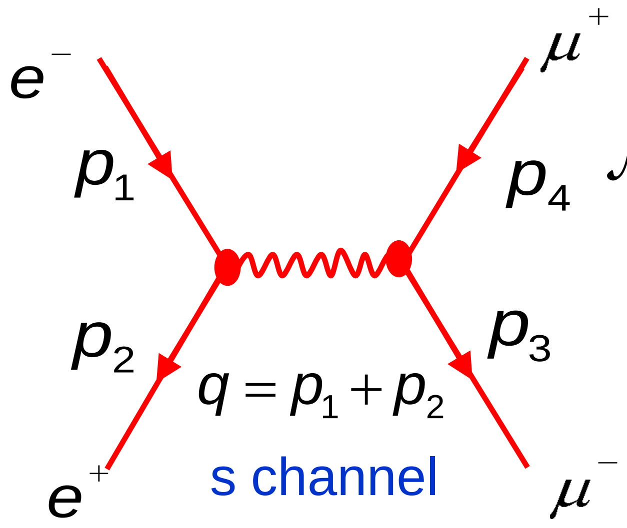
# 2. $e^+e^-$ scattering experiments

## 2.1 Fermion scattering

$$e^+e^- \rightarrow \mu^+\mu^-$$




$$\mathcal{M}_{fi} \sim -\frac{e^2}{q^2} \underbrace{\bar{v}_e(p_2)\gamma_\mu u_e(p_1) \cdot \bar{u}_\mu(p_3)\gamma^\mu v_\mu(p_4)}$$

Spinors decribe a specific spin state of the fermions

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections $\Rightarrow$ need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{\left|\mathcal{M}_{fi}\right|^2} = \frac{1}{4} \cdot \sum_{s_e, s'_e} \sum_{s_\mu, s'_\mu} \left|\mathcal{M}_{fi}\right|^2$$

Averaging and summing over spins of initial/final state:

$$\left|\overline{\mathcal{M}_{fi}}\right|^2 = \frac{1}{4}\frac{e^4}{s^2}\sum_{s,s',r,r'} |\bar{u}_{\mu,s}(p_3)\gamma^\nu v_{\mu,s'}(p_4)\ \bar{v}_{e,r}(p_2)\gamma_\nu u_{e,r'}(p_1)|^2$$

neglect masses

$$= 8\frac{e^4}{s^2}\left[(p_1p_4)(p_2p_3) + (p_2p_4)(p_1p_3)\right]$$

Lorentz invariant!

By using the Mandelstam variables in the relativistic limit

$$s = (p_1 + p_2)^2 = m^2 + m^2 + 2p_1p_2 \approx 2p_1p_2 \approx 2p_3p_4$$
$$t = (p_1 - p_3)^2 = m^2 + M^2 - 2p_1p_3 \approx -2p_1p_3 \approx -2p_2p_4$$
$$u = (p_1 - p_4)^2 = m^2 + M^2 - 2p_1p_4 \approx -2p_1p_4 \approx -2p_2p_3$$

if masses neglected

$$\overline{\left|\mathcal{M}_{fi}\right|^2} = 2e^4\,\frac{t^2 + u^2}{s^2}$$

Remember: matrix element squared can be expressed in s, u, t!

$$\overline{\left|\mathcal{M}_{fi}\right|^2}_{e^+e^-\to\mu^+\mu^-}(s,t,u) = 2e^4\,\frac{t^2+u^2}{s^2}$$

$$\Downarrow \qquad \frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\cdot\frac{1}{s}\cdot\overline{\left|\mathcal{M}_{fi}\right|^2}$$

$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2}\cdot\frac{1}{s}\cdot\frac{t^2+u^2}{s^2}$$

$$= \frac{e^4}{64\pi^2}\cdot\frac{1}{s}\cdot\left(1+\cos^2\theta\right)$$

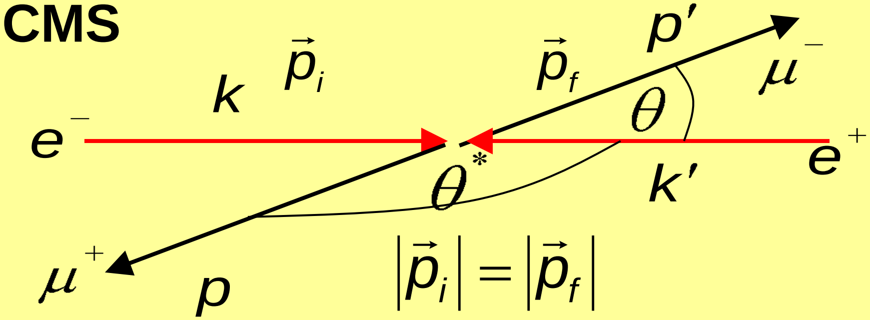
$$\Downarrow \qquad e^2 = 4\pi\alpha$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot\left(1+\cos^2\theta\right)$$

← 1/s dependence from flux factor

## Kinematics for high-relativistic particles

**CMS**




$$\left|\vec{p}_i\right| = \left|\vec{p}_f\right|$$

$$p^2 = p'^2 = k^2 = k'^2 = 0$$

$$s = (k+k')^2 \approx 4E_i^{\,2}$$

$$t = (k-p)^2 \approx -2kp \approx -2E_i^{\,2}\left(1-\cos\theta^*\right)$$

$$\approx -\frac{s}{2}\left(1+\cos\theta\right)$$

$$u = (k-p')^2 \approx -2kp' \approx -2E_i^{\,2}\left(1-\cos\theta\right)$$

$$\approx -\frac{s}{2}\left(1-\cos\theta\right)$$

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86\,\text{nb}\,\text{GeV}^2}{s}$$

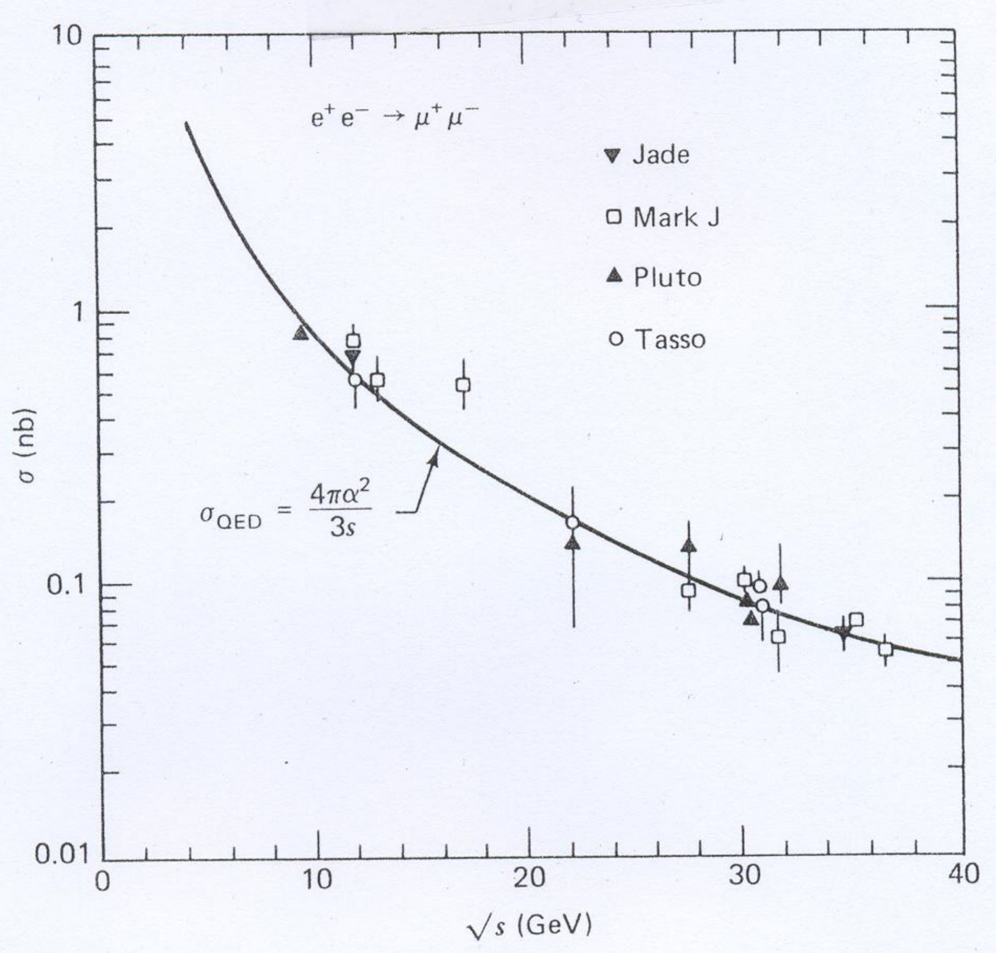
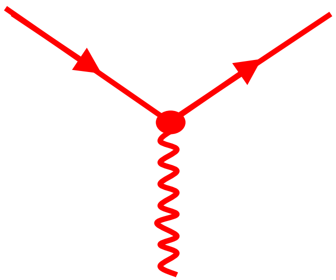



**Fig. 6.6** The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

# Comments about the angular distribution

Decomposition of the fermion current:

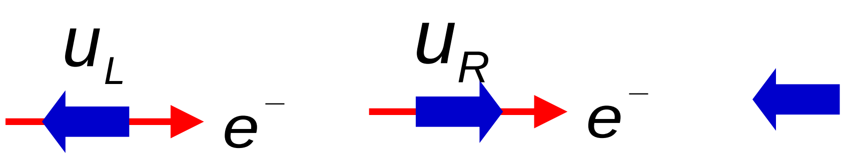


$$\bar{u}\gamma^{\mu}u = (\bar{u}_R + \bar{u}_L)\gamma^{\mu}(u_R + u_L)$$

$$= \bar{u}_R\gamma^{\mu}u_R + \bar{u}_L\gamma^{\mu}u_L$$

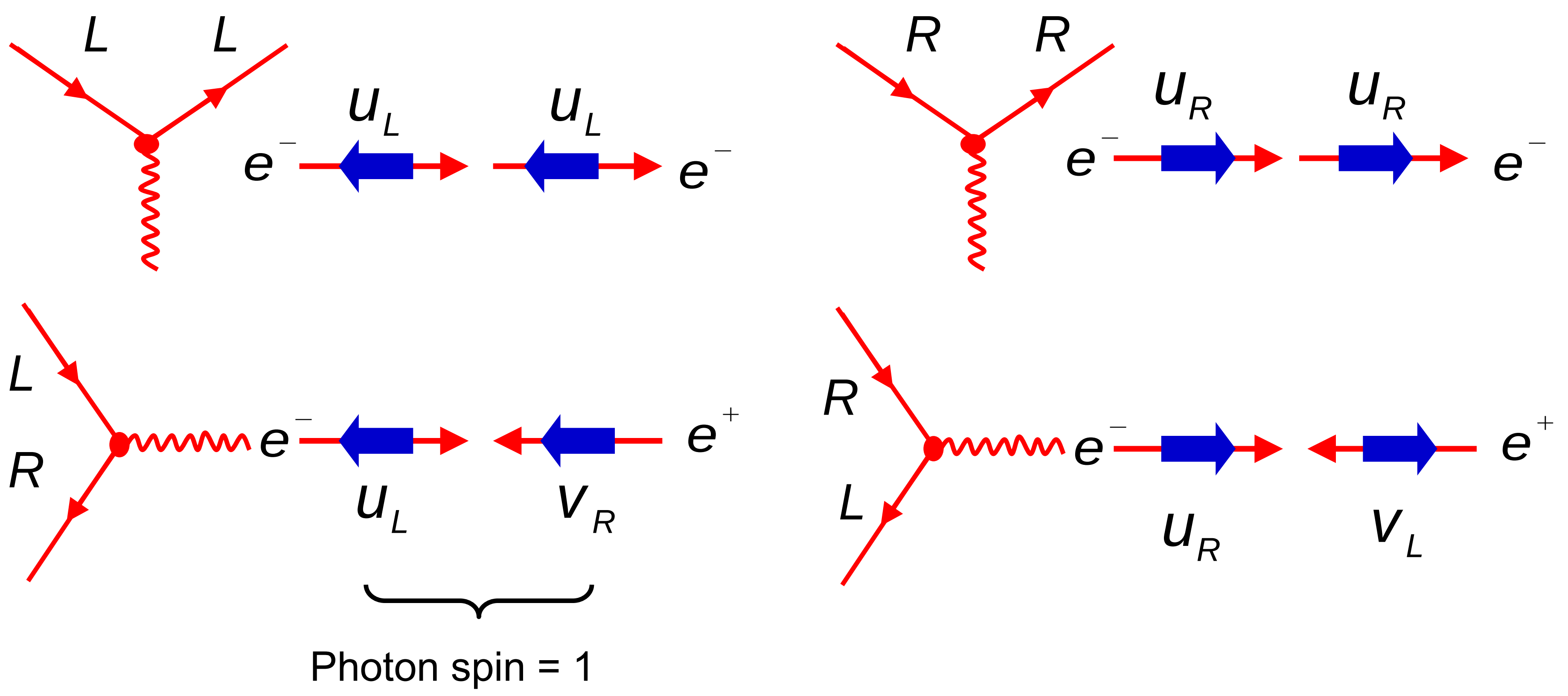
$$\{\gamma^5, \gamma^{\mu}\} = 0 \quad (\gamma^5)^2 = 1$$

For illustration:

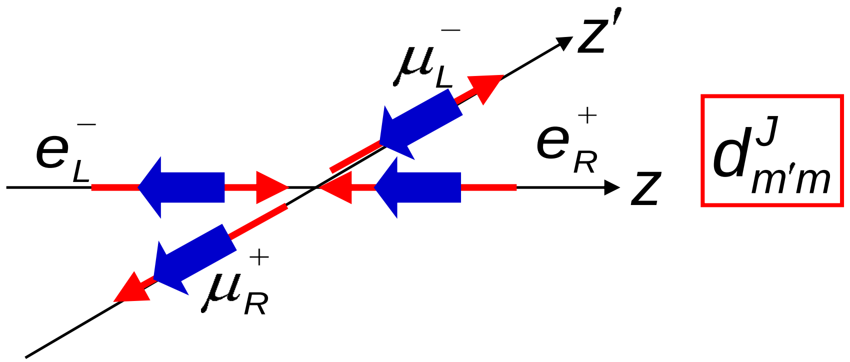


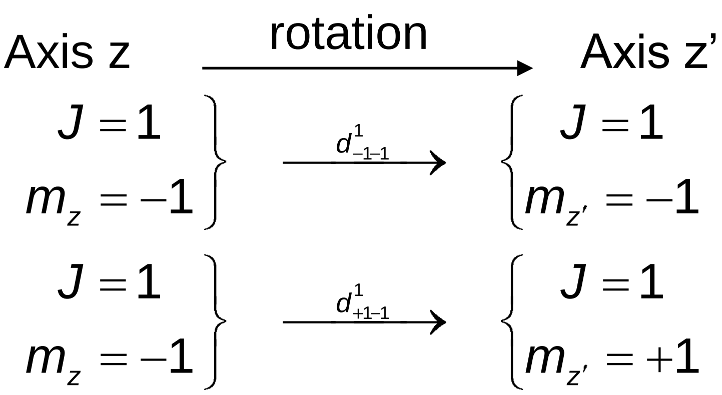

= Spin. Symbolically, correct only for massless fermions

## Vector current $ie\bar{u}\gamma^{\mu}u$:



# Angular distribution $e^+e^- \to \mu^+\mu^-$



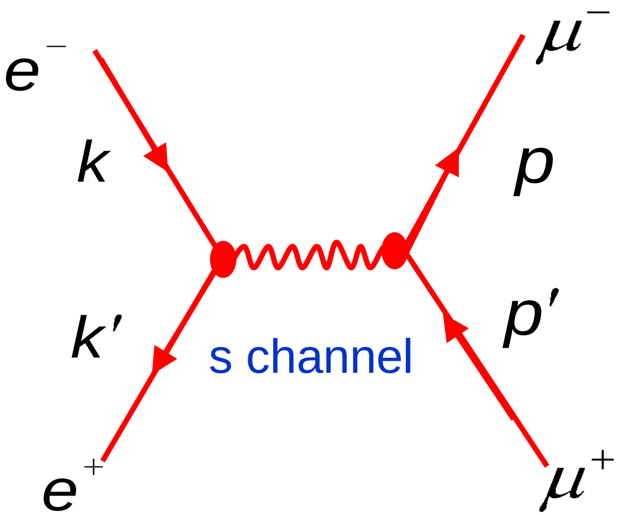




Scattering and can be treated as a change of the quantization axis.

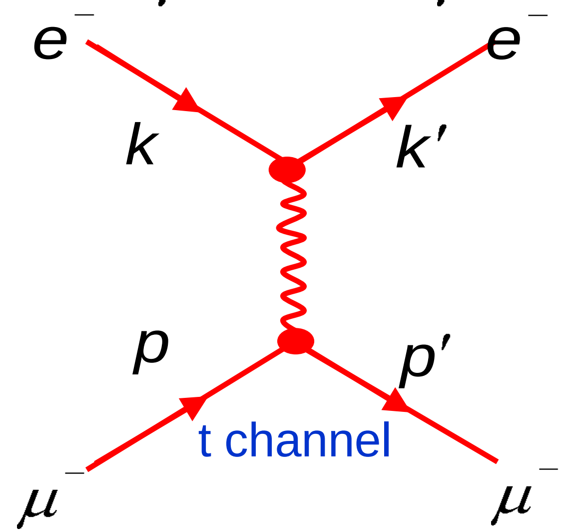
$$\frac{d\sigma}{d\Omega} \sim \left(d^1_{-1-1}\right)^2 + \left(d^1_{+1-1}\right)^2 \sim \frac{1}{4}(1+\cos\theta)^2 + \frac{1}{4}(1-\cos\theta)^2 \sim 1+\cos^2\theta$$

Angular distribution is an effect of vector $ie\gamma^\mu$ coupling!

$$e^+e^- \rightarrow \mu^+\mu^-$$




$$e^-\mu^- \rightarrow e^-\mu^-$$




s/t Crossing

$$k' \rightarrow -k' \qquad p \rightarrow -p$$

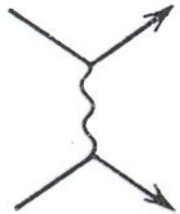
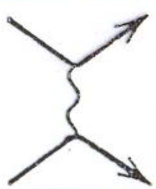
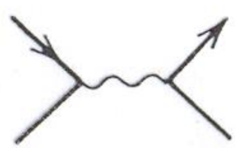
$$s = (k + k')^2 \quad \rightarrow \quad \tilde{t} = (k - k')^2$$

$$t = (k - p)^2 \quad \rightarrow \quad \tilde{s} = (k + p)^2$$

$$u = (k - p')^2 \quad \rightarrow \quad \tilde{u} = (k - p')^2 = u$$

$$\overline{|M|^2}_{e^+e^- \rightarrow \mu^+\mu^-}(s,t,u) = 2e^4 \frac{t^2 + u^2}{s^2} \quad \Rightarrow \quad \overline{|M|^2}_{e^-\mu^- \rightarrow e^-\mu^-}(\tilde{t},\tilde{s},\tilde{u}) = 2e^4 \frac{\tilde{s}^2 + \tilde{u}^2}{\tilde{t}^2}$$

# Fermion scattering - Summary

| | Feynman Diagrams | | $\overline{\vert\mathcal{M}\vert^2}/2e^4$ | | |
|---|---|---|---|---|---|
| | Forward peak | Backward peak | Forward | Interference | Backward |
| Møller scattering $e^-e^- \to e^-e^-$ |  |  | $\frac{s^2+u^2}{t^2}$ + | $\frac{2s^2}{tu}$ + | $\frac{s^2+t^2}{u^2}$ |
| | | | ($u \leftrightarrow t$ symmetric) | | |
| (Crossing $s \leftrightarrow u$) ↓ | Forward | "Time-like" | Forward | Interference | Time-like |
| Bhabha scattering $e^-e^+ \to e^-e^+$ |  |  | $\frac{s^2+u^2}{t^2}$ + | $\frac{2u^2}{ts}$ + | $\frac{u^2+t^2}{s^2}$ |
| $e^-\mu^- \to e^-\mu^-$ |  | | $\frac{s^2+u^2}{t^2}$ | Rutherford | |
| (Crossing ↓ $s \leftrightarrow t$) $e^-e^+ \to \mu^-\mu^+$ | |  | | | $\frac{u^2+t^2}{s^2}$ |

# 2.2 Experimental methods

## $e^+e^-$ accelerator (selection)

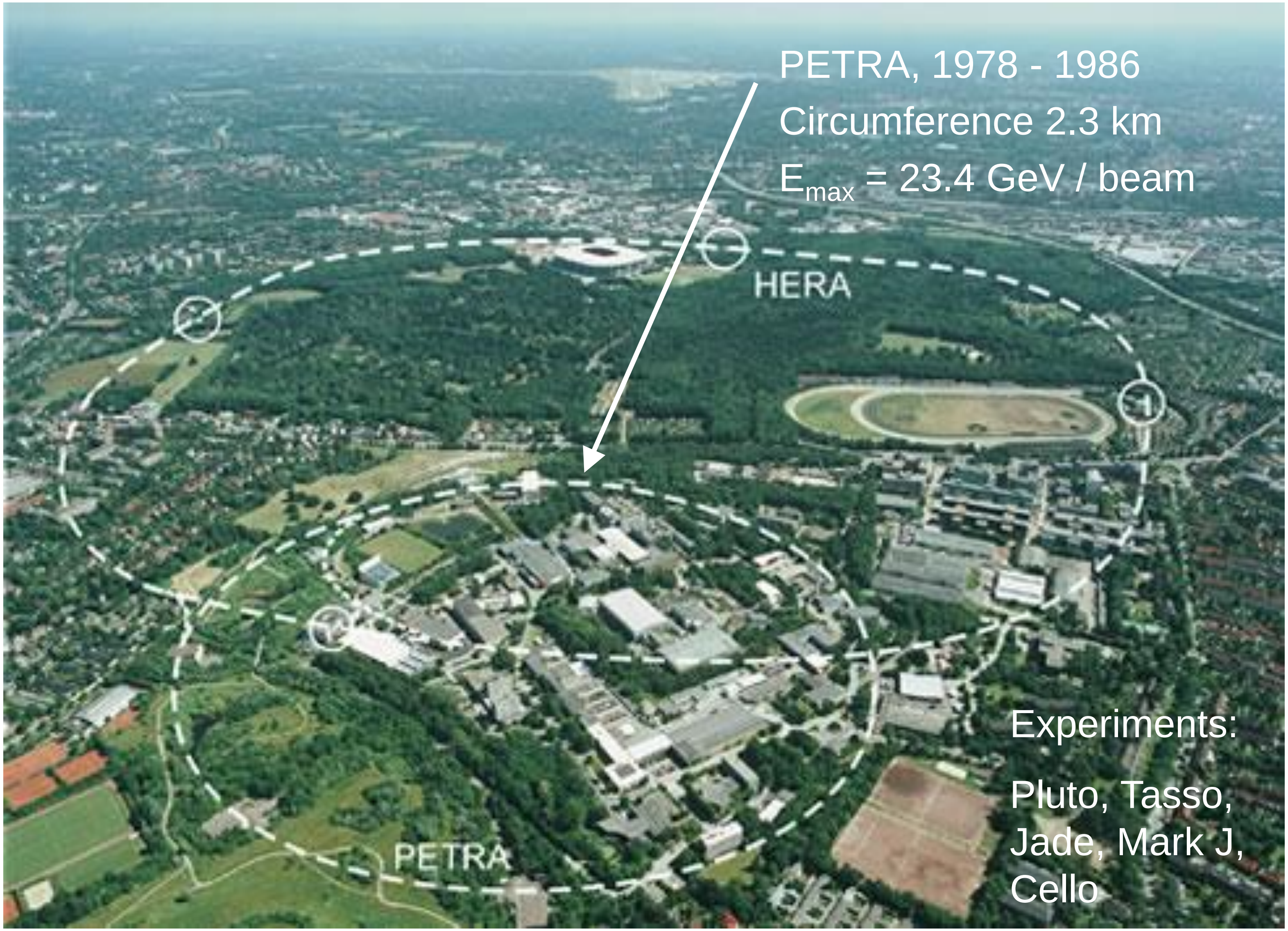
| Accelerator | Lab | $\sqrt{s}$ | $L_{int}$ / Exper. |
|---|---|---|---|
| SPEAR | SLAC | 2 – 8 GeV | |
| PEP | SLAC | →29 GeV | 220 - 300 $pb^{-1}$ |
| PETRA | DESY | 12 - 47 GeV | ~20 $pb^{-1}$ |
| TRISTAN | KEK | 50 – 60 GeV | ~20 $pb^{-1}$ |
| LEP | CERN | 90 GeV | ~200 $pb^{-1}$ |

Z physics

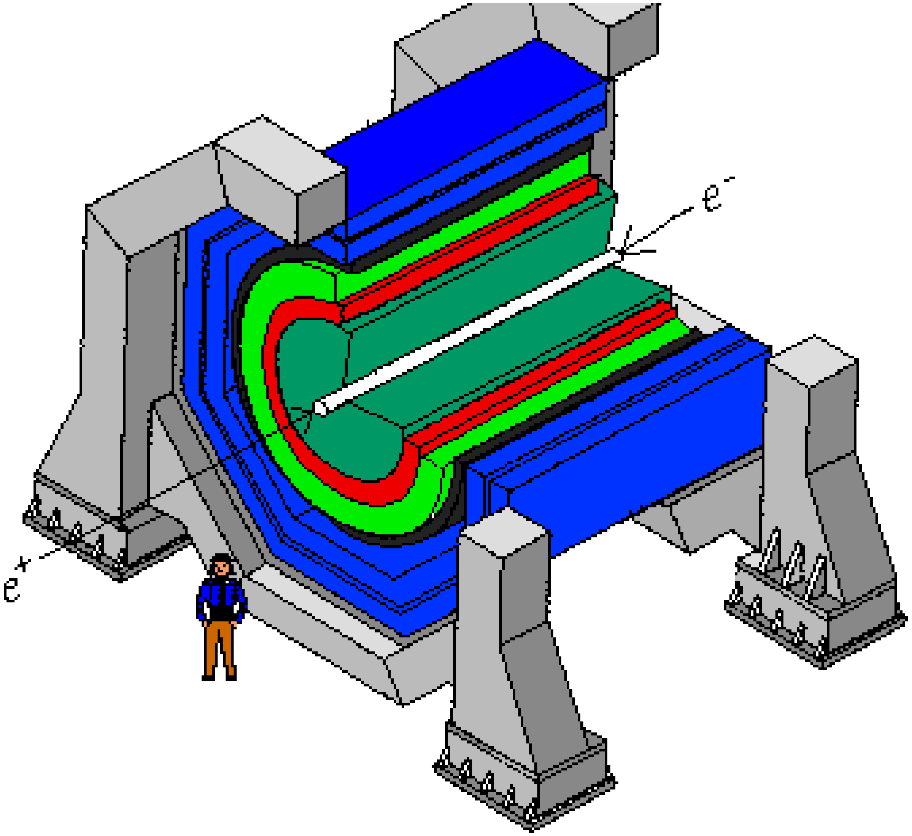
## Cross section (experimental definition)

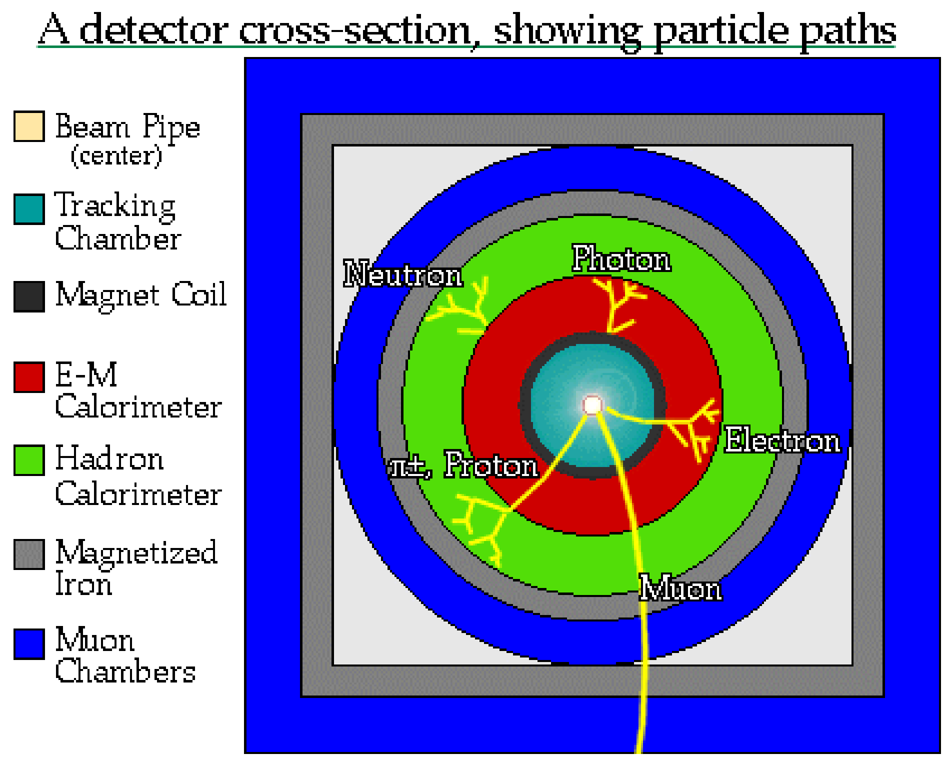
$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon\, L_{\text{int}}}$$

- $N_{ff}$ number of detected $e^+e^- \to f\bar{f}$ events
- b background fraction
- ε acceptance / efficiency
- $L_{int}$ integrated luminosity of collider

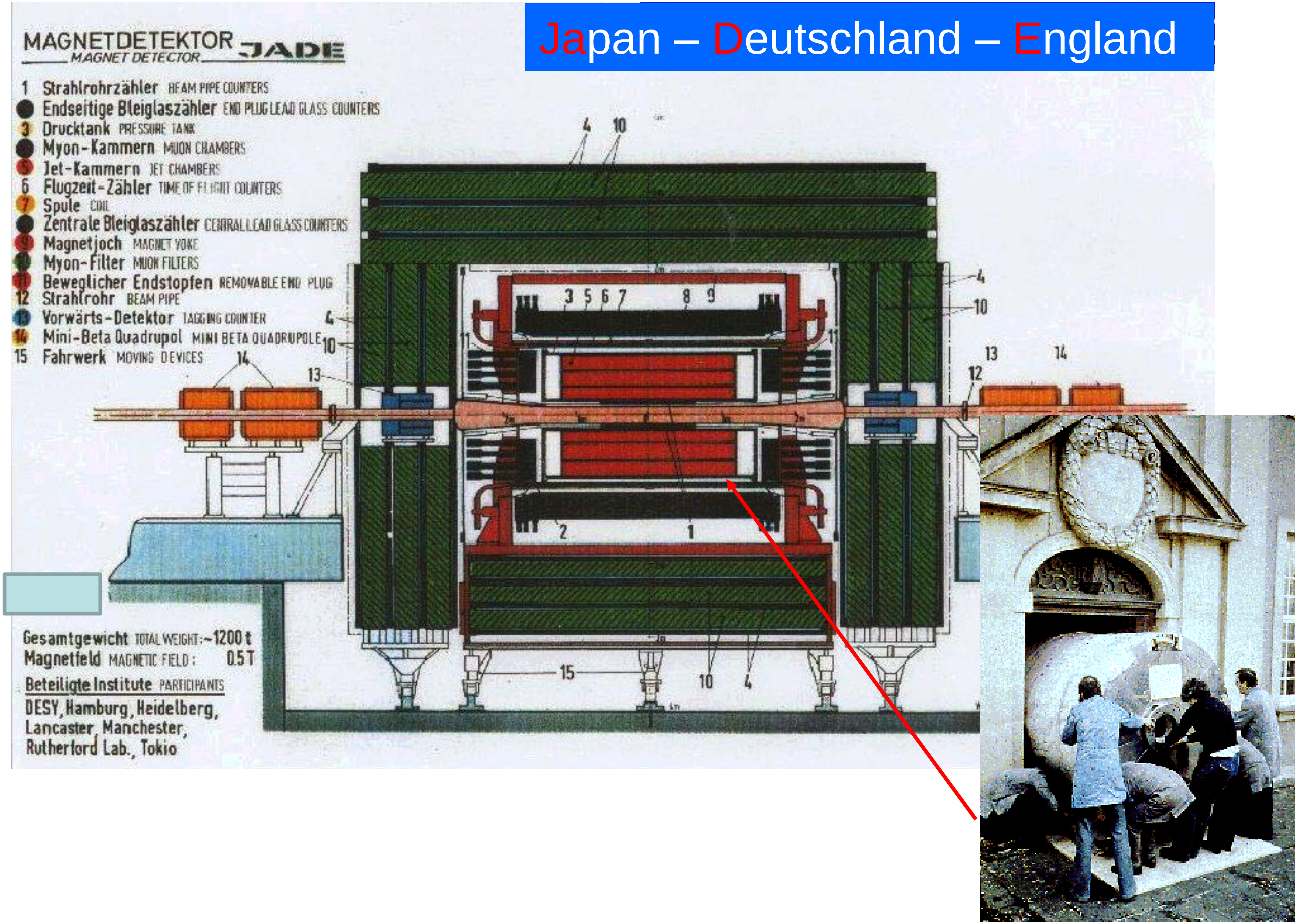

PETRA, 1978 - 1986
Circumference 2.3 km
$E_{max}$ = 23.4 GeV / beam
HERA
PETRA
Experiments:
Pluto, Tasso, Jade, Mark J, Cello

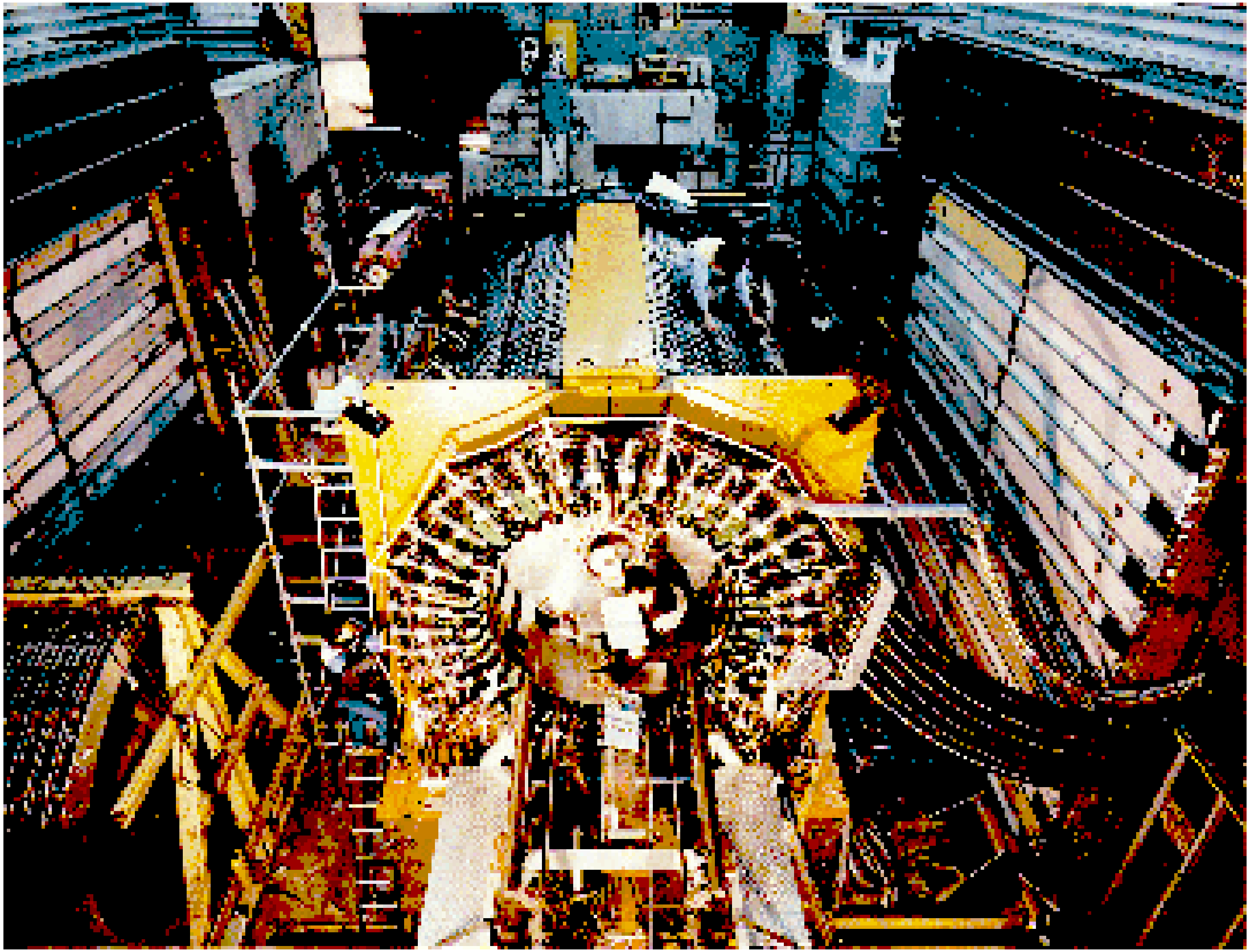
# Particle detectors



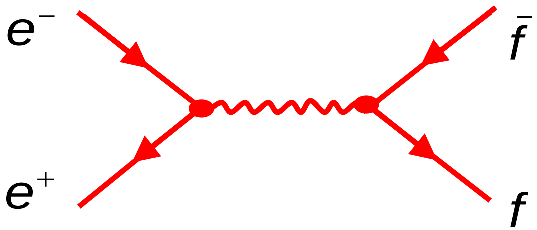
# Japan – Deutschland – England

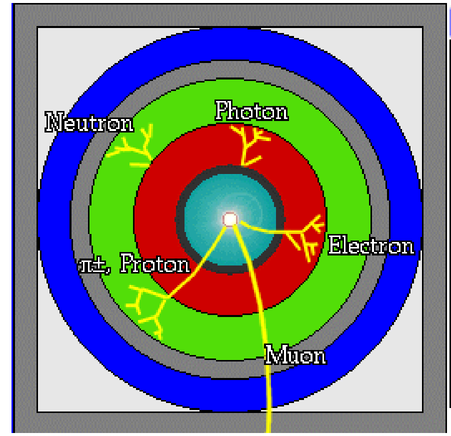


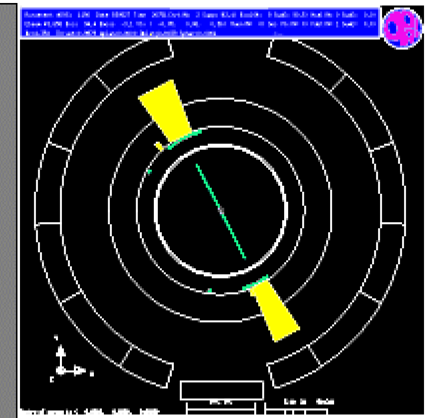


# Experimental Signatures:

**OPAL / LEP**



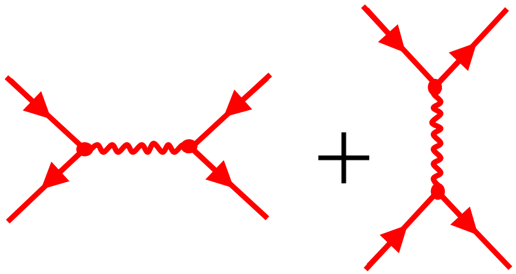



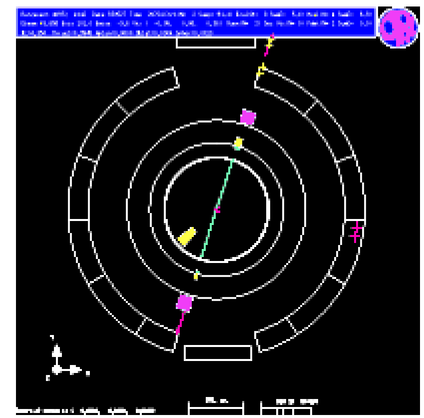



$e^-e^+$

$f\,\bar{f}=$

$e^-e^+$

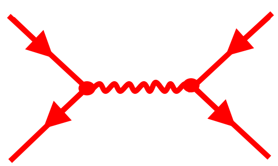


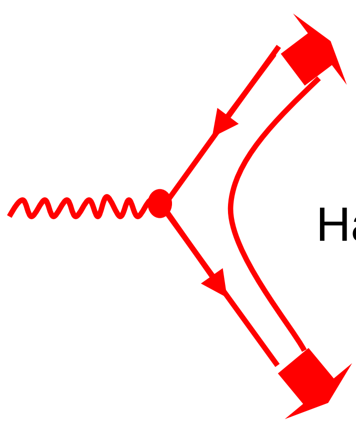


$\mu^-\mu^+$

$\mu^-\mu^+$

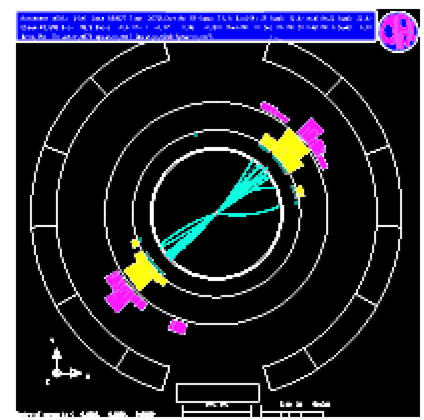
$\tau^-\tau^+$





Hadron jets

$q\,\bar{q}$



$q\,\bar{q}$ mit

$q=u,d,s,c,b,(t)$

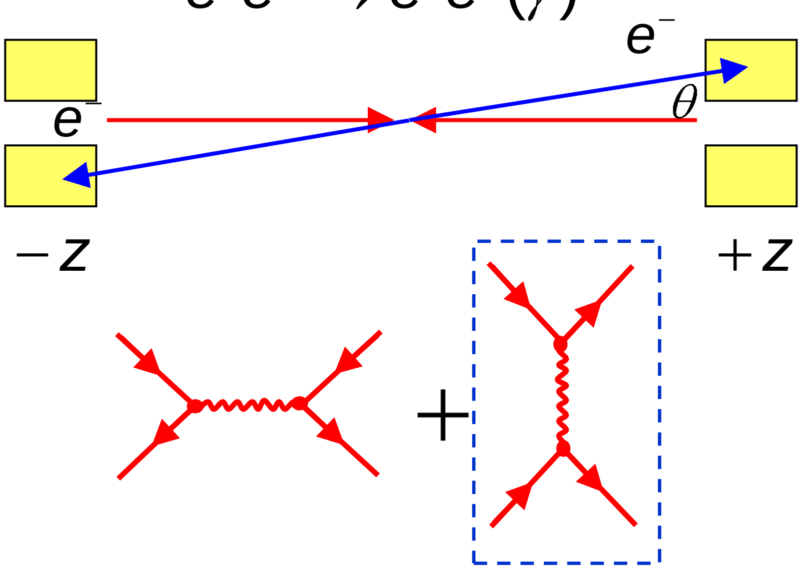
## Determination of integrated luminosity

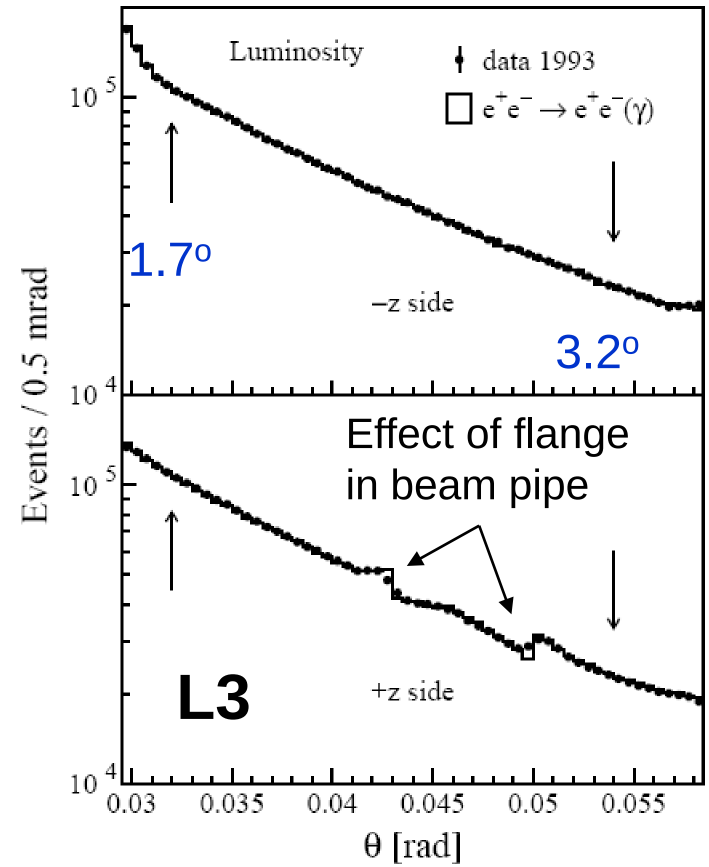
$$L_{\text{int}} = \int L_{ee}(t)\,dt$$

$$\sigma(e^+e^- \to f\,\bar{f}) = \frac{N_{ff}(1-b)}{\varepsilon\, L_{\text{int}}}$$

Use reference process to determine $L_{\text{int}}$ :
small angle Bhabha scattering
(low momentum transfer)

$$e^+e^- \to e^+e^-(\gamma)$$



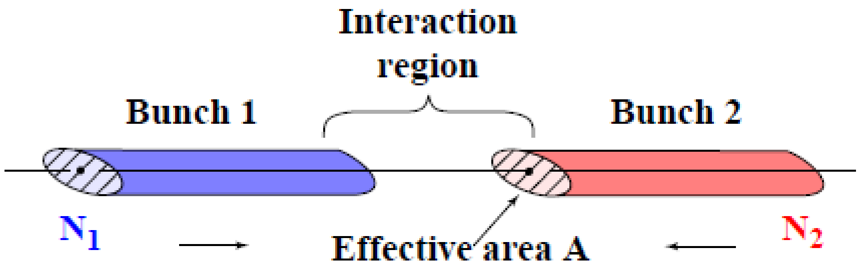

Small angle Bhabha scattering is t channel dominated: theoretical cross section $\sigma_{theo}$ well known.

$$L_{\text{int}} = \frac{N_{ee}}{\sigma_{theo}\,\varepsilon}$$

At LEP:
typ. errors < 0.5%

# Luminosity determination at LHC:

Problem: Cross section for nearly all reference processes at LHC rely on the knowledge of the proton structure functions and cross sections for well understood processes are small!
$\Rightarrow$ Need: Process independent luminosity determination!




Luminosity from first principles:

$$L = \frac{N_1 N_2 f N_b}{A} = \frac{N_1 N_2 f N_b}{4\pi\sigma_x\sigma_y}$$

- $N_{1,2}$ = number of protons in bunches
- $N_b$ = numer of colliding bunches
- f = revolution frequency
- A = effective beam transverse A

A can be calculated from overlap of two transerse beam distributions $g_i(x,y)$:

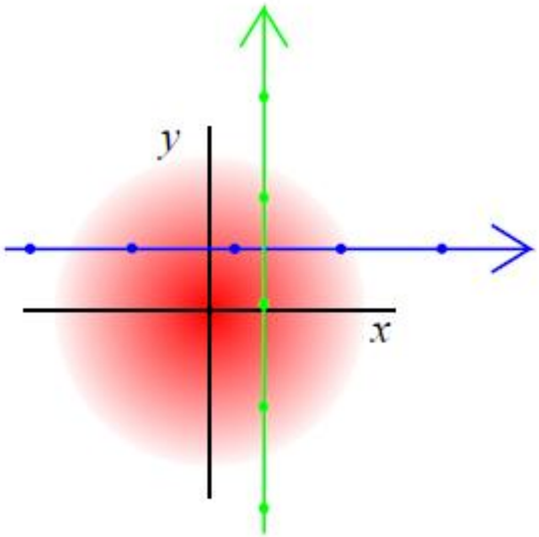
$$\frac{1}{A_{\rm eff}} = \int g_1(x,y)\, g_2(x,y)\, dx\, dy\,.$$

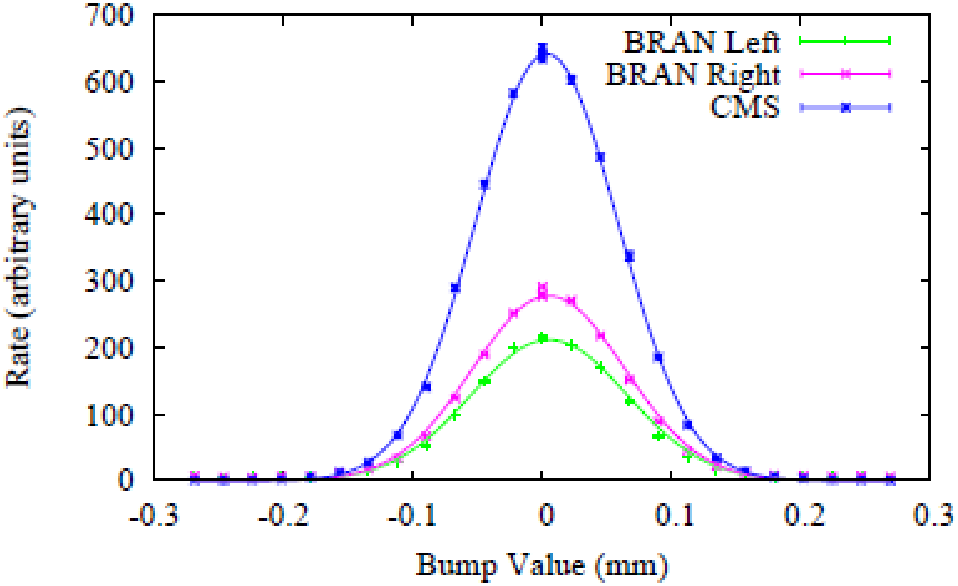
With equal gaussians:

$$g_1 = g_2 = \frac{1}{2\pi\sigma_x\sigma_y} \exp\left[-\frac{x^2}{2\sigma_x^2} - \frac{y^2}{2\sigma_y^2}\right]$$

$N_{1,2}$, $N_b$, f known by the operations crew.
Only unknown: beam profiles at the collision points.

**Van der Meer scan:** Separate beams by known amount and measure the change of particle rate.



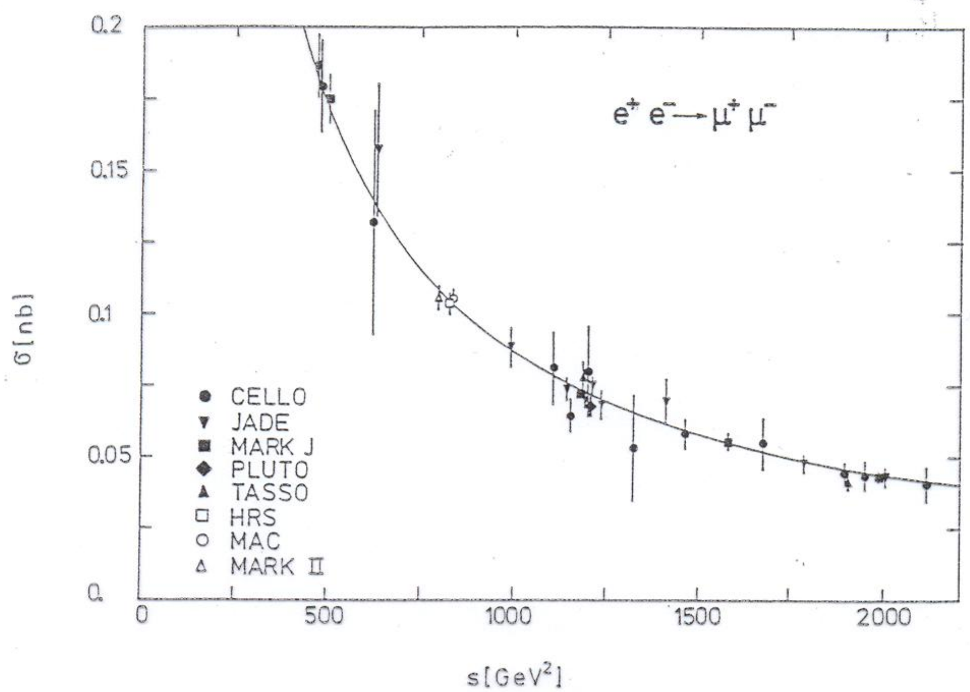




*S. White et al., First Luminosity Measurement scans in the LHC, IPAC 2010, Kyoto.*

Table 1: Effective beam size derived from the scans.

| | $\sigma_{x\mathrm{eff}}$ (mm) | $\sigma_{y\mathrm{eff}}$ (mm) |
|---|---|---|
| IP1 | 0.0473±1.314E-3 | 0.0550±1.289E-3 |
| IP5 | 0.0546±0.567E-3 | 0.0693±1.526E-3 |
| IP8 | 0.0466±1.177E-3 | 0.0517±2.007E-3 |

# 2.3 $e^+e^- \to \mu^+\mu^-$

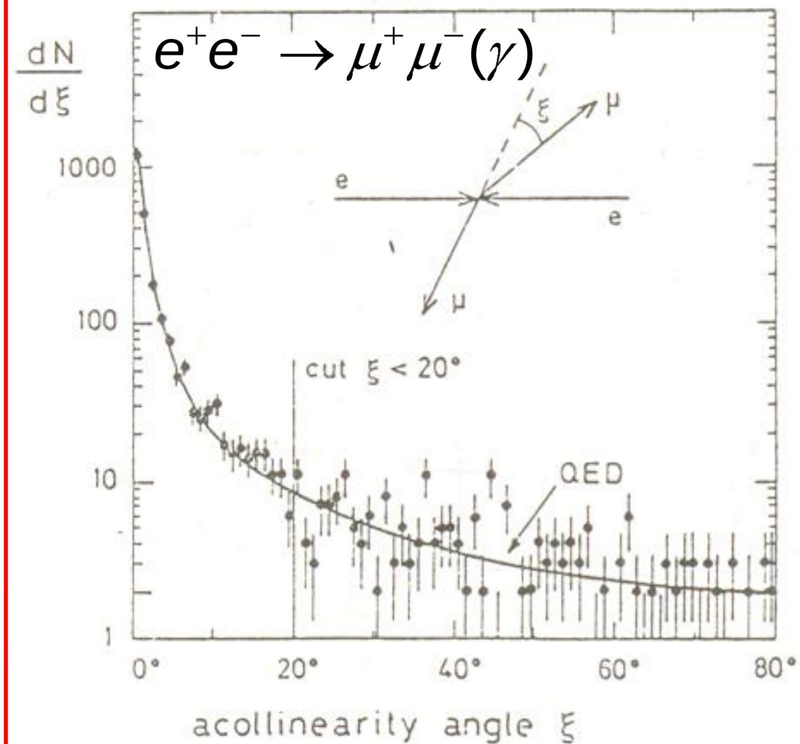



Good agreement with QED!

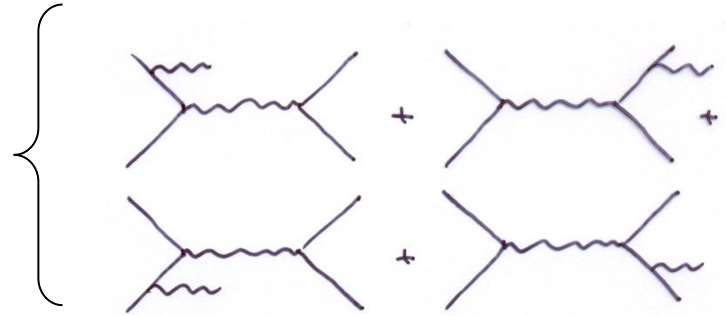
Quantitative limit for new physics ?

There will always be additional photons

Acollinearity

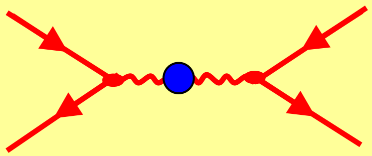



Effect of bremsstrahlung:



## Possible deviation from QED:

- additonal heavy photon



Modifies propagator

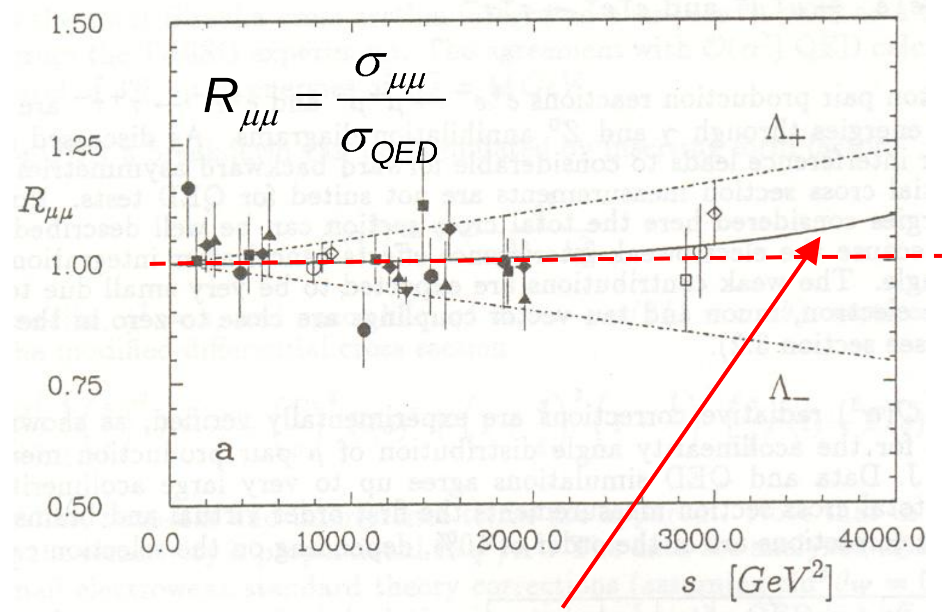
$$\frac{1}{q^2} \rightarrow \frac{1}{q^2} - \frac{1}{q^2 - \Lambda^2} = \frac{1}{q^2}\left(1 - \frac{q^2}{q^2 - \Lambda^2}\right)$$

$$\approx \frac{1}{q^2}\underbrace{\left(1 + \frac{q^2}{\Lambda^2}\right)}_{\text{FF - like } F(q^2)}$$

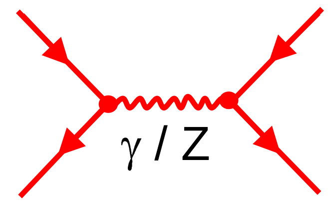
$\Lambda$ corresponds to the mass of new photon

To also account for possible lower cross sections:

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2}\left(1 \mp \frac{q^2}{q^2 - \Lambda_\pm^2}\right)$$




$$\frac{\sigma_{\mu\mu}^{\gamma+Z}}{\sigma_{QED}}$$




Additional heavy photon:

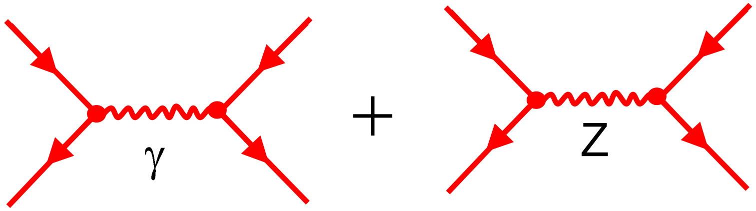
$$\sigma_{\mu\mu} = \frac{4\pi\alpha^2}{s}\left(1 \mp \frac{s}{s - \Lambda_\pm^2}\right)^2$$

$\rightarrow \Lambda_\pm > 200\,\text{GeV}$

Confirms "Coulomb law" down to $10^{-18}$ m

# Effect of Z boson exchange

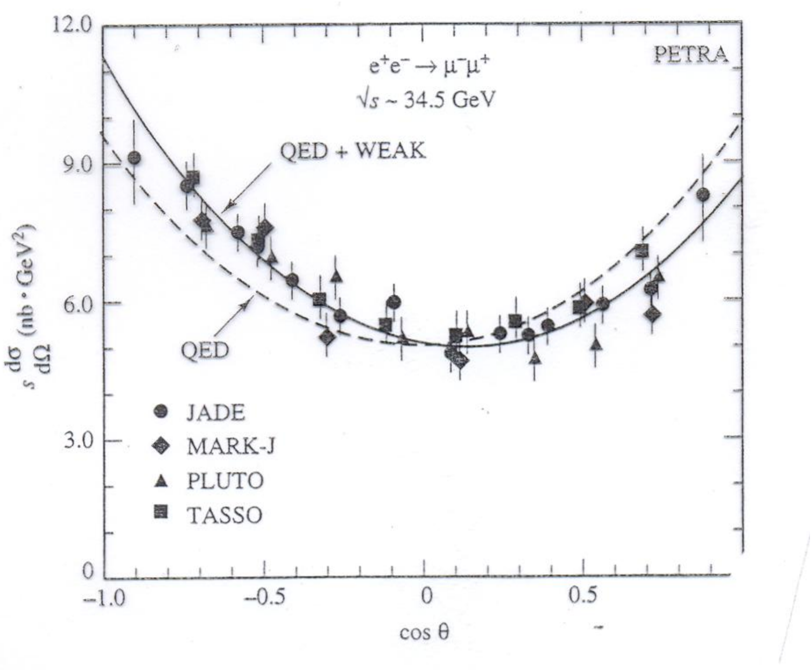
„heavy photon w/ different couplings“




$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2\theta)\Big|_{QED}$$

Vector coupling!

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{4s} \cdot (1 + \cos^2\theta + A\cos\theta)\Big|_{\gamma+Z}$$

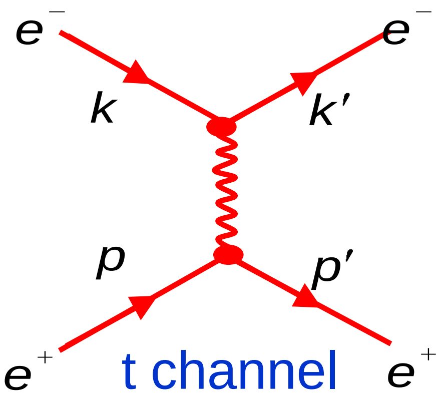


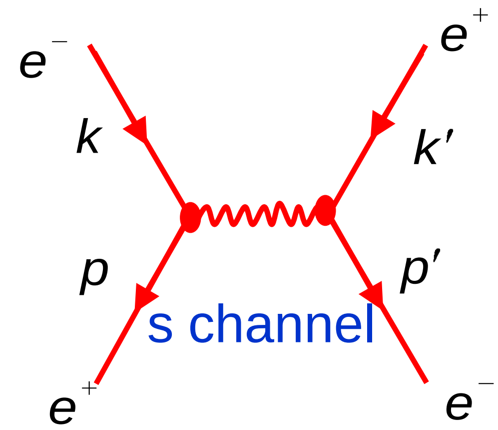

The effect of the “heavy” Z boson is already seen at low energies!

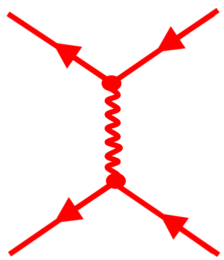
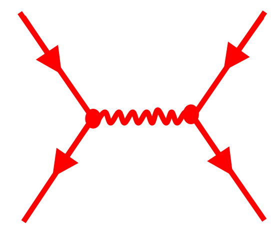
Clear deviation from QED:

⇒Effect of electro-weak γ/Z interference

# 2.4 Bhabha scattering $e^+e^- \to e^+e^-$

$M =$ 

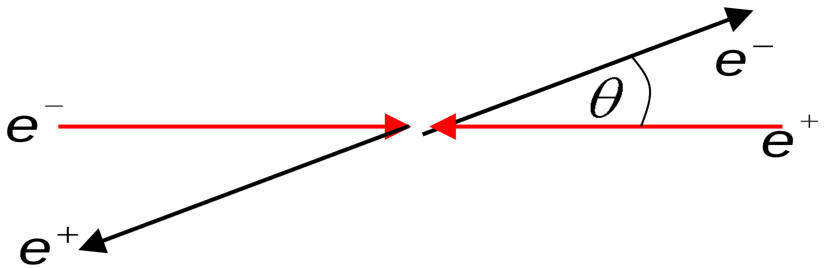
$+$ 


$\overline{|M|^2} = \Big|$  $\Big|^2$ + interference + $\Big|$  $\Big|^2$

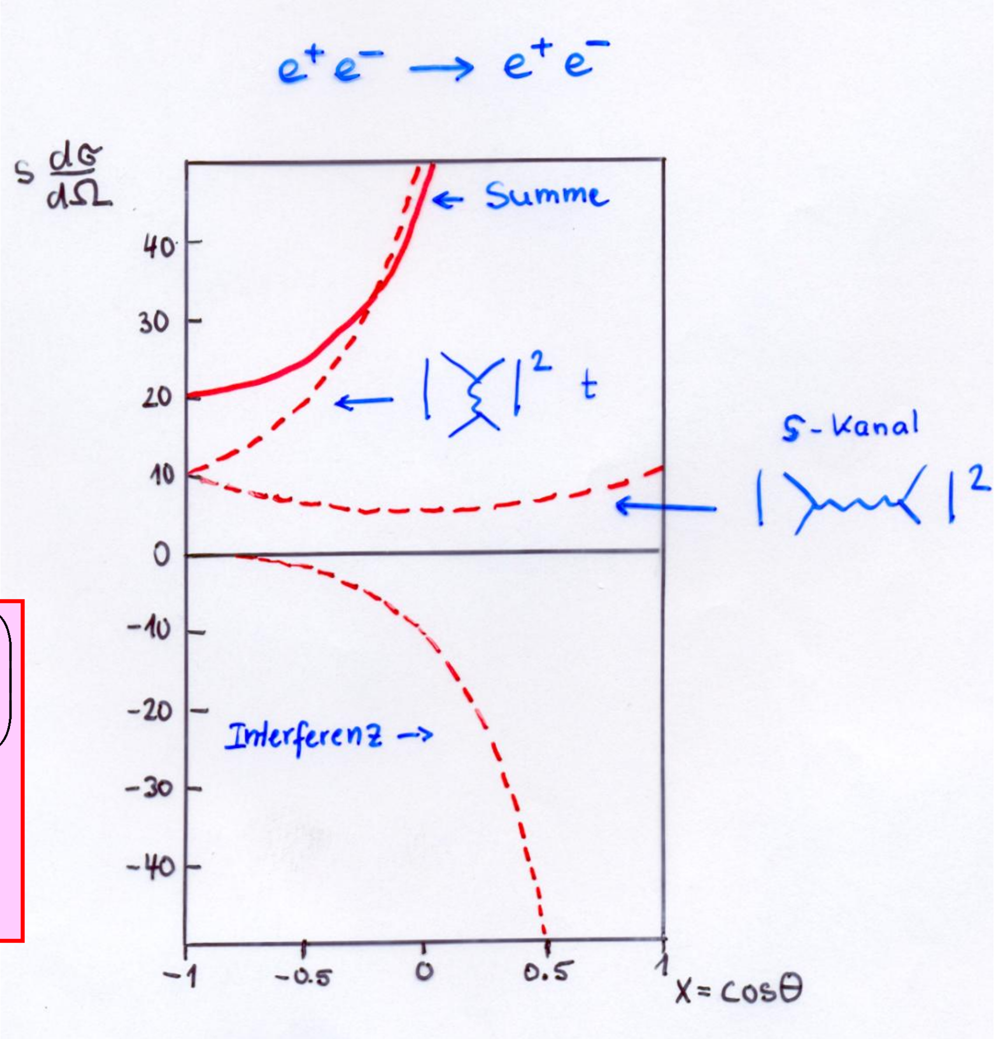
(first term: $e^-\mu^- \to e^-\mu^-$; last term: $e^+e^- \to \mu^+\mu^-$)

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}\right)$$

CM system:




with x=cosθ

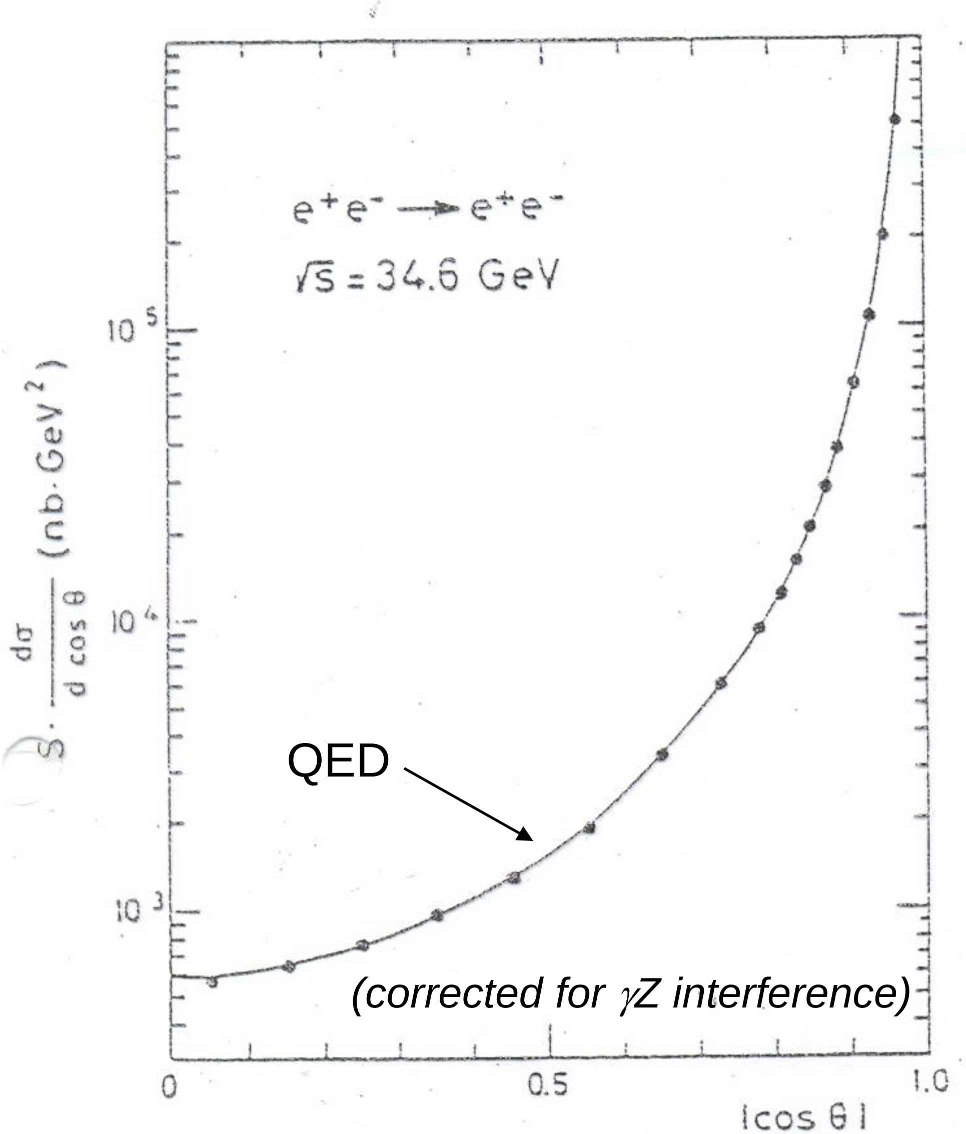



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{4+(1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2}\right)$$

$$= \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

divergent for cosθ→1




Additional „heavy photon“:

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2}\left(1 \mp \frac{q^2}{q^2-\Lambda_\pm^2}\right)$$

⇒ Can be described by form factor:

$$F(q^2) = 1 \mp \frac{q^2}{q^2-\Lambda_\pm^2}$$

Form factor modifies differential cross section:

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{u^2+s^2}{t^2}\left|F(t)\right|^2 + \frac{2u^2}{ts}\left|F(t)F(s)\right| + \frac{u^2+t^2}{s^2}\left|F(s)\right|^2\right)$$

Fit to combined PETRA $e^+e^-$ data:

$\Lambda$+ >435 GeV @ 95% CL

$\Lambda$- > 590 GeV

In the "space picture" form factor corresponds to modified Coulomb potential at small distances:

$$\frac{1}{r} \rightarrow \frac{1}{r}(1-e^{-\Lambda r})$$

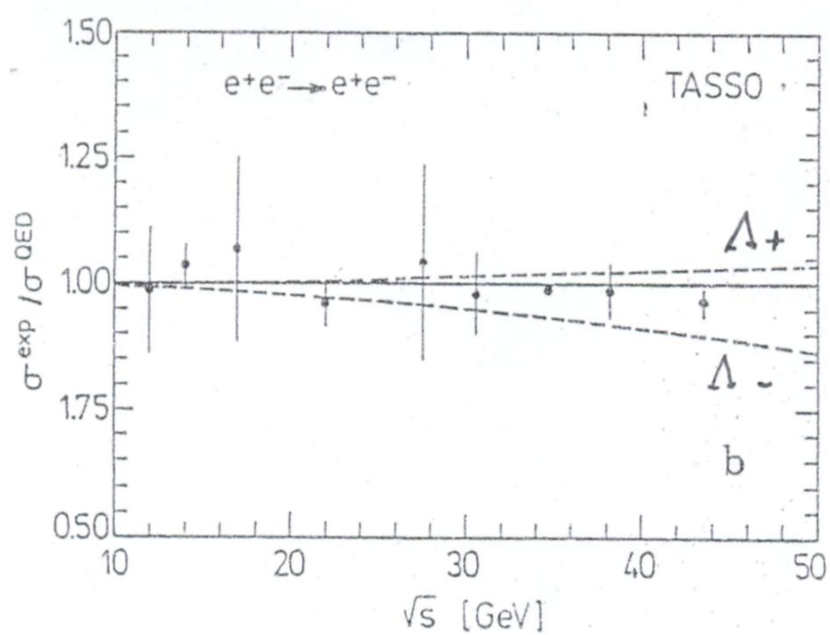
i.e. $\Lambda$ measures point-like nature of e$\gamma$ interaction (size of electron).

$\Lambda$> ~500 GeV $\Leftrightarrow$ $r_e$< 0.197/500 fm

Electr. substructure < 0.5 $\times 10^{-18}$ m

Tasso: $\Lambda_+$ >370 GeV

$\Lambda_-$ > 190 GeV



# 2.5 Discovery of the Tau-Lepton

*MARK I (SLAC), 1975, M.Perl et al.*

*Nobel Prize 1995 for M.Perl*

Evidence for Anomalous Lepton Production in $e^+ - e^-$ Annihilation*

M. L. Perl, G. S. Abrams, A. M. Boyarski, M. Breidenbach, D. D. Briggs, F. Bulos, W. Chinowsky, J. T. Dakin,† G. J. Feldman, C. E. Friedberg, D. Fryberger, G. Goldhaber, G. Hanson, F. B. Heile, B. Jean-Marie, J. A. Kadyk, R. R. Larsen, A. M. Litke, D. Lüke,‡ B. A. Lulu, V. Lüth, D. Lyon, C. C. Morehouse, J. M. Paterson, F. M. Pierre,§ T. P. Pun, P. A. Rapidis, B. Richter, B. Sadoulet, R. F. Schwitters, W. Tanenbaum, G. H. Trilling, F. Vannucci,‖ J. S. Whitaker, F. C. Winkelmann, and J. E. Wiss

*Lawrence Berkeley Laboratory and Department of Physics, University of California, Berkeley, California 94720, and Stanford Linear Accelerator Center, Stanford University, Stanford, California 94305*

(Received 18 August 1975)

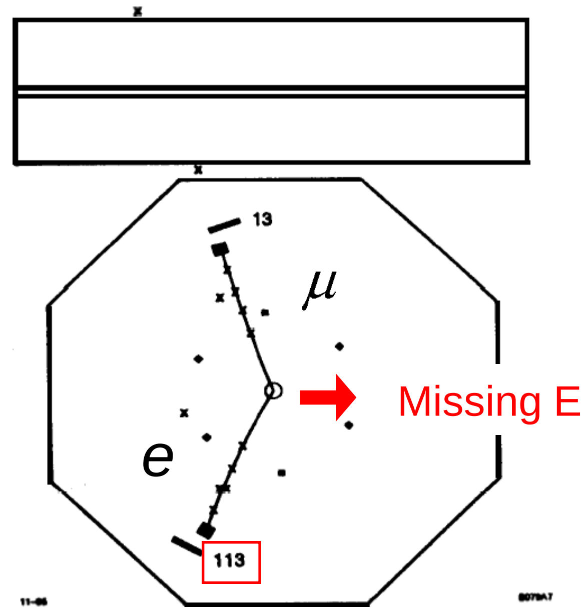
We have found events of the form $e^+ + e^- \rightarrow e^\pm + \mu^\mp +$ missing energy, in which no other charged particles or photons are detected. Most of these events are detected at or above a center-of-mass energy of 4 GeV. The missing-energy and missing-momentum spectra require that at least two additional particles be produced in each event. We have no conventional explanation for these events.

We have found 64 events of the form

$$e^+ + e^- \rightarrow e^\pm + \mu^\mp + \geq 2 \text{ undetected particles} \quad (1)$$

for which we have no conventional explanation. The undetected particles are charged particles or photons which escape the 2.6π sr solid angle of the detector, or particles very difficult to detect such as neutrons, $K_L^0$ mesons, or neutrinos. Most of these events are observed at center-of-mass energies at, or above, 4 GeV. These events were found using the Stanford Linear Accelerator Center–Lawrence Berkeley Laboratory (SLAC-

1489




*Relative amount of EM energy*

Explanation:

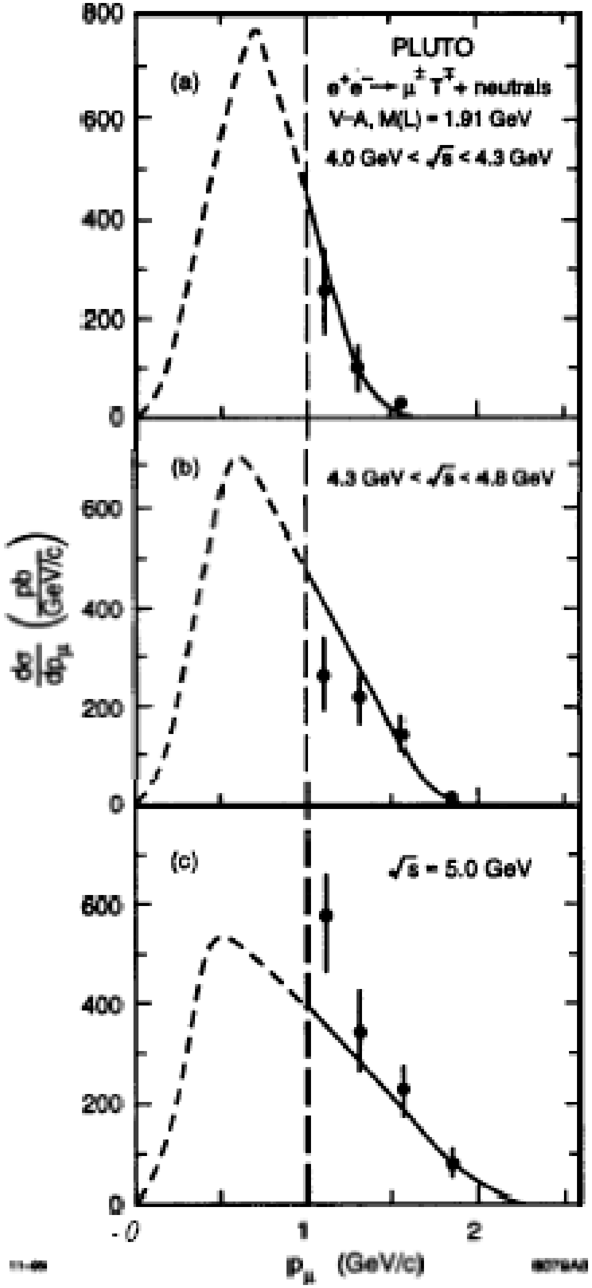
$$e^+e^- \rightarrow \tau^+ \tau^-$$

$$\tau^- \rightarrow e^- \, \bar{\nu}_e \nu_\tau$$

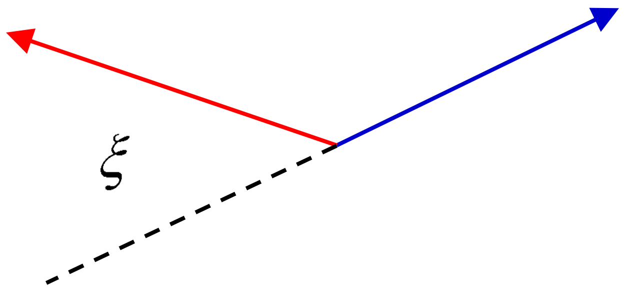
$$\tau^+ \rightarrow \mu^+ \, \nu_\mu \bar{\nu}_\tau$$

A lot of Discussions in 1975:

Are these events really decays of a new 3rd generation of heavy lepton ?




1) Large acollinearity confirms tau hypothesis




2) Anomalous “single muon events” predicted:

Expectation: $BR(\tau \to e(\mu)\nu\bar{\nu}) \approx 20\%$

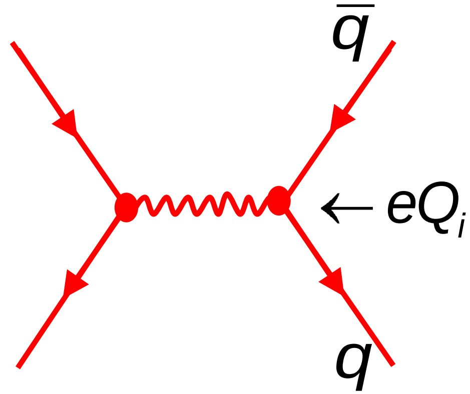
$BR(\tau \to h + \nu) \approx 60\%$

➡ $e^+ + e^- \to \mu^\pm + h^\mp + \text{missing E}$

⬅ PLUTO (DESY, 1976) confirms the anomalous “single muon events”. Muon spectrum consistent with 3-body tau decay.

# 2.6 $e^+e^- \rightarrow$ hadrons

$e^+e^-$ annihilation to a pair of quarks with subsequent hadronization.




$$Q_i = \begin{cases} +\frac{2}{3} \\ -\frac{1}{3} \end{cases}$$

Quarks have fractional charges and carry “color” as additional quantum number.

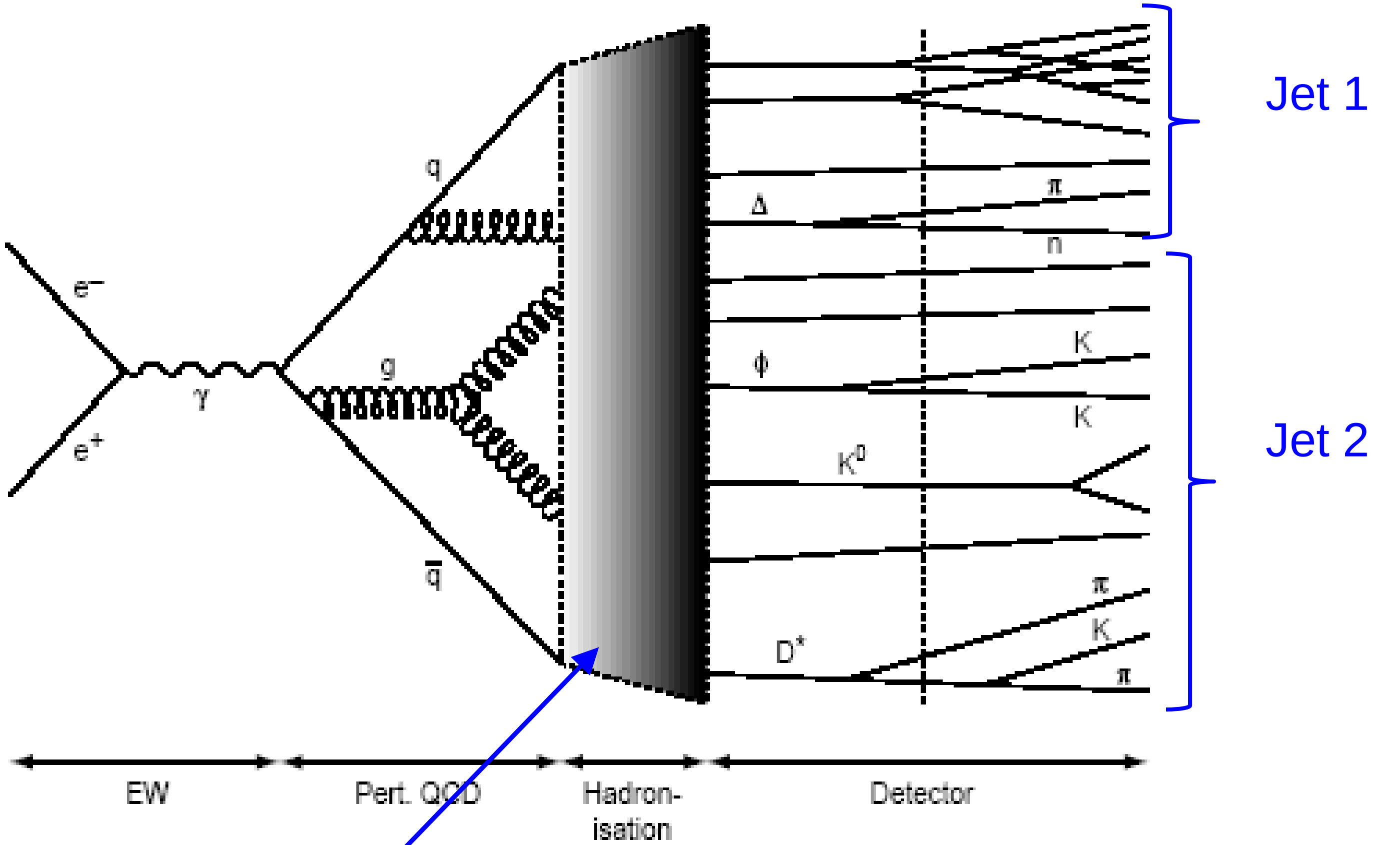
Additional color factor $N_C$

$$\left.\frac{d\sigma}{d\Omega}\right|_{ee\rightarrow hadrons} = \frac{\alpha^2}{4s} \cdot N_C \cdot \sum_{quarks\,i} Q_i^2 (1+\cos^2\theta)$$

Sum over kinematically possible quark flavors:
$4m_q^2 < s$

| $\sqrt{s}$ | Quarks |
|---|---|
| < ~3 GeV | uds |
| < ~10 GeV | udsc |
| < ~350 GeV | udscb |
| > ~350 GeV | udscbt |

# From Quarks to Jets

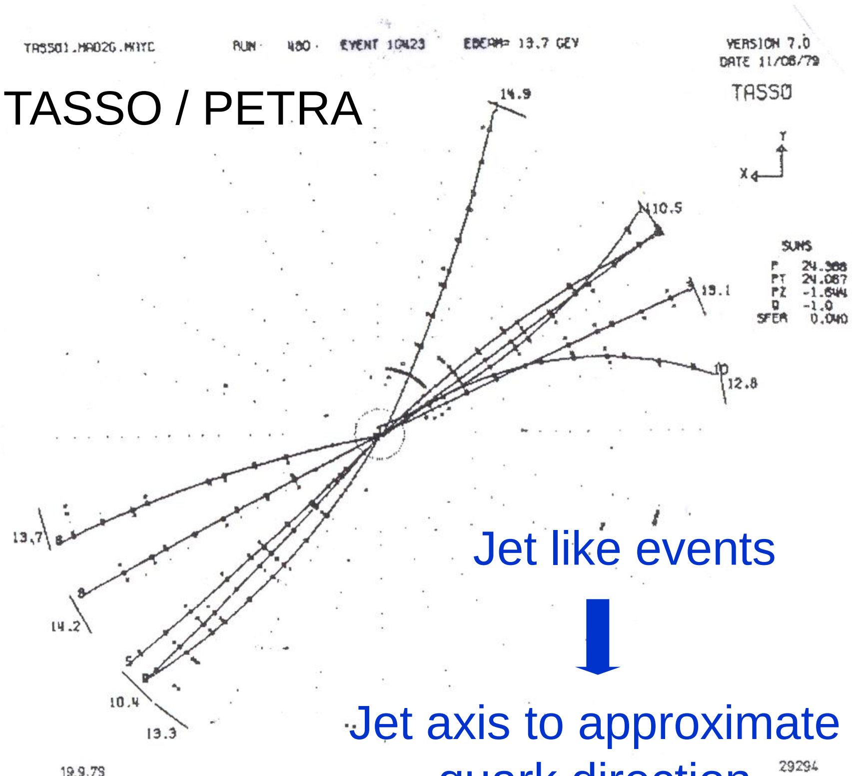



Described successfully by different phenomenological fragmentation models realized as Monte Carlo programs:
**PHYTIA, HERWIG, SHERPA**

*~ 20 particles at 90 GeV*

# Quark jets and angular distribution

TASSO / PETRA

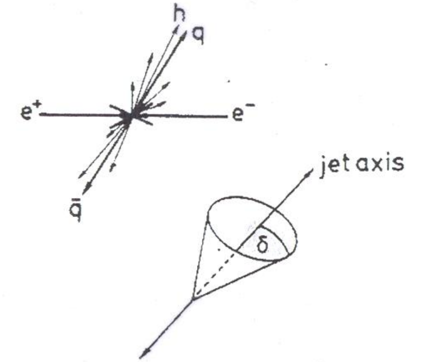



Jet like events

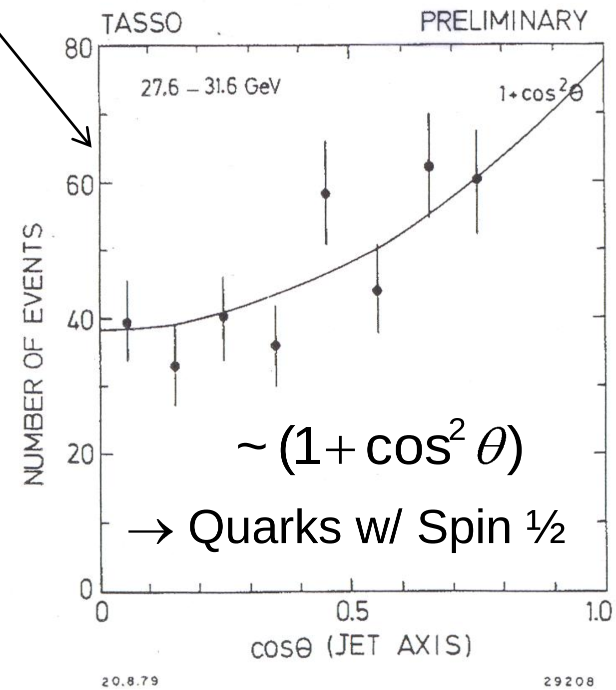
⬇

Jet axis to approximate quark direction

Fig.2 A typical multihadron event at 27.4 GeV recorded in the central detector. The inner 4 layers belong to the proportional chamber, the following 9 are zero degree layers of the drift chamber. The solid bars at the periphery mark time-of-flight counters.




Not very convincing



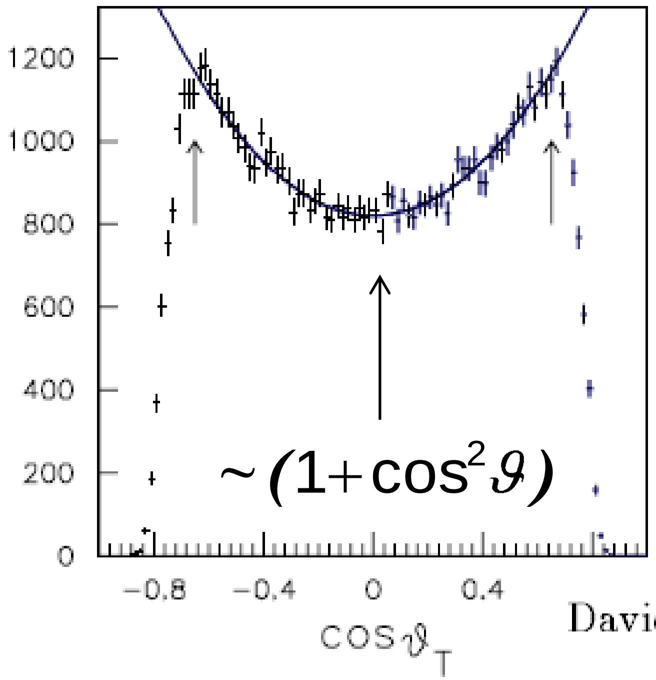

$\sim (1+\cos^2\theta)$

$\rightarrow$ Quarks w/ Spin ½

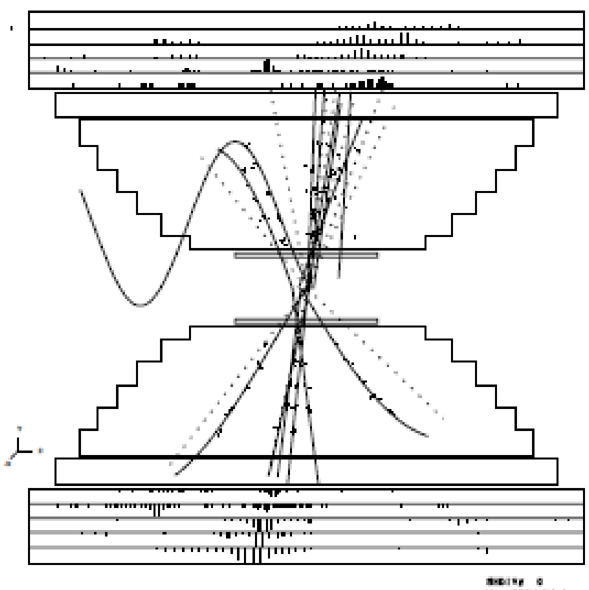
Fig.7 Angular distribution of the jet axis with respect to the beam.

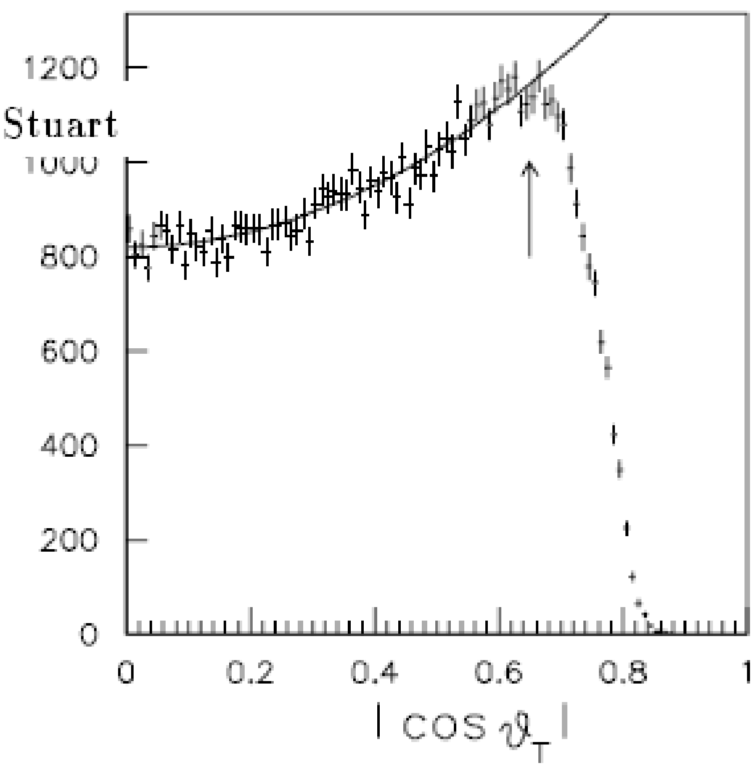
*From a thesis by D.D.Stuart UC Davis, 1992. AMY (Tristan) Data*

Using the thrust axis as “jet axis”: works also for n-jet events.




David Donald Stuart

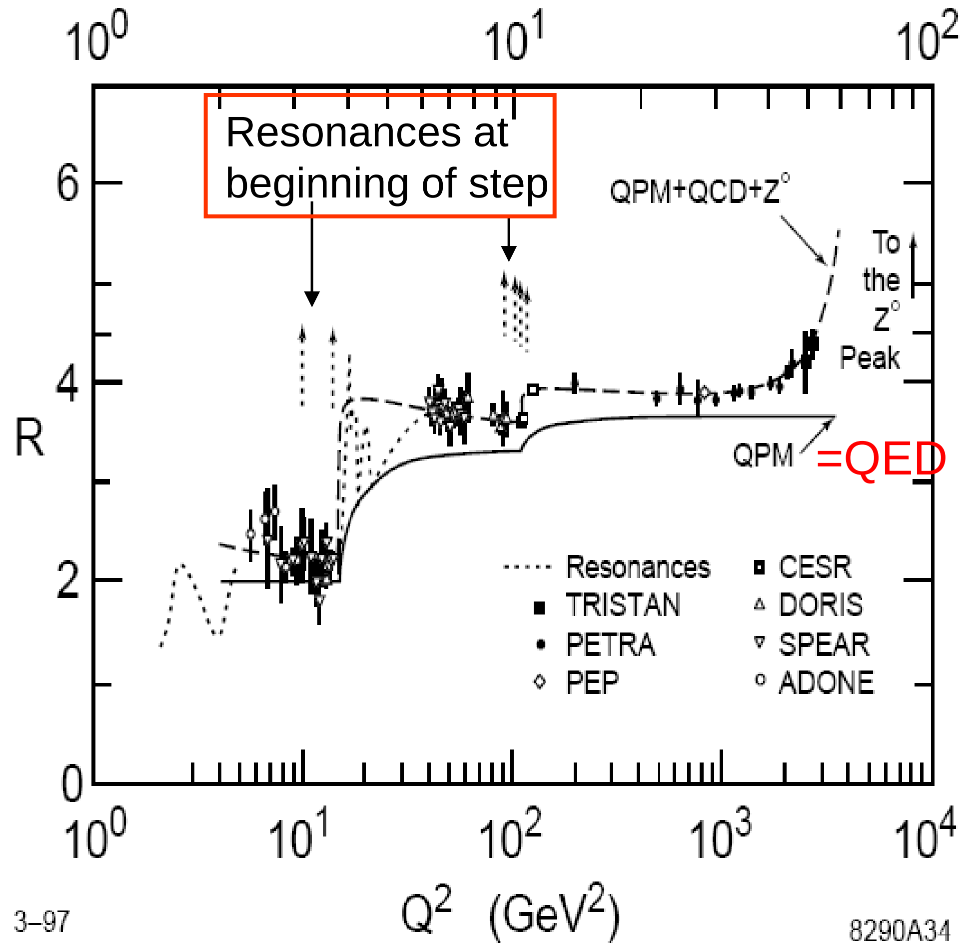




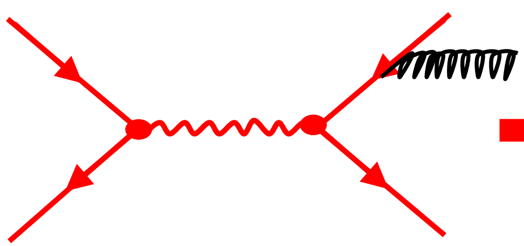
## Definition:

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = 3 \cdot \sum_i Q_i^2$$

| $\sqrt{s}$ | Quarks | $R_{had} = 3 \cdot \sum_i Q_i^2$ |
|---|---|---|
| < ~3 GeV | uds | 3·6/9=2.00 |
| < ~10 GeV | udsc | 3·10/9=3.33 |
| < ~350 GeV | udscb | 3·11/9=3.67 |
| > ~350 GeV | udscbt | 3·15/9=5.00 |




Data lies systematically higher than the prediction from Quark Parton Model (QPM) $\rightarrow$ gluon bremsstrhl.



$$\sigma^{qq}(s) = \sigma^{qq}_{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \dots\right]$$

~ 7%