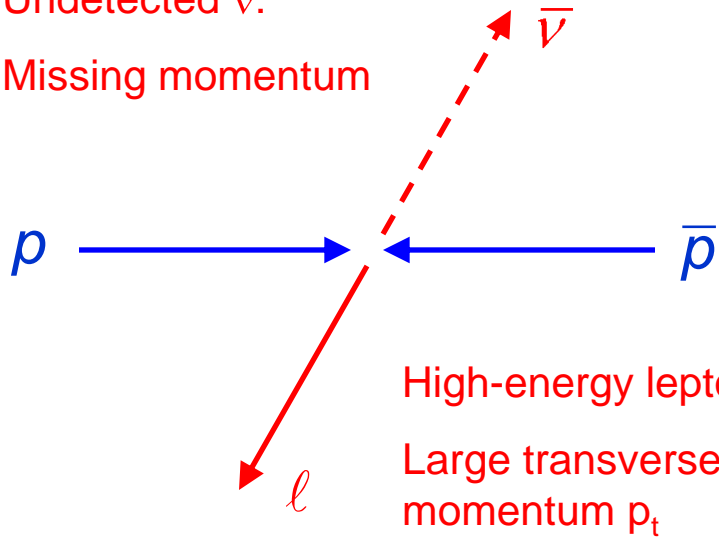


$M_Z \approx 91 \text{ GeV}$

1.3 Event signature: $p\bar{p} \rightarrow W \rightarrow \ell \bar{\nu}_\ell + X \quad W^- \rightarrow e \bar{\nu}$

Undetected ν :

Missing momentum



High-energy lepton:

Large transverse momentum p_t

How can the W mass be reconstructed ?

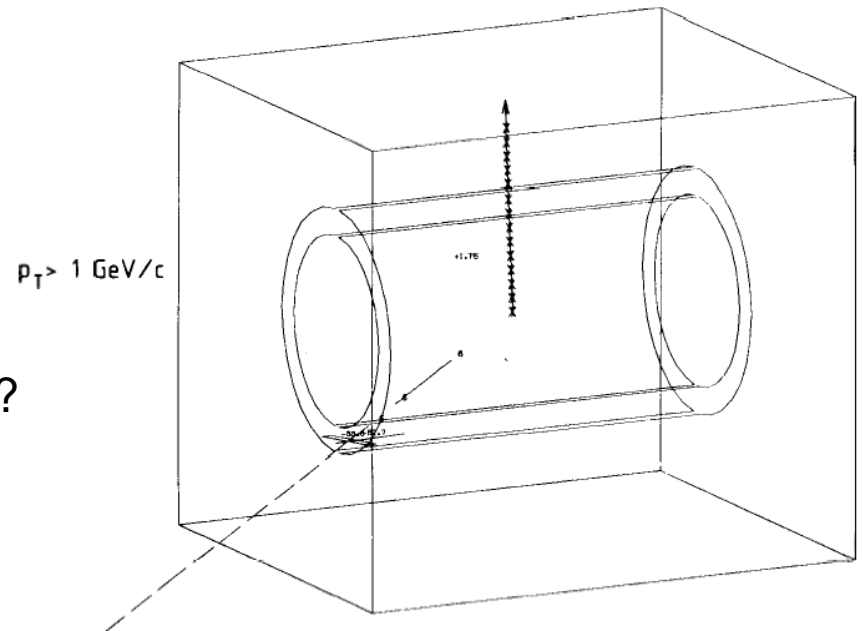
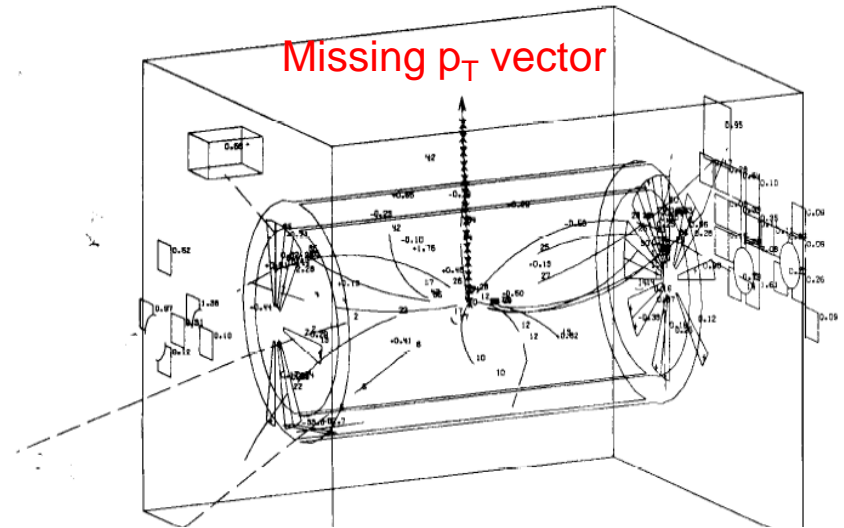
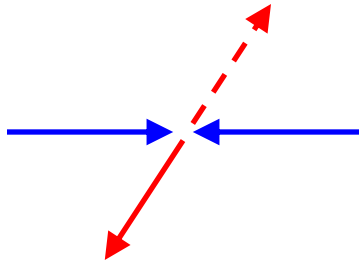


Fig. 16b. The same as picture (a), except that now only particles with $p_T > 1$ GeV/c and calorimeters with $E_T > 1$ GeV are shown.

W mass measurement



In the W rest frame:

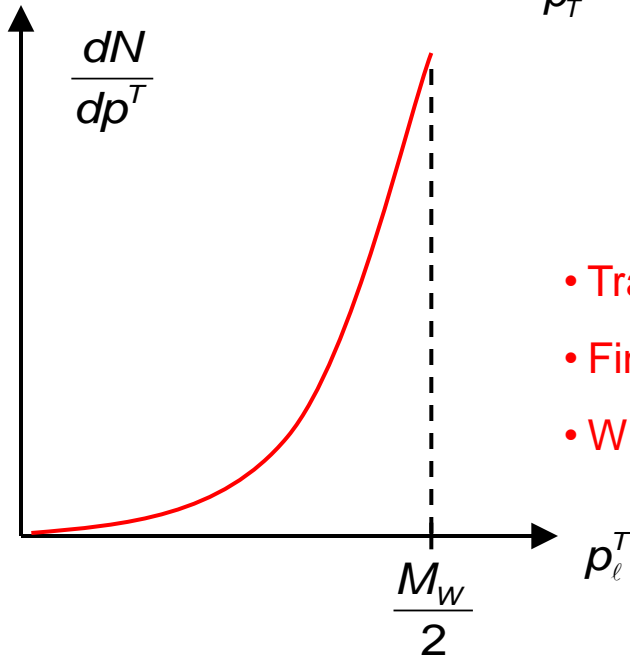
- $|\vec{p}_\ell| = |\vec{p}_\nu| = \frac{M_W}{2}$
- $|p_\ell^T| \leq \frac{M_W}{2}$

In the lab system:

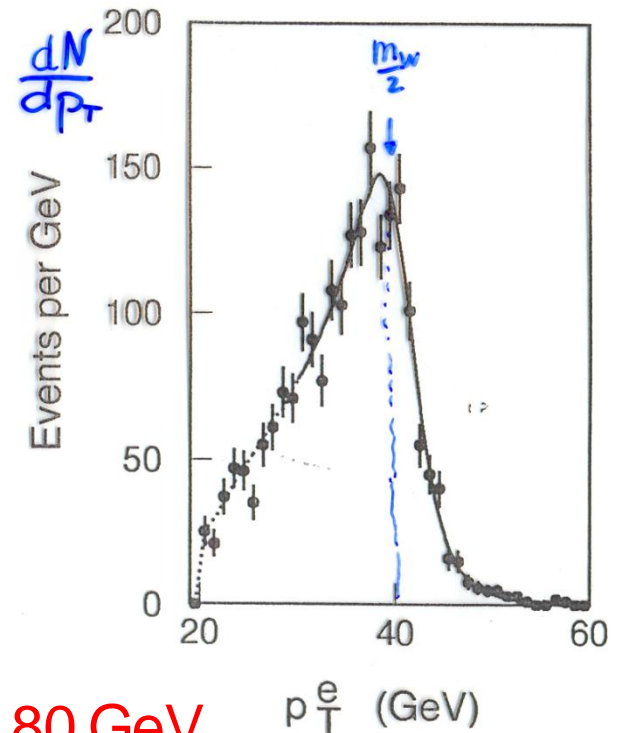
- W system boosted only along z axis
- p_T distribution is conserved: maximum $p_T = M_W / 2$

Jacobian Peak:

$$\frac{dN}{p_T} \sim \frac{2p_T}{M_W} \cdot \left(\frac{M_W^2}{4} - p_T^2 \right)^{-1/2}$$



- Trans. Movement of the W
- Finite W decay width
- W decay not isotropic

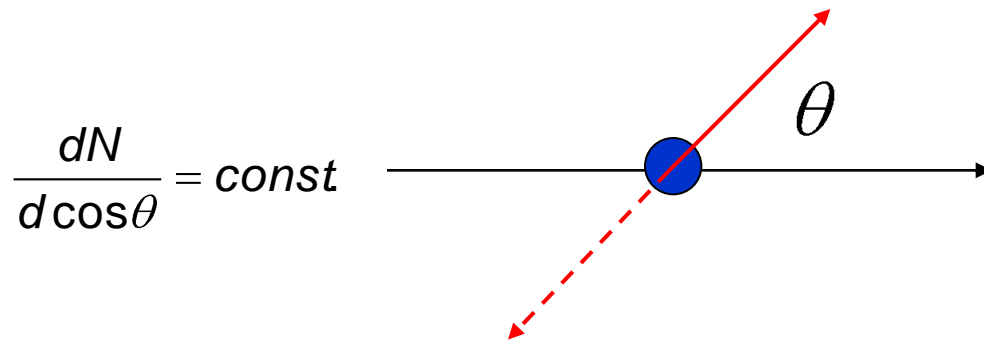


$$M_W \approx 80 \text{ GeV}$$

Jacobian Peak

Assume isotropic decay of the W boson in its CM system:

(Not really correct: W boson has spin=1 → decay is not isotropic!)



$$\frac{dN}{d\cos\theta} = \text{const}$$

$$\sin\theta = \frac{p_T}{p} = \frac{p_T}{M_W/2}$$

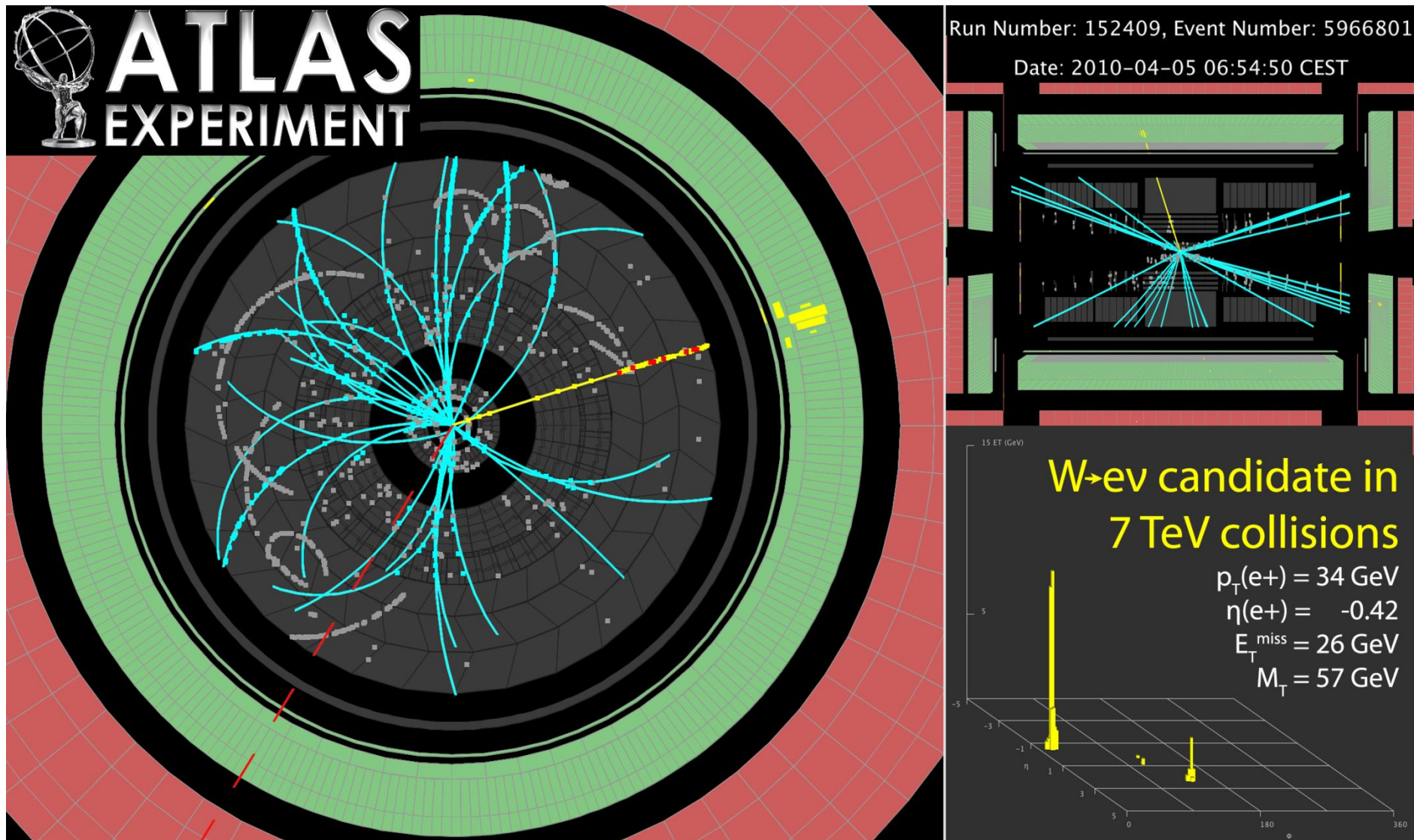
$$1 - \cos^2\theta = \left(\frac{p_T}{M_W/2}\right)^2$$

$$d\cos\theta \sim \frac{p_T}{M_W/2} \frac{dp_T}{\cos\theta}$$

$$\frac{dN}{dp_T} = \left(\frac{dN}{d\cos\theta}\right) \cdot \left(\frac{d\cos\theta}{dp_T}\right) \sim \frac{2p_T}{M_W} \cdot \left(\frac{M_W^2}{4} - p_T^2\right)^{-1/2}$$

Jacobian

W candidate from the LHC – they still exist!



$$u \bar{d} \rightarrow W^+ \rightarrow e^+ \nu_e$$

Anti-quarks from the sea!

Z bosons are also produced at the LHC

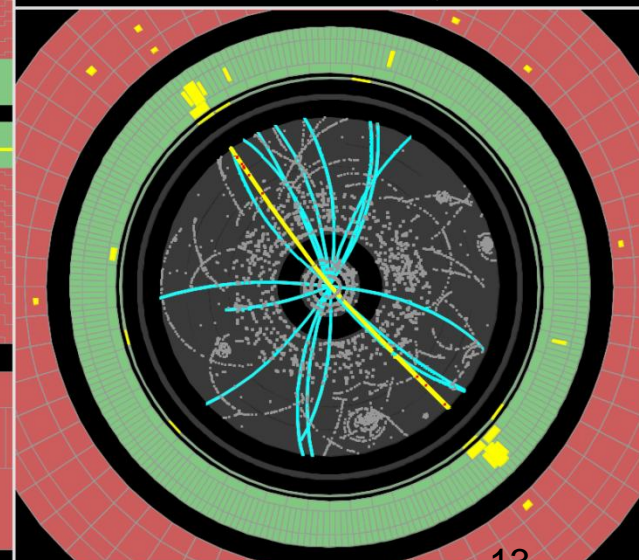
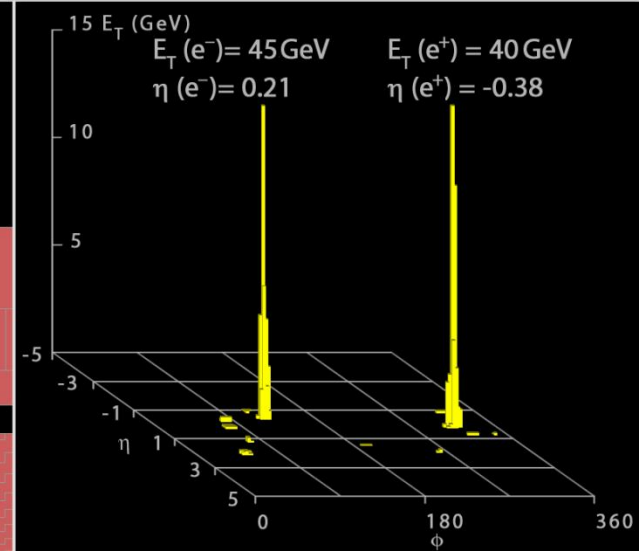
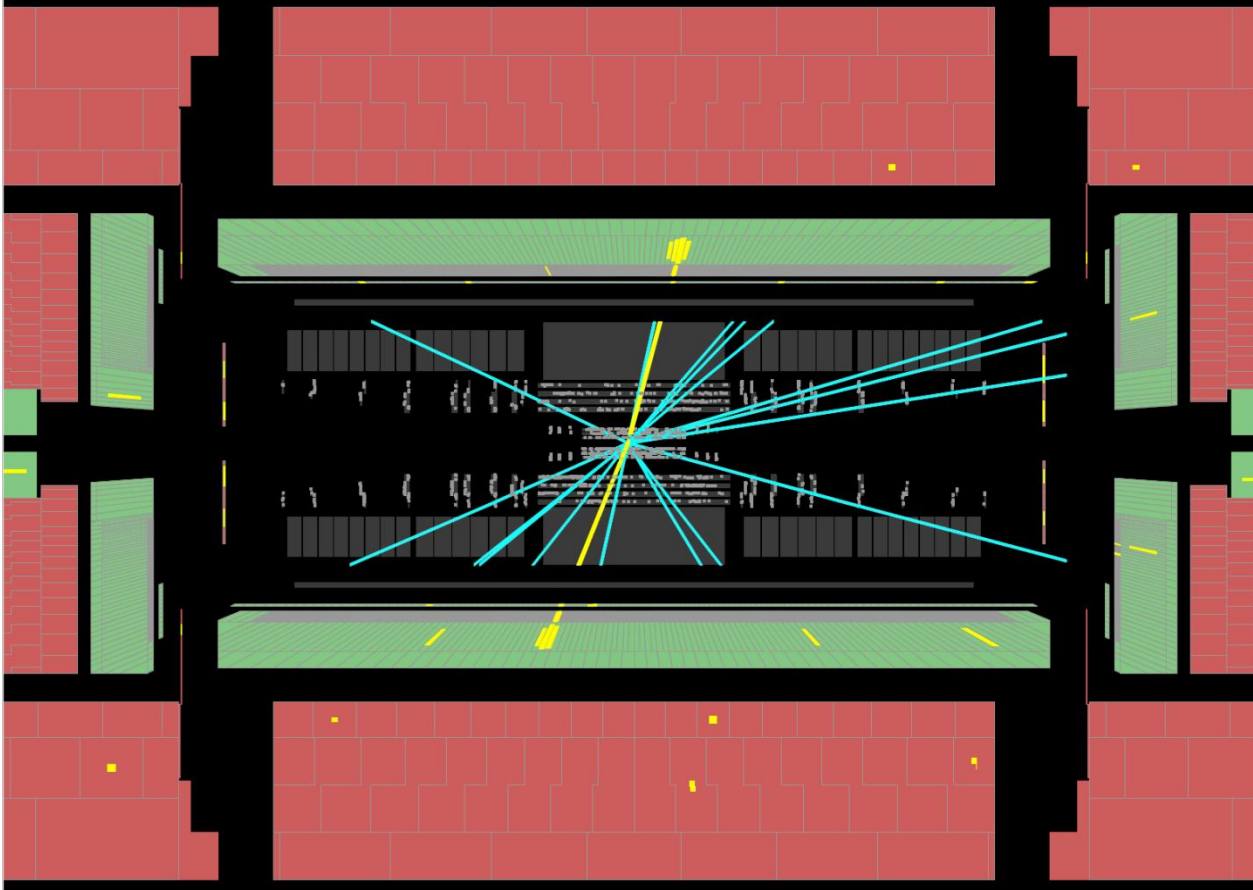


Run Number: 154817, Event Number: 968871

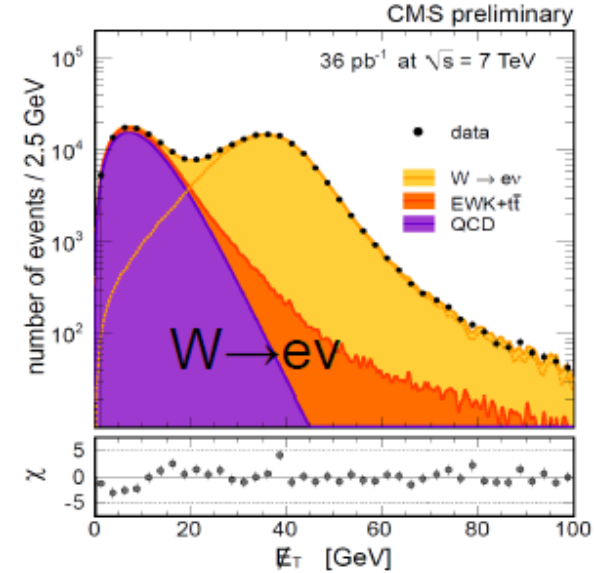
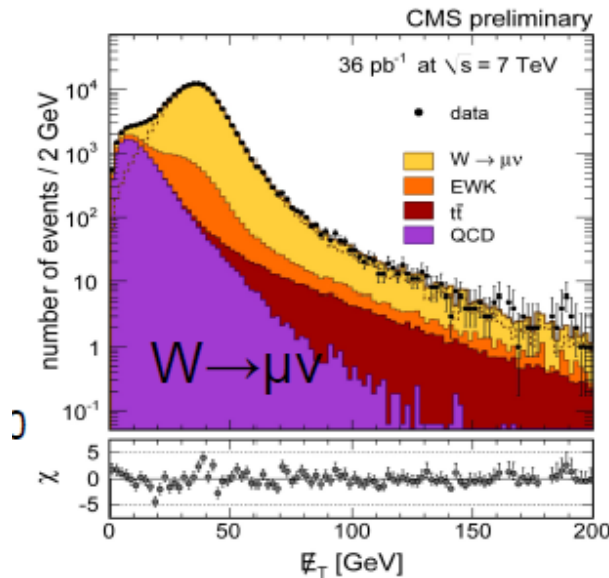
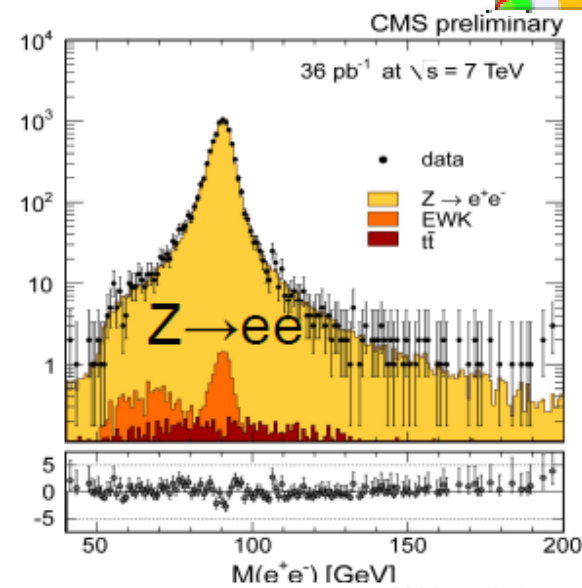
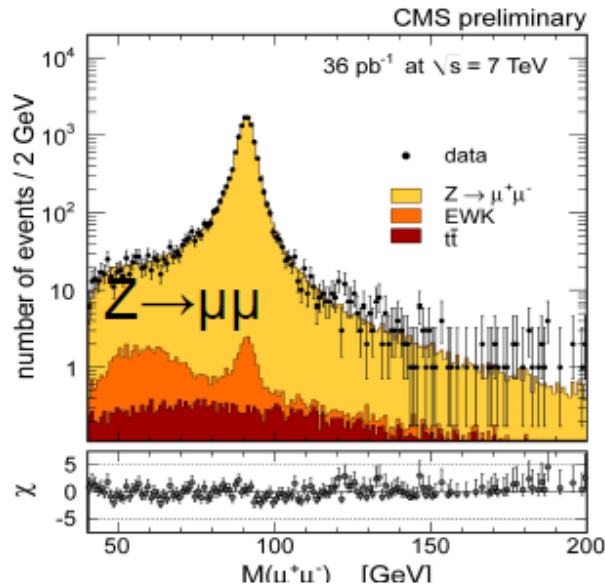
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$M_{ee} = 89 \text{ GeV}$

Z \rightarrow ee candidate in 7 TeV collisions



Z and W production at LHC

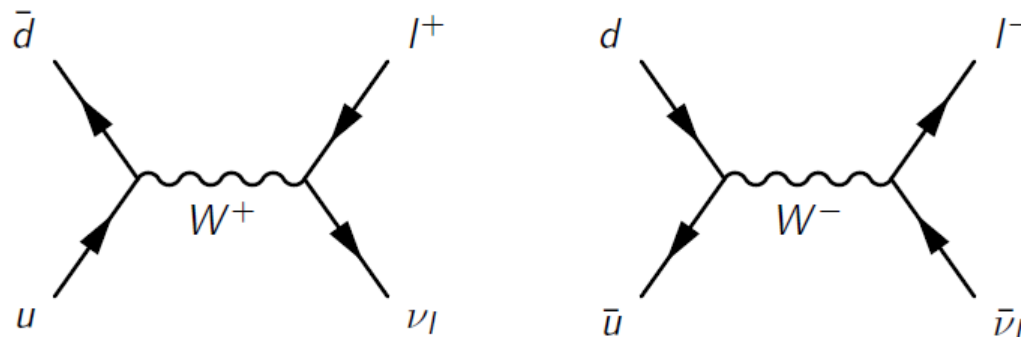


*P.Harris,
Moriond 2011*

Instead of E_{eT}
use E_T (i.e. $E_{\nu T}$)

W-boson production at LHC

Valence quark +
sea quark

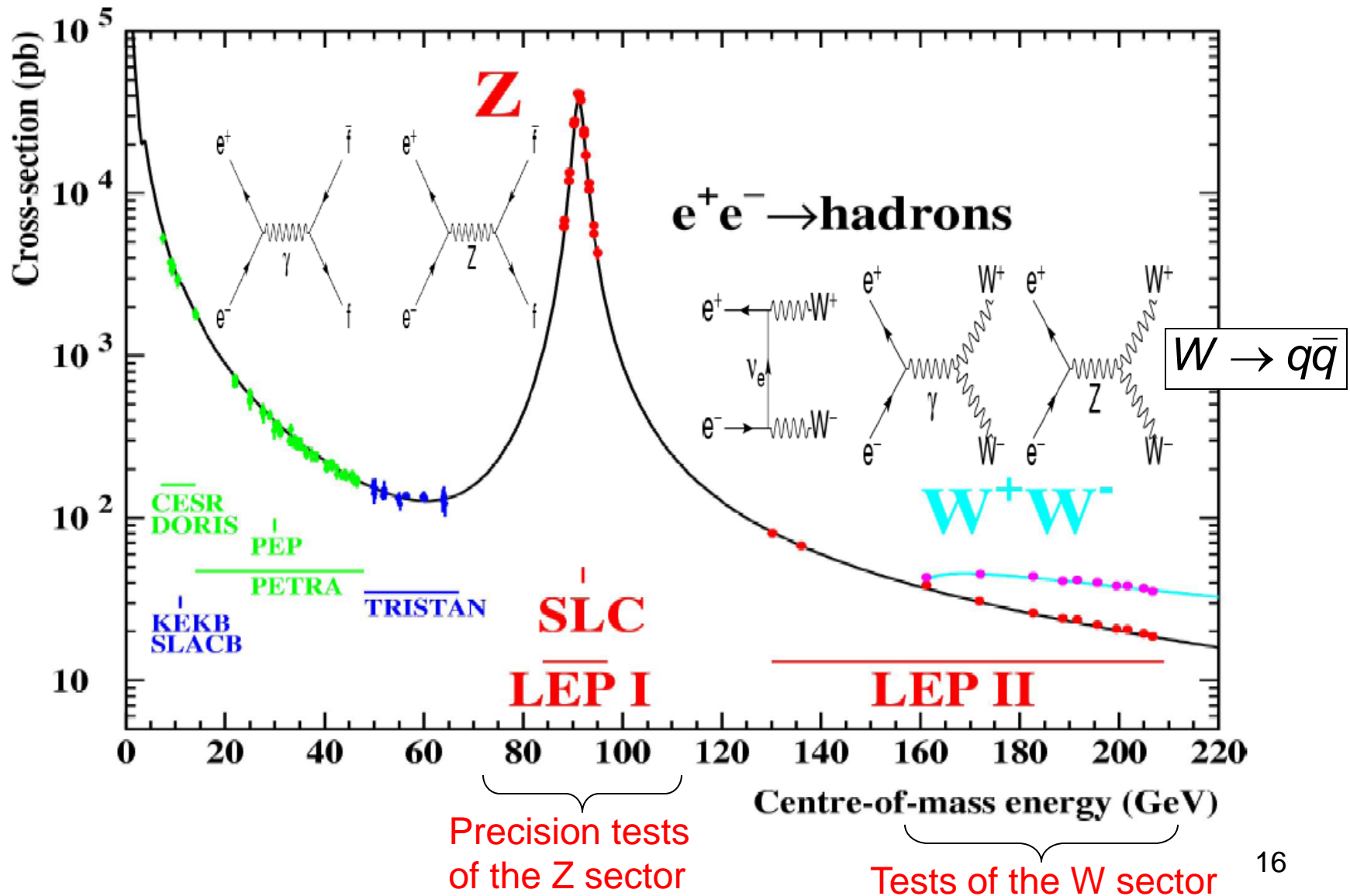


valence quark ratio $u/d = 2 \Rightarrow$ more W^+ than W^-

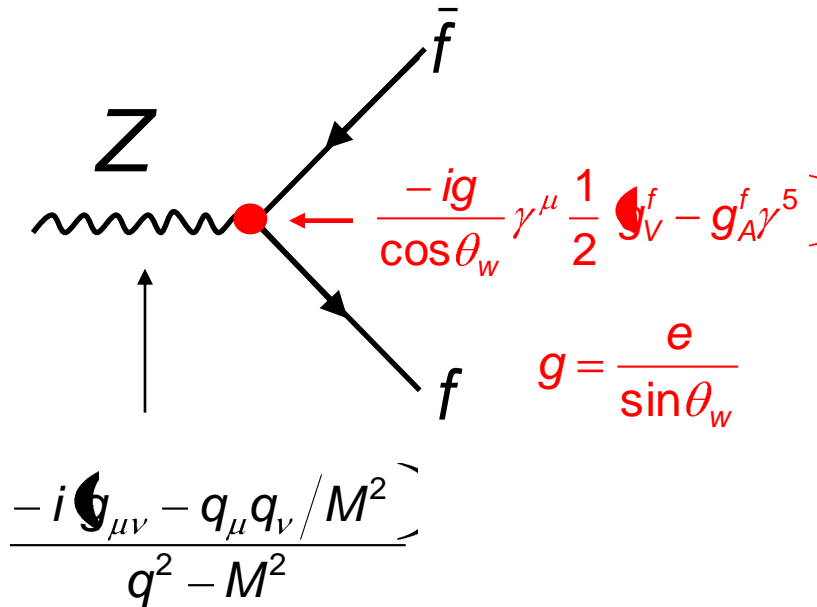
ATLAS 2010:

[nb]	Data
W^+	$6.257 \pm 0.017(\text{sta}) \pm 0.152(\text{sys}) \pm 0.213(\text{lum}) \pm 0.188(\text{acc})$
W^-	$4.149 \pm 0.014(\text{sta}) \pm 0.102(\text{sys}) \pm 0.141(\text{lum}) \pm 0.124(\text{acc})$
W	$10.391 \pm 0.022(\text{sta}) \pm 0.238(\text{sys}) \pm 0.353(\text{lum}) \pm 0.312(\text{acc})$

1.4 Production of Z and W bosons in e^+e^- annihilation



2. Precision tests of the Z sector (LEP and SLC)



Standard Model

$$g_V = T_3 - 2Q\sin^2\theta_W \quad \text{and} \quad g_A = T_3$$

$$g_L = \frac{1}{2}(g_V + g_A) \quad g_R = \frac{1}{2}(g_V - g_A)$$

$$\frac{g_V}{g_A} = 1 - 2\frac{Q}{T_3}\sin^2\theta_W = 1 - 4|Q|\sin^2\theta_W$$

$$\sin^2\theta_w = 1 - \frac{M_w^2}{M_Z^2}$$

	g_V	g_A
ν	$\frac{1}{2}$	$\frac{1}{2}$
l^-	$-\frac{1}{2} + 2\sin^2\theta_w$	$-\frac{1}{2}$
u - quark	$+\frac{1}{2} - \frac{4}{3}\sin^2\theta_w$	$\frac{1}{2}$
d - quark	$-\frac{1}{2} + \frac{2}{3}\sin^2\theta_w$	$-\frac{1}{2}$

Cross section for $e^+ e^- \rightarrow \gamma / Z \rightarrow f \bar{f}$

$$|M|^2 = \left| \text{diagram with } \gamma \text{ propagator} + \text{diagram with } Z \text{ propagator} \right|^2$$

for $e^+ e^- \rightarrow \mu^+ \mu^-$

$$M_\gamma = -ie^2 (\bar{u}_\mu \gamma^\nu v_\mu) \frac{g_{\rho\nu}}{q^2} (\bar{v}_e \gamma^\rho u_e)$$

$$M_Z = -i \frac{g^2}{\cos^2 \theta_W} \left[\bar{u}_\mu \gamma^\nu \frac{1}{2} (g_V^\mu - g_A^\mu \gamma^5) v_\mu \right] \underbrace{\frac{g_{\rho\nu} - q_\rho q_\nu / M_Z^2}{(q^2 - M_Z^2) + iM_Z \Gamma_Z}}_{\text{Z propagator considering a finite Z width (real particle)}} \left[\bar{v}_e \gamma^\rho \frac{1}{2} (g_V^e - g_A^e \gamma^5) u_e \right]$$

Z propagator considering a finite Z width (real particle)

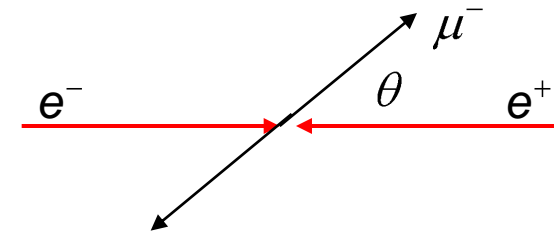
With a “little bit” of algebra similar as for M_γ

One finds for the differential cross section:

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{2s} \left[F_\gamma(\cos\theta) + F_{\gamma Z}(\cos\theta) \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} + F_Z(\cos\theta) \frac{s^2}{(s - M_Z^2)^2 + M_Z^2\Gamma_Z^2} \right]$$

known
 γ
 γ/Z interference
Z

Vanishes at $\sqrt{s} \approx M_Z$



$$F_\gamma(\cos\theta) = Q_e^2 Q_\mu^2 (1 + \cos^2\theta) = (1 + \cos^2\theta)$$

$$F_{\gamma Z}(\cos\theta) = \frac{Q_e Q_\mu}{4 \sin^2\theta_W \cos^2\theta_W} \left[g_V^e g_V^\mu (1 + \cos^2\theta) + 4 g_A^e g_A^\mu \cos\theta \right]$$

$$F_Z(\cos\theta) = \frac{1}{16 \sin^4\theta_W \cos^4\theta_W} \left[(g_V^e{}^2 + g_A^e{}^2)(g_V^\mu{}^2 + g_A^\mu{}^2)(1 + \cos^2\theta) + 8 g_V^e g_A^e g_V^\mu g_A^\mu \cos\theta \right]$$

At the Z-pole $\sqrt{s} \approx M_Z \rightarrow$ Z contribution is dominant
 \rightarrow interference vanishes

$$\sigma_{tot} \approx \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16 \sin^4 \theta_w \cos^4 \theta_w} \cdot \left[(g_V^e)^2 + (g_A^e)^2 \right] \left[(g_V^\mu)^2 + (g_A^\mu)^2 \right] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$$

Forward-backward asymmetry

$$\frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta$$

$$\text{with } \begin{cases} A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \\ \sigma_{F(B)} = \int_{0^{(-1)}}^{1^{(0)}} \frac{d\sigma}{d\cos\theta} d\cos\theta \end{cases}$$

$$A_{FB} = 3 \cdot \frac{g_V^e g_A^e}{(g_V^e)^2 + (g_A^e)^2} \cdot \frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2}$$

$$\sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16 \sin^4 \theta_w \cos^4 \theta_w} \cdot \left[(g_V^e)^2 + (g_A^e)^2 \right] \left[(g_V^\mu)^2 + (g_A^\mu)^2 \right] \cdot \frac{s^2}{(s - M_Z^2)^2 + (M_Z \Gamma_Z)^2}$$



Breit-Wigner Resonance:
BW description is very general

$$\sigma_Z \left(\sqrt{s} = M_Z \right) = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

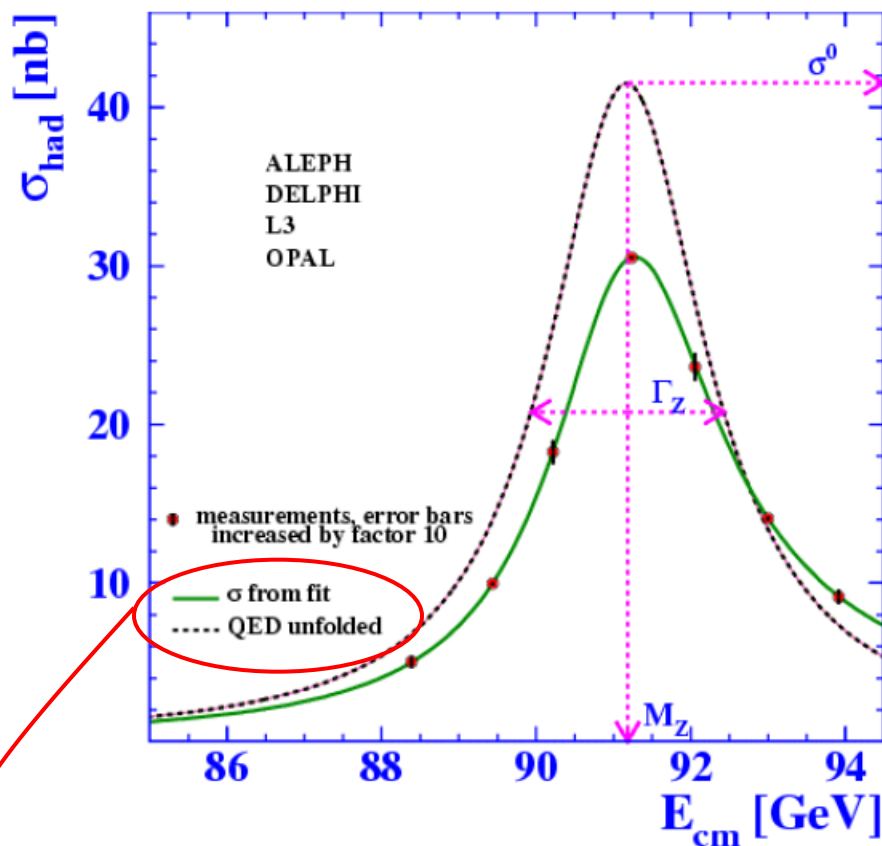
With partial and total widths:

$$\Gamma_f = \frac{\alpha M_Z}{12 \sin^2 \theta_w \cos^2 \theta_w} \cdot \left[(g_V^f)^2 + (g_A^f)^2 \right]$$

$$\Gamma_Z = \sum_i \Gamma_i \quad BR(Z \rightarrow ii) = \frac{\Gamma_i}{\Gamma_Z}$$

Cross sections and widths
can be calculated within the
Standard Model if all
parameters are known

2.2 Measurement of the Z lineshape



Z Resonance curve:

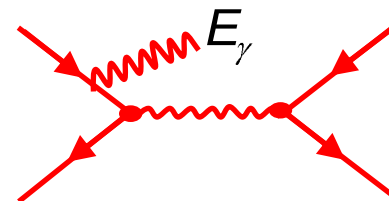
$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_\mu}{M_Z^2} \cdot \frac{s}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2}$$

Peak:
$$\sigma_0 = \frac{12\pi}{M_Z^2} \frac{\Gamma_e \Gamma_\mu}{\Gamma_Z^2}$$

- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_\mu$
- Width $\rightarrow \Gamma_Z$

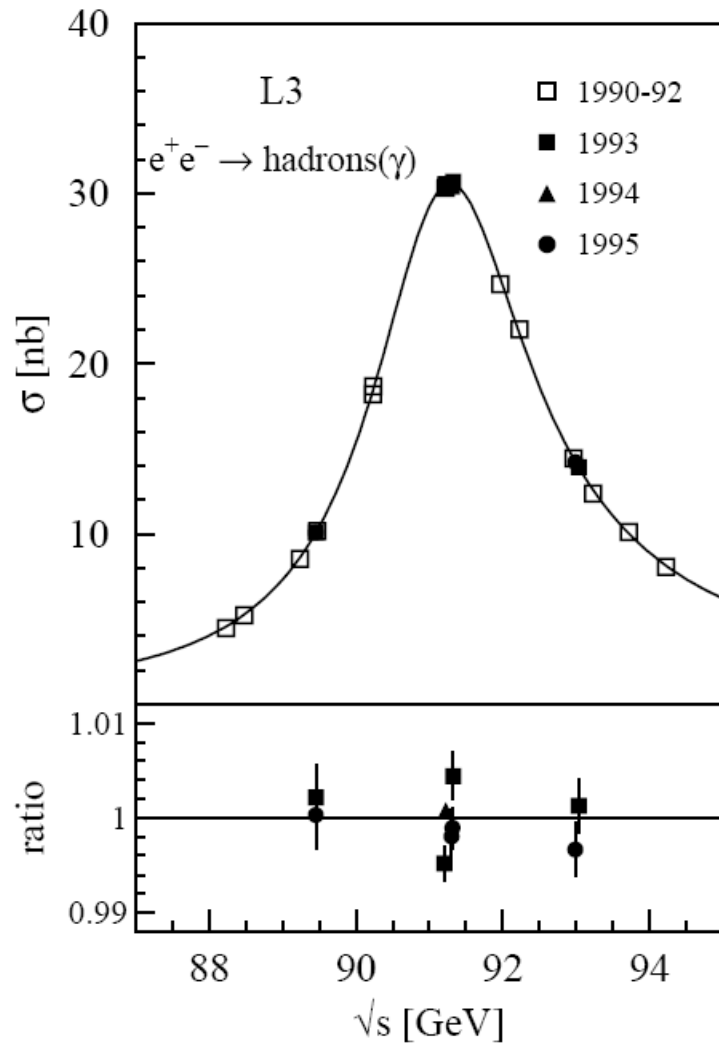
Initial state Bremsstrahlung corrections

$$\sigma_{ff(\gamma)} = \int_{4m_f^2/s}^1 G(z) \sigma_{ff}^0(zs) dz \quad z = 1 - \frac{2E_\gamma}{\sqrt{s}}$$

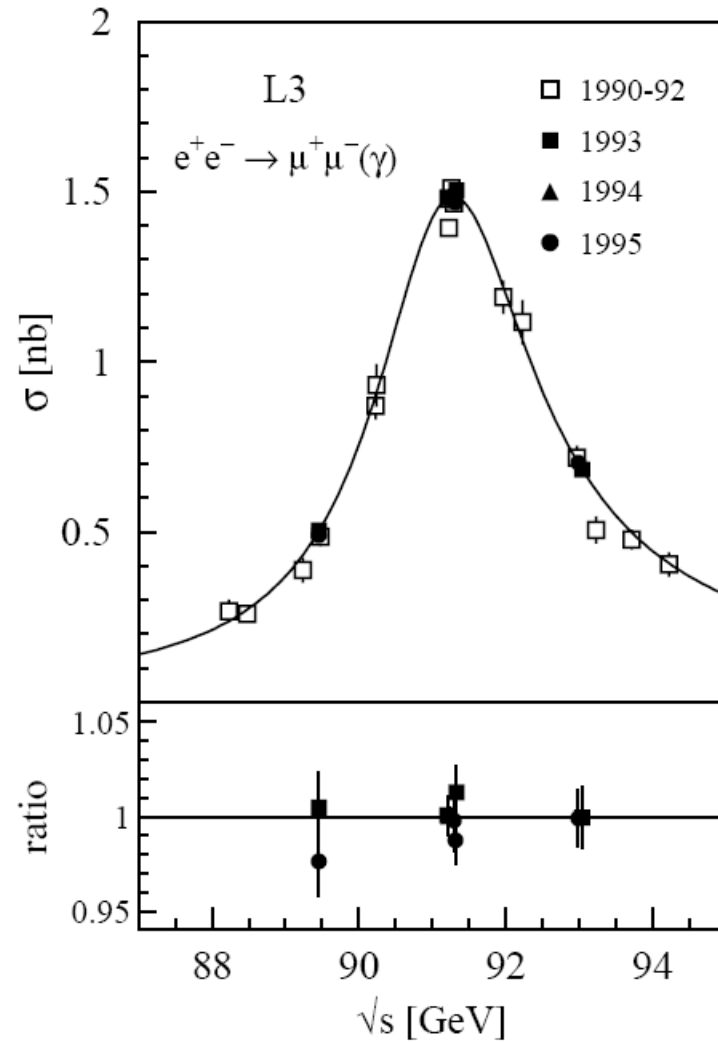


Leads to a deformation of the resonance: large (30%) effect !

$e^+ e^- \rightarrow \text{hadrons}$

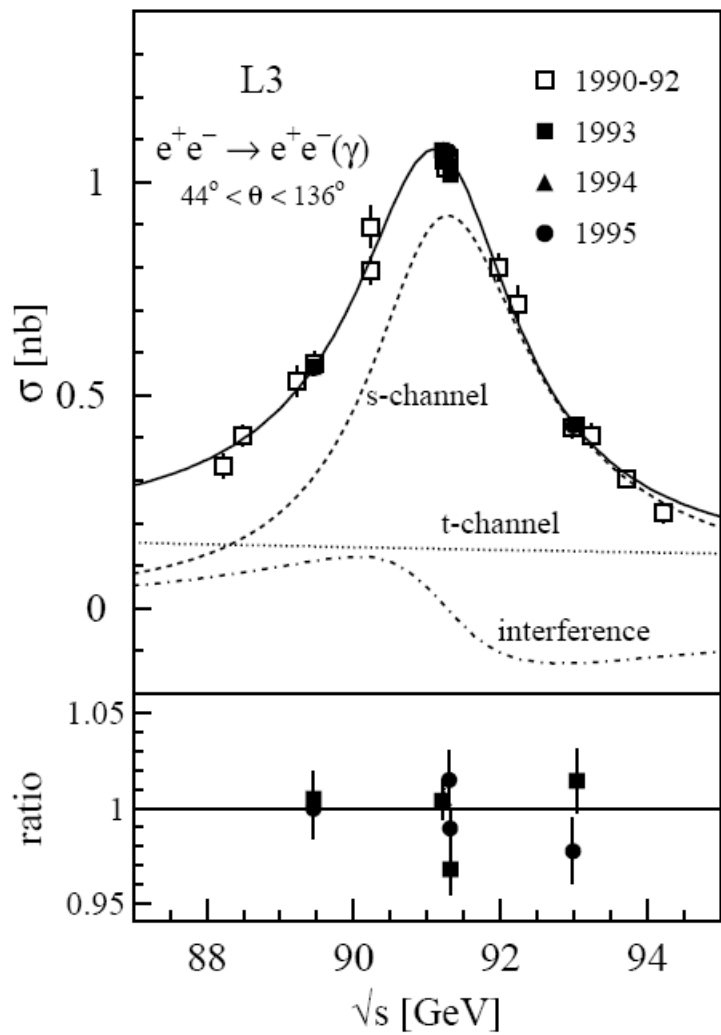


$e^+ e^- \rightarrow \mu^+ \mu^-$



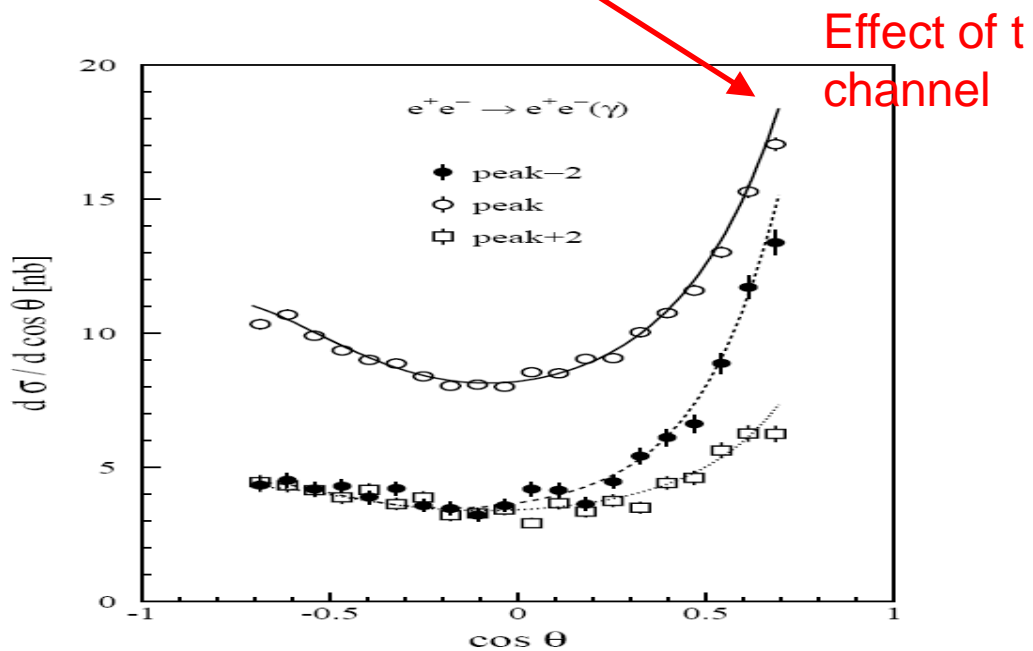
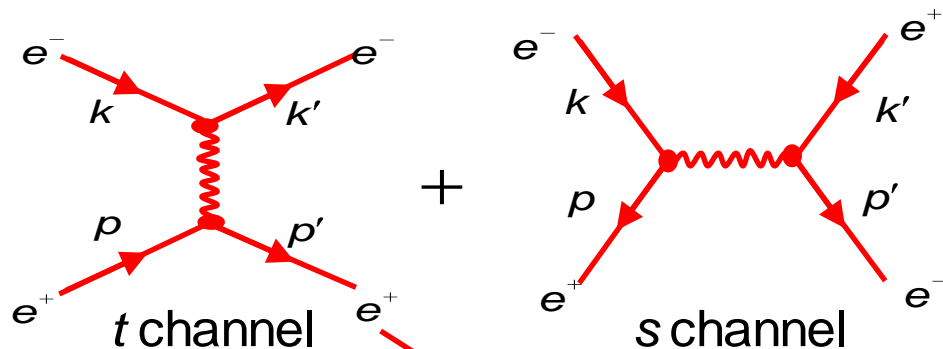
Resonance shape is the same, independent of final state: Propagator the same!

$$e^+ e^- \rightarrow e^+ e^-$$



$$\text{s-channel contribution} \sim (\Gamma_e)^2$$

t channel contribution \rightarrow forward peak



Z line shape parameters (LEP average)

M_Z	=	91.1876 ± 0.0021 GeV	± 23 ppm (*)
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$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\Gamma_{\text{had}} = 1.7458 \pm 0.0027 \text{ GeV}$$

$$\Gamma_e = 0.08392 \pm 0.00012 \text{ GeV}$$

$$\Gamma_\mu = 0.08399 \pm 0.00018 \text{ GeV}$$

$$\Gamma_\tau = 0.08408 \pm 0.00022 \text{ GeV}$$

$\pm 0.09\%$

3 leptons are treated independently



test of lepton universality

$$\Gamma_Z = 2.4952 \pm 0.0023 \text{ GeV}$$

$$\Gamma_{\text{had}} = 1.7444 \pm 0.0022 \text{ GeV}$$

$$\Gamma_e = 0.083985 \pm 0.000086 \text{ GeV}$$

Assuming lepton universality: $\Gamma_e = \Gamma_\mu = \Gamma_\tau$

(predicted by SM: g_A and g_V are the same)

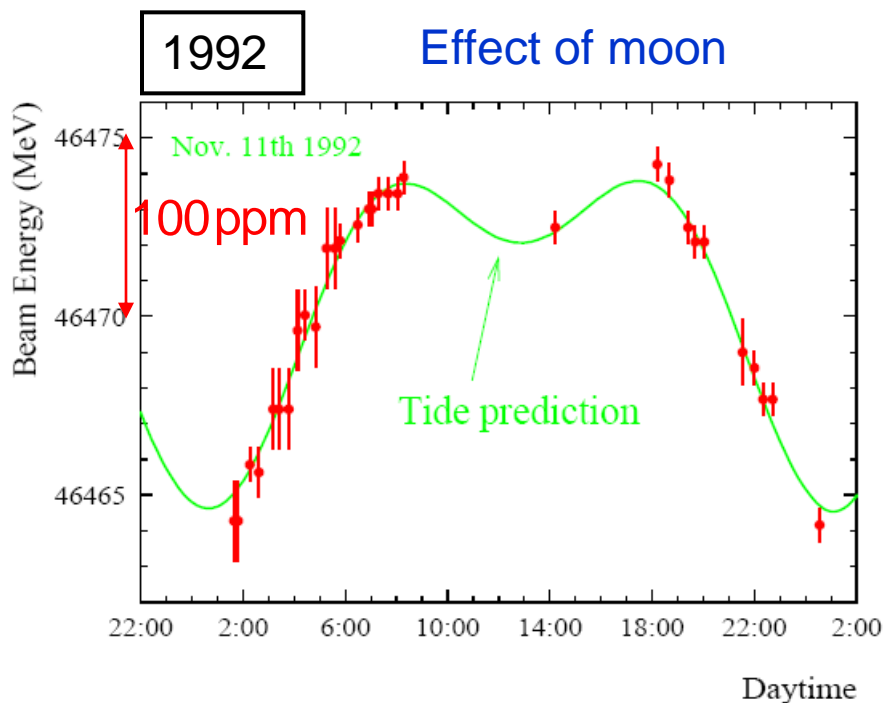
*) error of the LEP energy determination: ± 1.7 MeV (19 ppm)

LEP energy calibration: Hunting for ppm effects

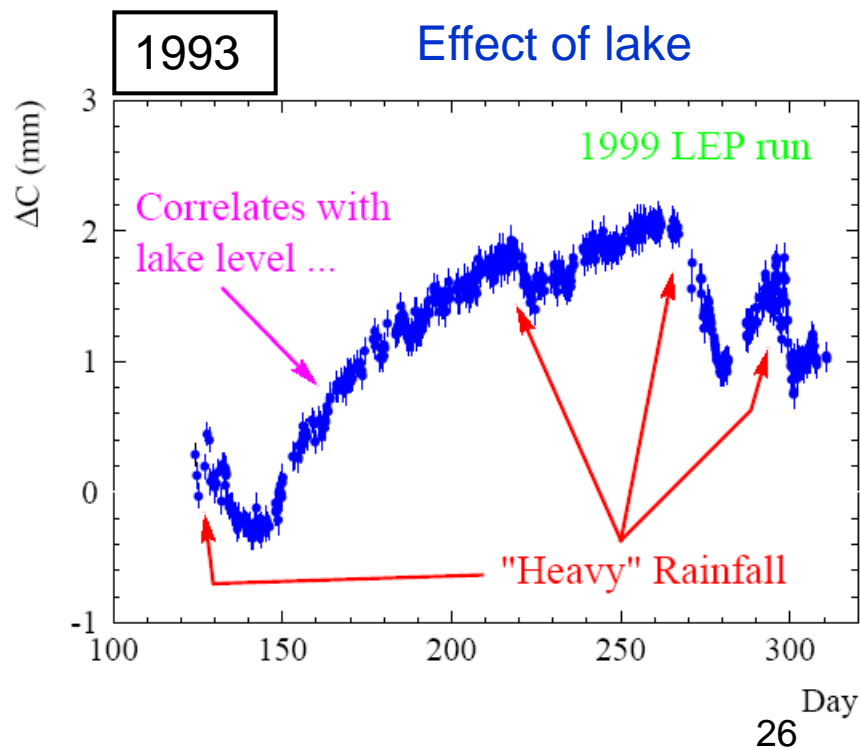
Changes of the circumference of the LEP ring changes the energy of the electrons and thus the CM energy (shifts M_Z) :

- tide effects
- water level in lake Geneva

Changes of LEP circumference $\Delta C = 1 \dots 2 \text{ mm} / 27 \text{ km}$ ($4 \dots 8 \times 10^{-8}$)

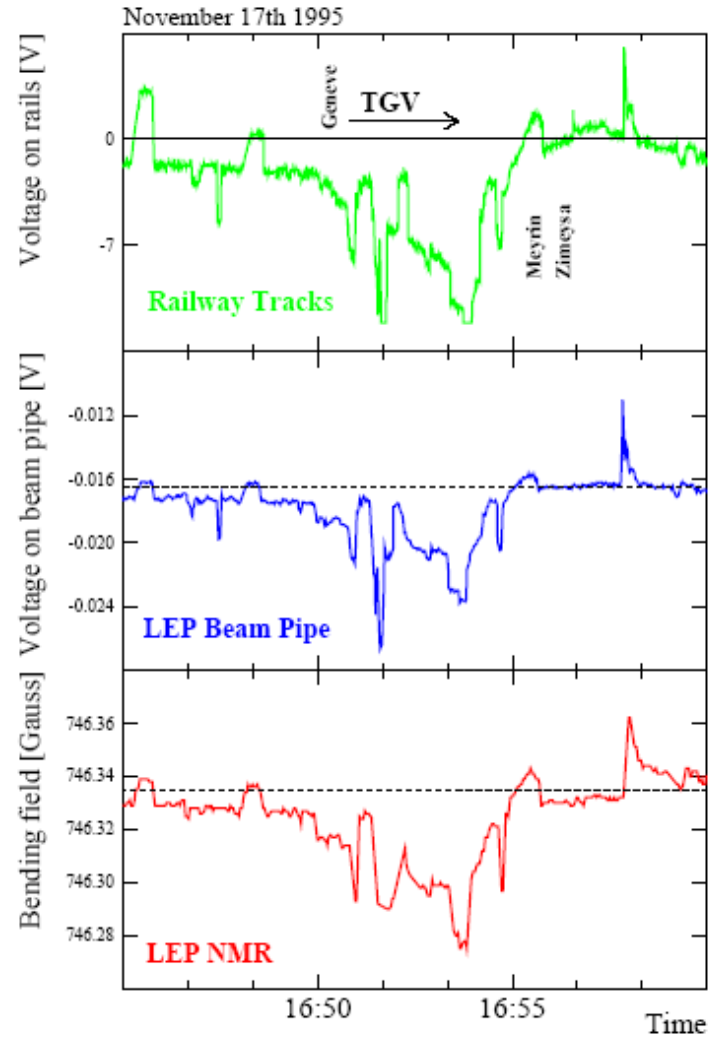
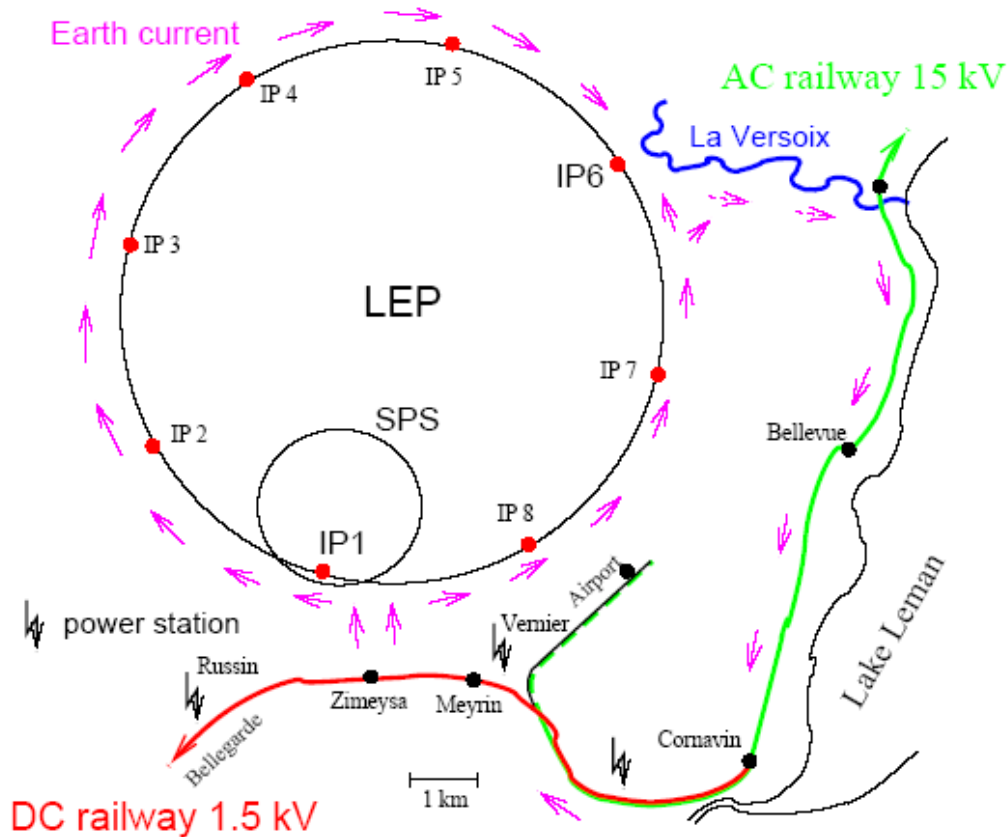


The total strain is 4×10^{-8} ($\Delta C = 1 \text{ mm}$)



Effect of the French "Train a Grande Vitesse" (TGV)

Vagabonding currents from trains



In conclusion: Measurements at the ppm level are difficult to perform. Many effects must be considered!