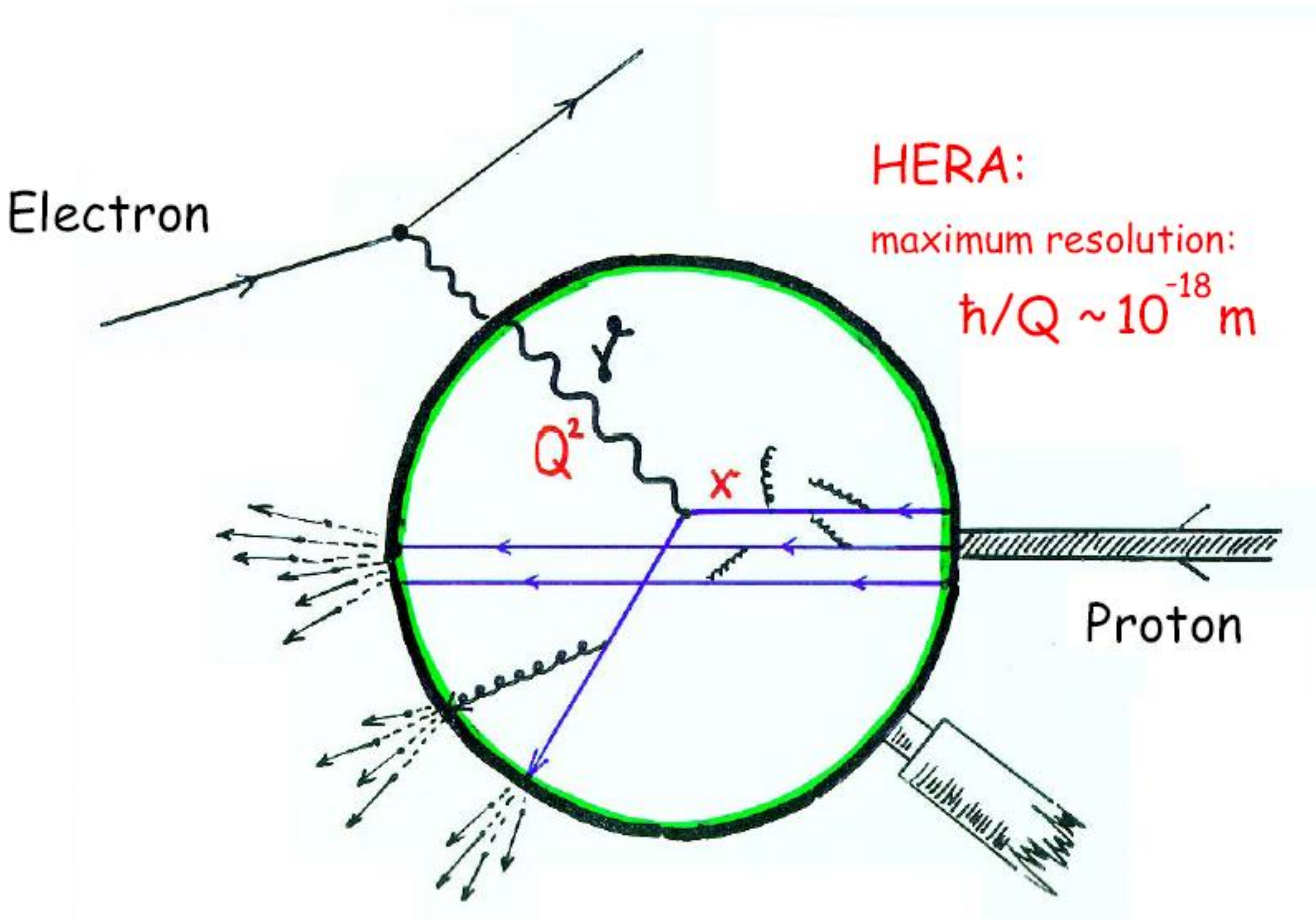
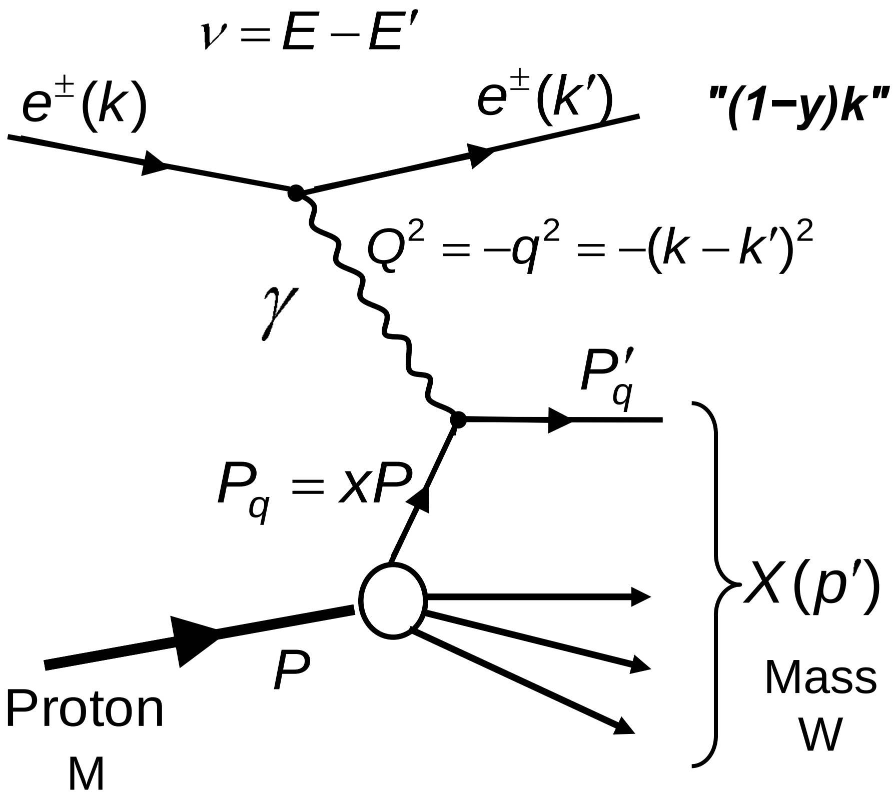
3. Study of QCD in deep inelastic scattering (DIS)



Courtesy: H.C. Schultz-Coulon

3.1 DIS in the quark parton model (QPM)



- Elastic scattering: W = M

  $\Rightarrow$ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W $\neq$ M

  $\Rightarrow$scattering described by
  2 independent variables

$$(E,\nu),\ (Q^2,x),\ (x,y),\ \ldots$$

x = fractional momentum of struck quark

y = Pq/Pk = elasticity, fractional energy transfer in proton rest frame

ν = E - E′ = energy transfer in lab

$$y = \frac{P\cdot q}{P\cdot k}$$

$$x = \frac{Q^2}{2M\nu} = \frac{Q^2}{2P\cdot q} \quad \text{(Bjorken x)}$$

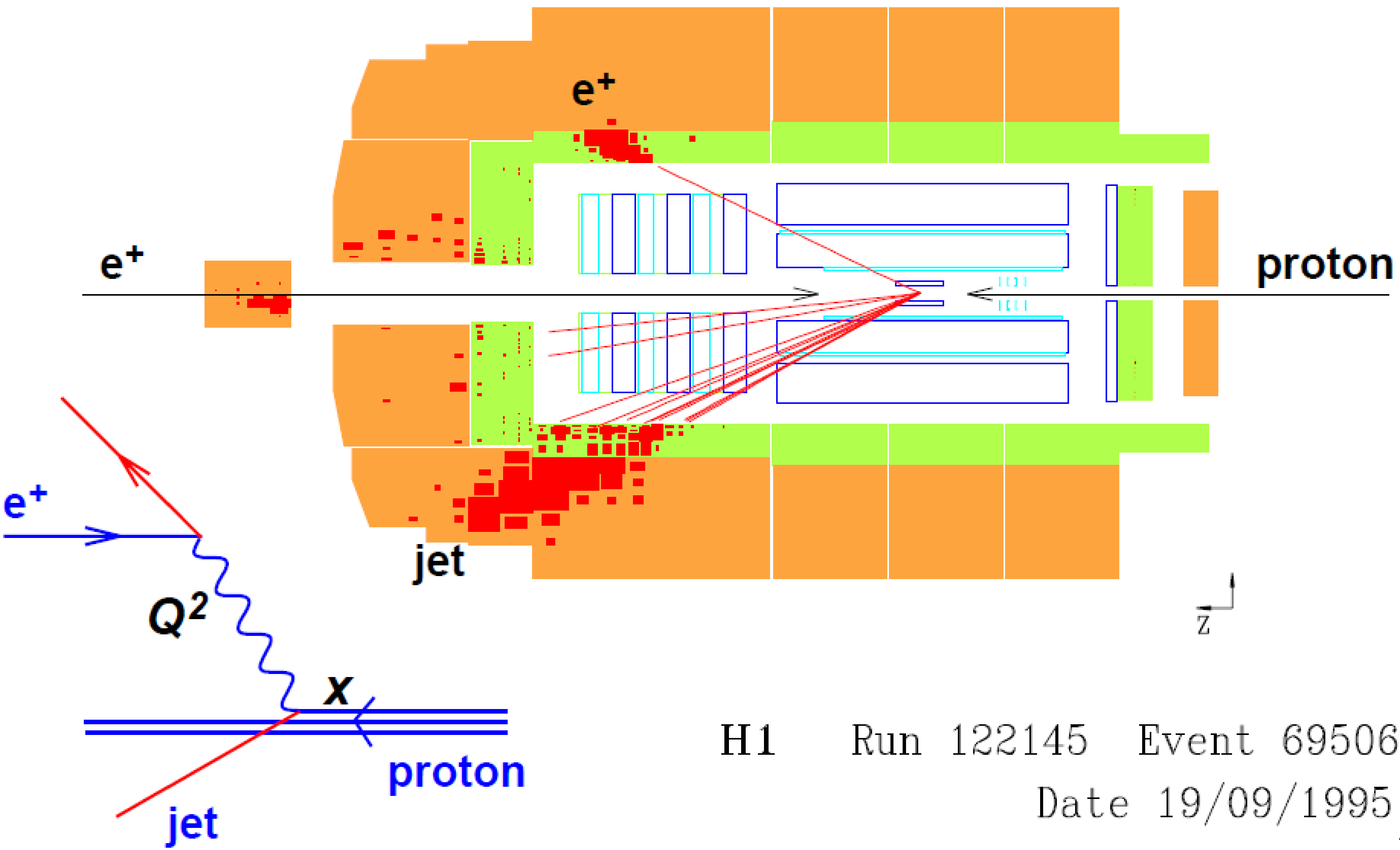
$$Q^2 = sxy$$ s = CMS energy

HERA

$e^{\pm}$ → 30 GeV ← p 900 GeV

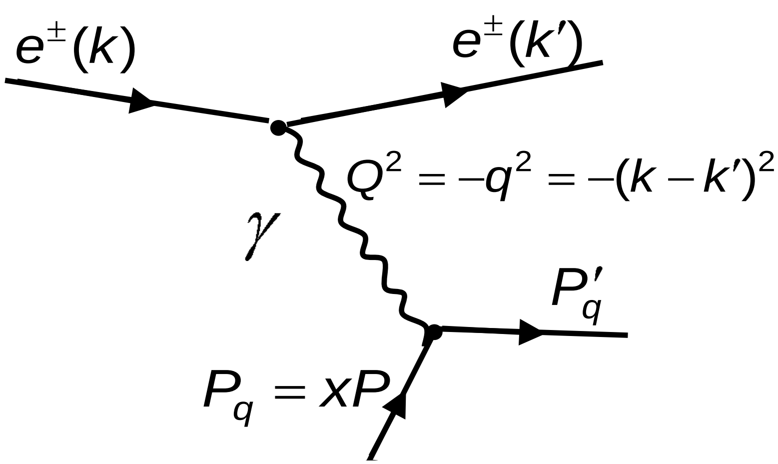
$$s = 4E_eE_p \approx 10^5 \,\mathrm{GeV}^2$$

$Q^2 = 25030\ \mathrm{GeV}^2$, $y = 0.56$, $x = 0.50$



Cross section in quark parton model (QPM)

Elastic scattering on single quark



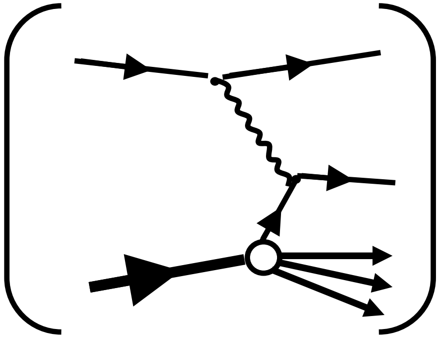
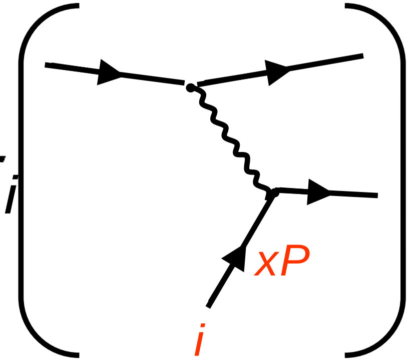
Starting point:
electron muon scattering

$$\ldots_{e\mu\to e\mu}^{elastic} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

Electron-quark scattering (quark momentum fraction x):

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot e_i^2\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

↑ Charge of struck quark

σ  $= \sum_i q_i(x)\,\sigma_i$ 

Parton density $q_i(x)dx$: Probability to find parton i in momentum interval [x, x+dx]

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\sum_i\int_0^1 e_i^2\cdot q_i(\xi)\cdot\delta(x-\xi)\,d\xi\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

Parton distribution function PDF:

$$F_2(x) = x\sum_i\int_0^1 e_i^2 q_i(\xi)\cdot\delta(x-\xi)\,d\xi = x\sum_i e_i^2 q_i(x)$$

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\left(\frac{F_2(x)}{x}\cos^2\frac{\theta}{2} + 2F_1(x)\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

Kinematical relations

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{xQ^4}\right)\cdot\left[(1-y)F_2(x) + xy^2F_1(x)\right]$$

Deep inelastic electron-proton scattering:

- Free partons: $F_2=F_2(x)$ ⇔ “scaling” (F_2 only function of x)
- Spin ½ partons: $2xF_1(x) = F_2(x)$ (Callan-Gross relation)

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{xQ^4}\right)\cdot\left(\frac{1+(1-y)^2}{2}F_2(x)\right)$$

$+\mathcal{O}(\alpha_s)$

Parton level, i.e. ignoring QCD corrections

Parton distribution functions

(ignoring see quarks)

Proton

$$F_2 = x(e_u^2 u(x) + e_d^2 d(x)) = x\left(\frac{4}{9}u(x) + \frac{1}{9}d(x)\right)$$

Neutron

$$F_2^n = \frac{4}{9}u_n(x) + \frac{1}{9}d_n(x) \simeq \frac{4}{9}d_p(x) + \frac{1}{9}u_p(x)$$

↑ isospin symmetry

Considering QCD corrections: Valence quarks + see quarks

Isoscalar Target: #n=#p

$$F_2^N = \frac{1}{2}\left[F_2^p + F_2^n\right] = \frac{5}{18}x\cdot[u+\bar{u}+d+\bar{d}] + \frac{1}{9}x\cdot[s+\bar{s}]$$

Charged-current ($W^{\pm}$) scattering by using neutrinos instead of electrons, allows to determine the valence quark distributions.

PDF for Neutrino Scattering

Additional PDF F_3 to account for parity violation

$$F_2^{\nu p} = 2x[d + \bar{u}] \qquad xF_3^{\nu p} = 2x[d - \bar{u}]$$

$$F_2^{\nu n} = 2x[d^n + \bar{u}^n] \qquad xF_3^{\nu n} = 2x[d^n - \bar{u}^n]$$

$$= 2x[u + \bar{d}] \qquad = 2x[u - \bar{d}]$$

$$F_2^{\nu N} = x[u + \bar{u} + d + \bar{d}] \qquad xF_3^{\nu N} = x[(u + d) - (\bar{u} + \bar{d})]$$

Iso-scalar target →

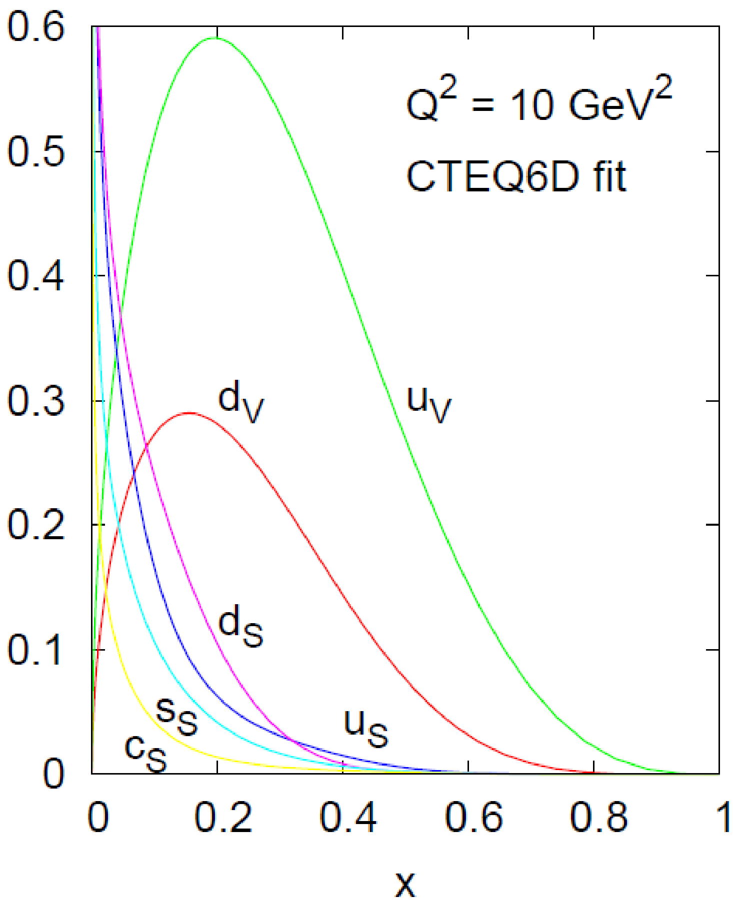
$$F_2^{\nu N} = x[Q(x) + \bar{Q}(x)] \qquad xF_3^{\nu N} = x[Q(x) - \bar{Q}(x)]$$

Measures sum of quarks and anti-quarks

Measures valence quarks

Measurement: $F_2^{\nu N} + xF_3^{\nu N} = 2xQ(x)$ ➡ Sea and valence quarks

$F_2^{\nu N} - xF_3^{\nu N} = 2x\bar{Q}(x)$ ➡ Sea quarks



Definition of PDFs: $\sum_i \int dx\, x q_i(x) = 1$

q_i	momentum
d_V	0.111
u_V	0.267
d_S	0.066
u_S	0.053
s_S	0.033
c_S	0.016
total	0.546

46% of nucleon momentum not carried by quarks

Gluons have been neglected so far.

Sum rules

$$\int_0^1 u(x) - \bar{u}(x)\,dx = \int_0^1 u_v(x)\,dx = 2$$

$$\int_0^1 d(x) - \bar{d}(x)\,dx = \int_0^1 d_v(x)\,dx = 1$$

Valence quarks

$$\int_0^1 (q_s(x) - \bar{q}_s(x))\,dx = 0$$

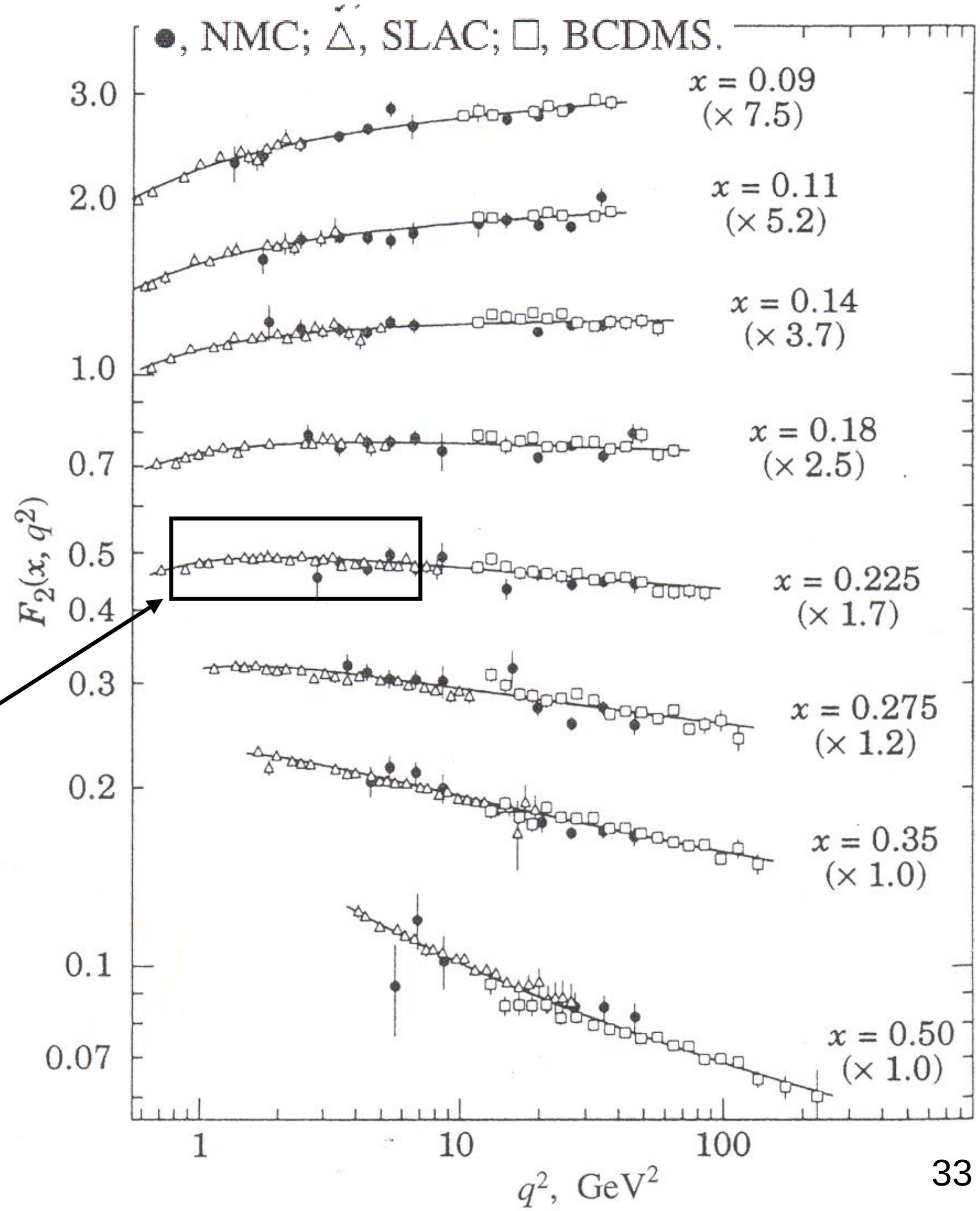
Sea quarks: s, c, …

3.2 Scaling violation

Structure function:

$$F_2 = F_2(x,Q^2) = x\sum_i e_i^2 q_i(x)$$

PDFs are functions of Q^2 as expected when considering QCD corrections.

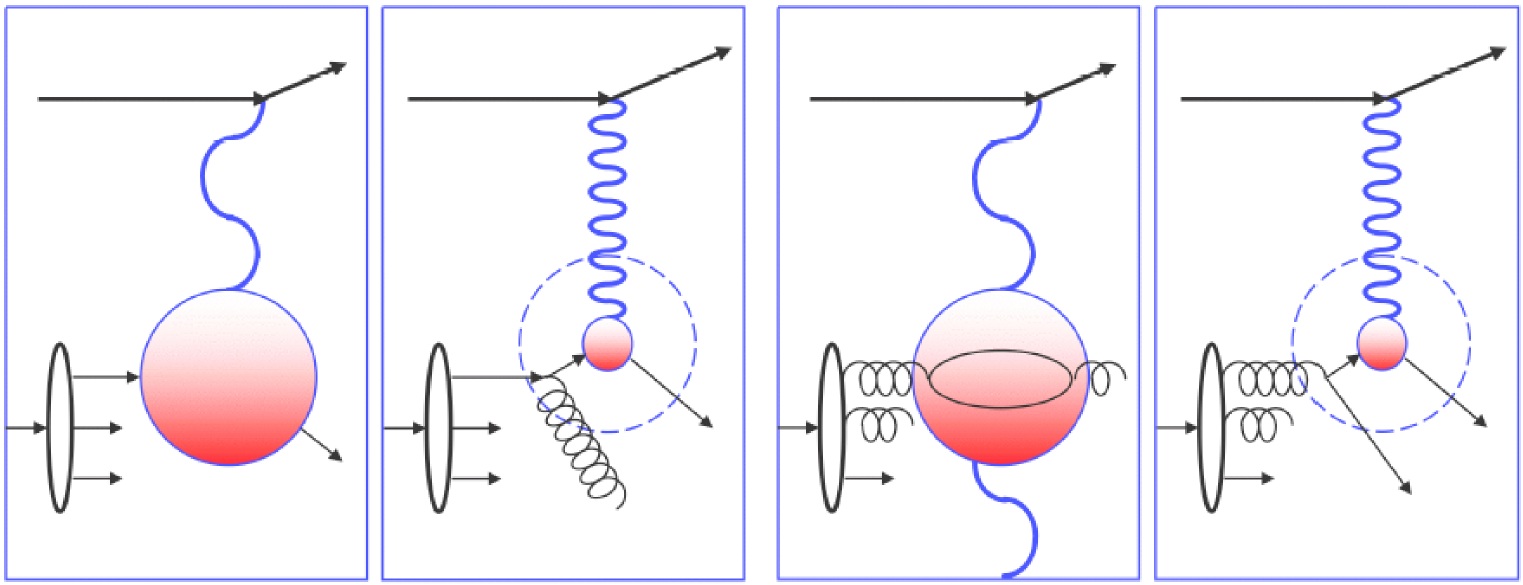


Region of 1st SLAC measurement (1972):

Scaling: $F_2 = F_2(x)$, PDFs are independent of Q^2 (QPM).

QCD explanation for scaling violation

Large x: valence quark scattering   Small x: sea quark scattering



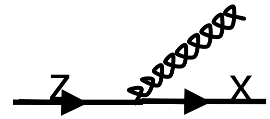
$Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x $Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed (small) x

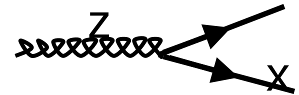
Scaling violation is one of the clearest manifestation of radiative effects predicted by QCD. PDFs depend on Q^2 (structure functions)

Evolution of parton densities

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **Al**tarelli, **P**arisi)

evolution of quark density with $\ln Q^2$



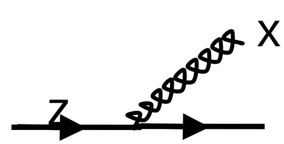


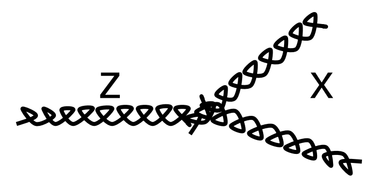
$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left[q(z,Q^2) P_{qq}\left(\frac{x}{z}\right) + g(z,Q^2) P_{qg}\left(\frac{x}{z}\right) \right]$$

Splitting function P_{qq}:
Probability for $q(z) \rightarrow q(x) + g$

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi} \int_x^1 \frac{dz}{z} \left[q(z,Q^2) P_{gq}\left(\frac{x}{z}\right) + g(z,Q^2) P_{gg}\left(\frac{x}{z}\right) \right]$$

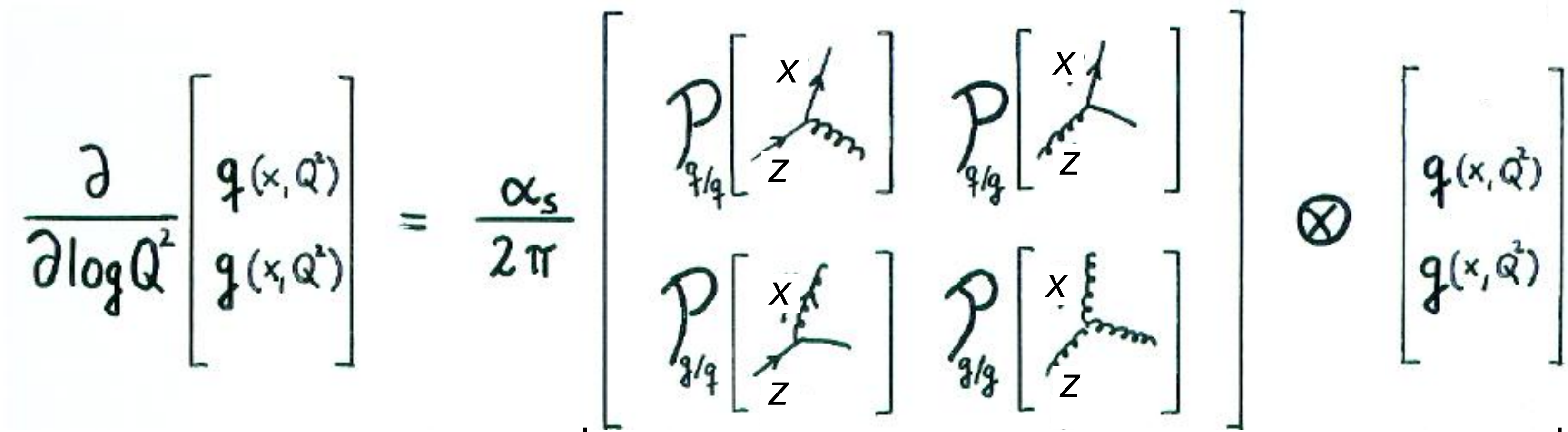
evolution of gluon density with $\ln Q^2$



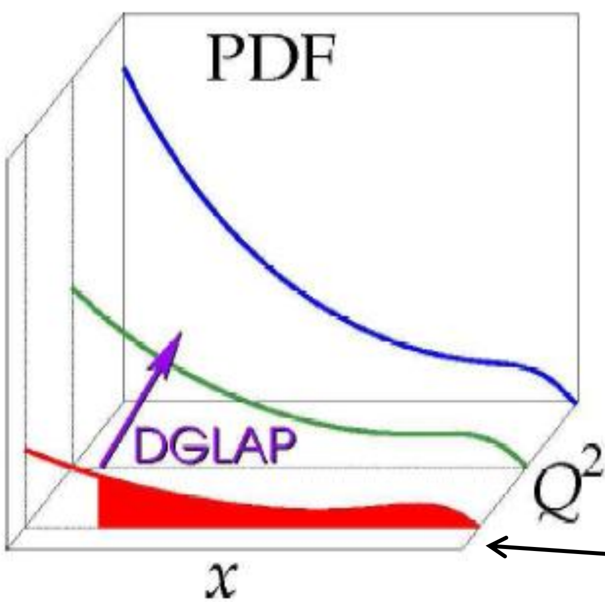


Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction $\varepsilon = x/z$ of the parent parton.

DGLAP Evolution ("symbolic"):



$$P \otimes f(x,Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2)$$



<u>QCD evolution:</u>

QCD predicts evolution of the PDF along the scale Q^2.

QCD cannot predict the shape of PDF, PDF must be measured!

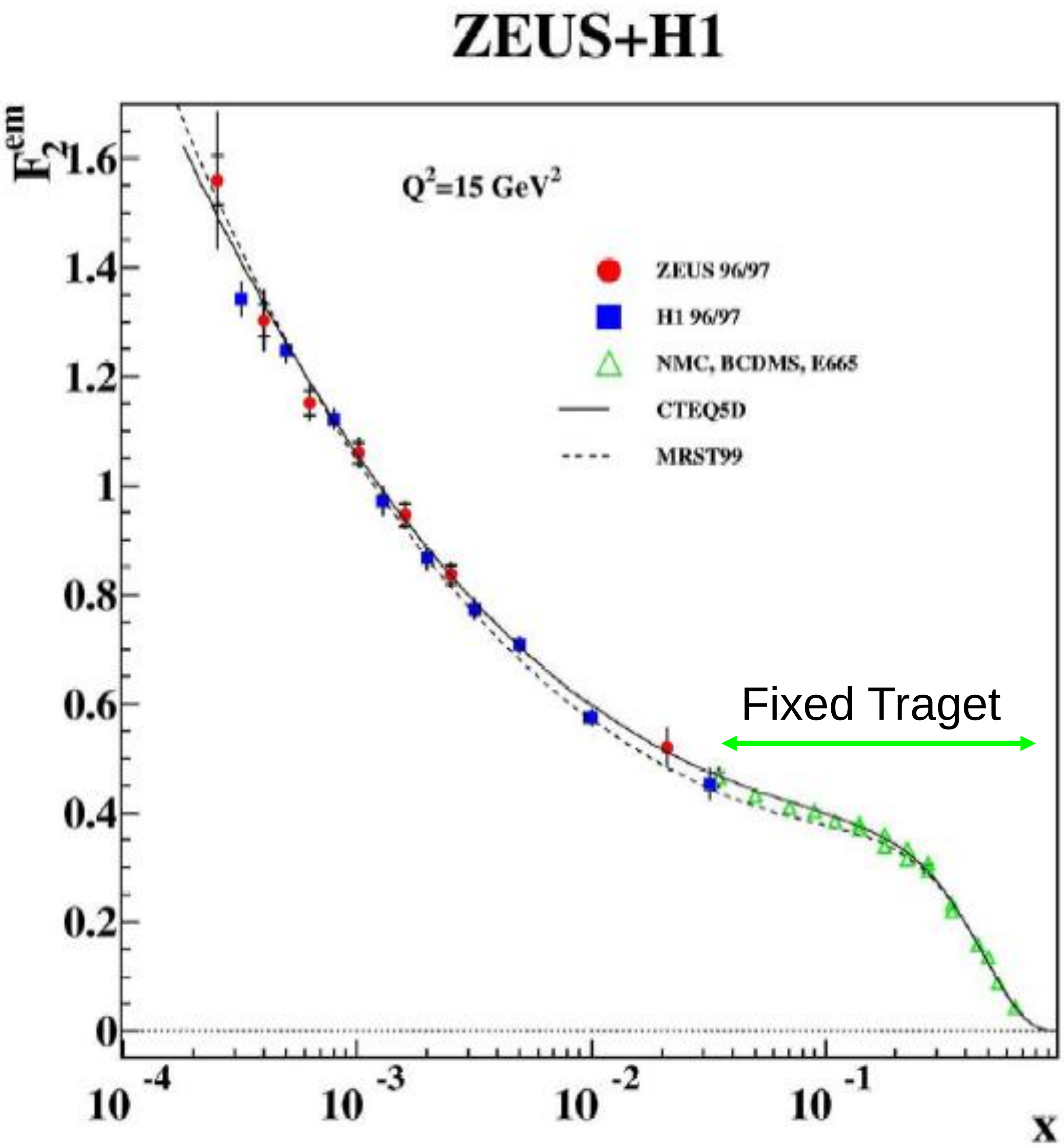
Measurement of the structure function $F_2(x,Q^2)$

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot\left[2\cdot(1-y)F_2(x,Q^2) + y^2F_2(x,Q^2)\right]$$

e.g. for y=1 $\quad Q^2 = sxy$

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot F_2(x,Q^2)$$

$$F_2(x,Q^2) = x\sum_q e_q^2\left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$



$F_2(x)$

Large increase of $F_2(x)$ for very small x - unexpected

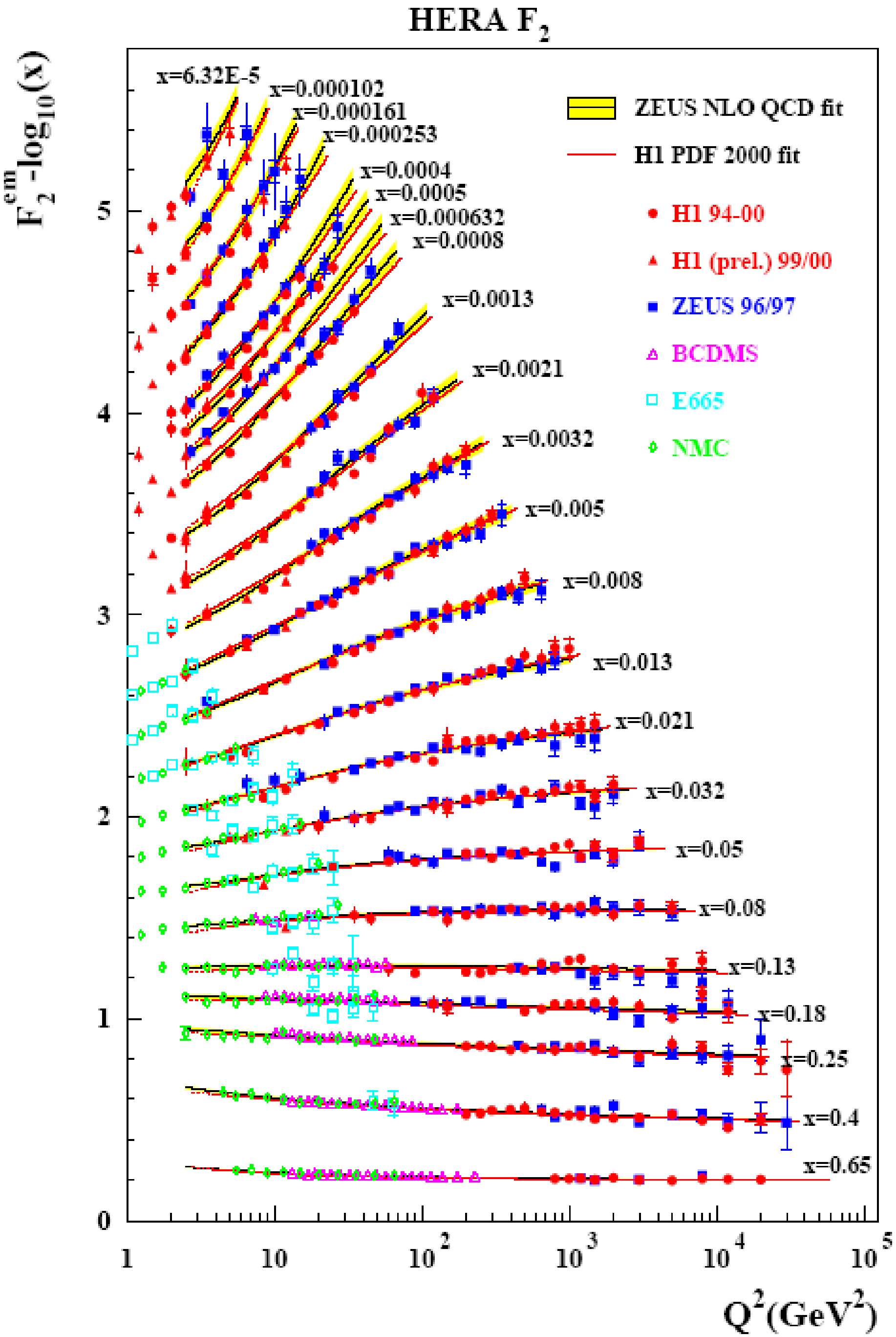
$F_2(x,Q^2)$

↓

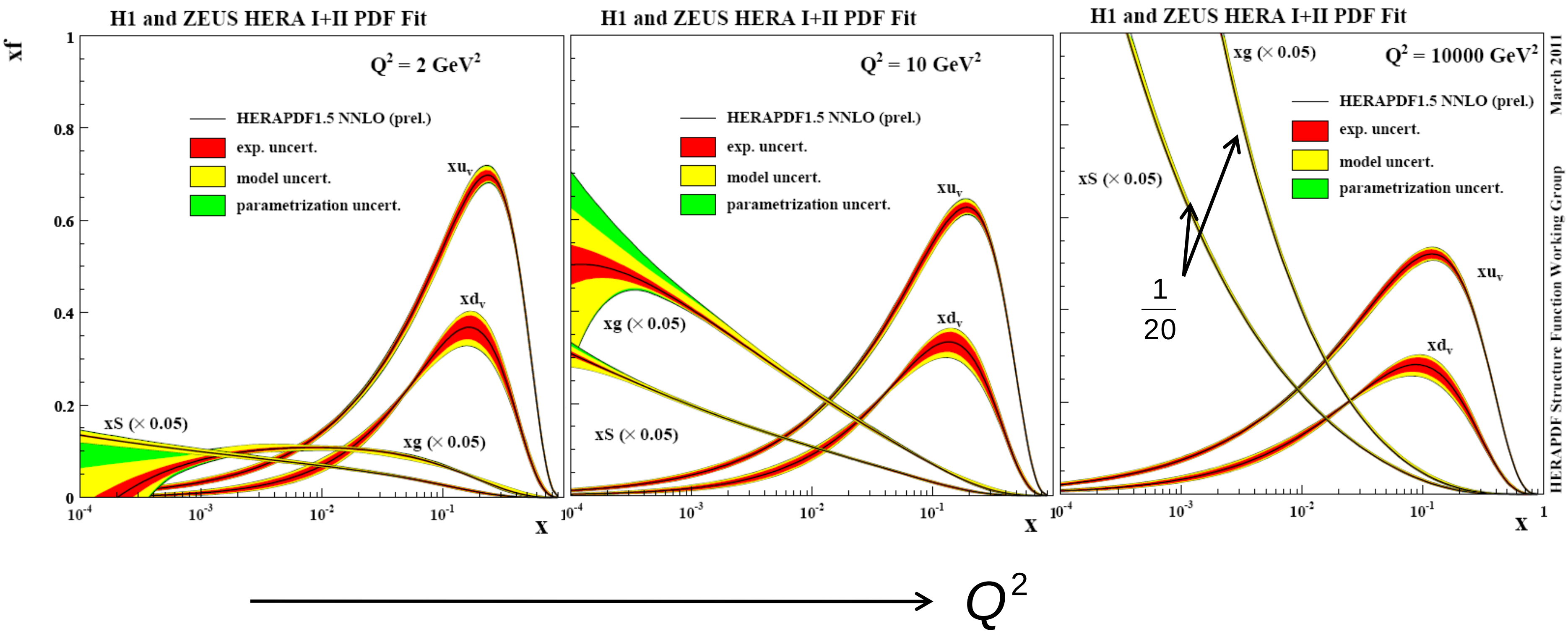
Q^2 dependence is correctly described by QCD evolution

↓

Determine the PDFs

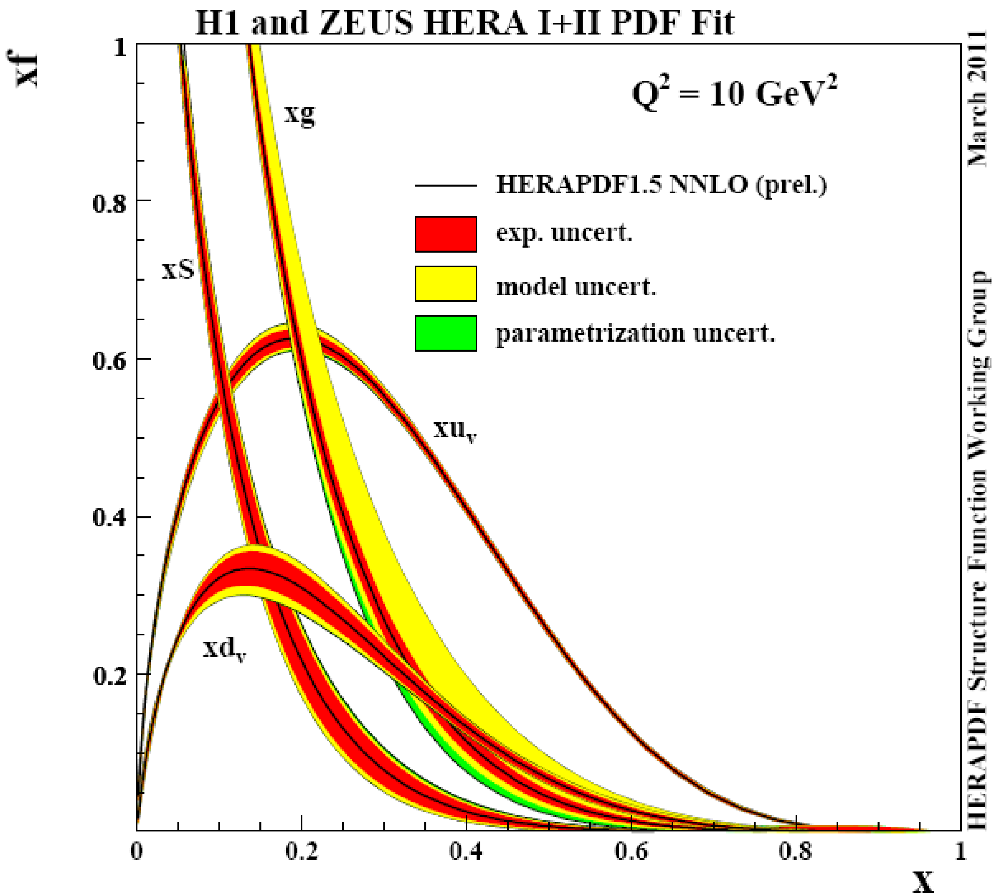


Proton PDFs as seen by HERA



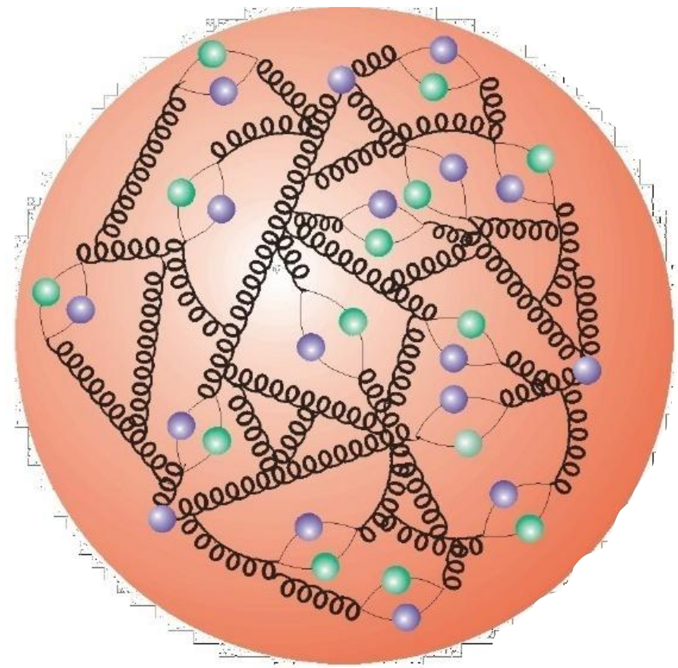
https://www.desy.de/h1zeus/combined_results/index.php?do=proton_structure

Structure of the proton



$$\# \text{ Valenzquarks} = \int u_v(x) + d_v(x)\, dx = 3$$

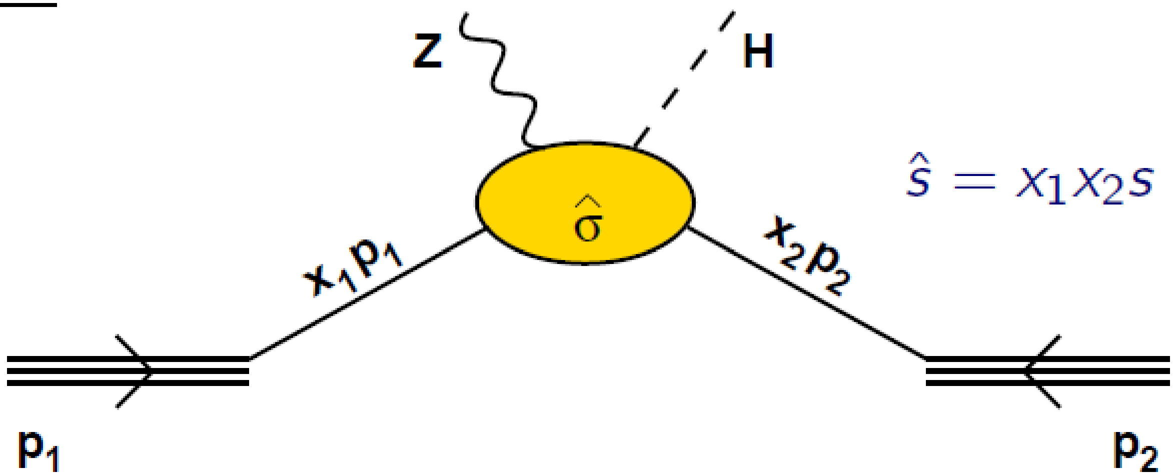
$$\# \text{ Gluonen} = \int g(x)\, dx > 30$$



The most dramtic of these [experimental consequences], that the protons viewed at ever higher resolution would appear more and more as field energy (soft glue), was only clearly verified at HERA ...   F. Wilczek [Nobel Prize 2004]

4. Hadron-hadron collisions

Factorization



$$\sigma = \int dx_1 f_{q/p}(x_1, \mu^2) \int dx_2 f_{\bar{q}/\bar{p}}(x_2, \mu^2)\, \hat{\sigma}(x_1 p_1, x_2 p_2, \mu^2)$$

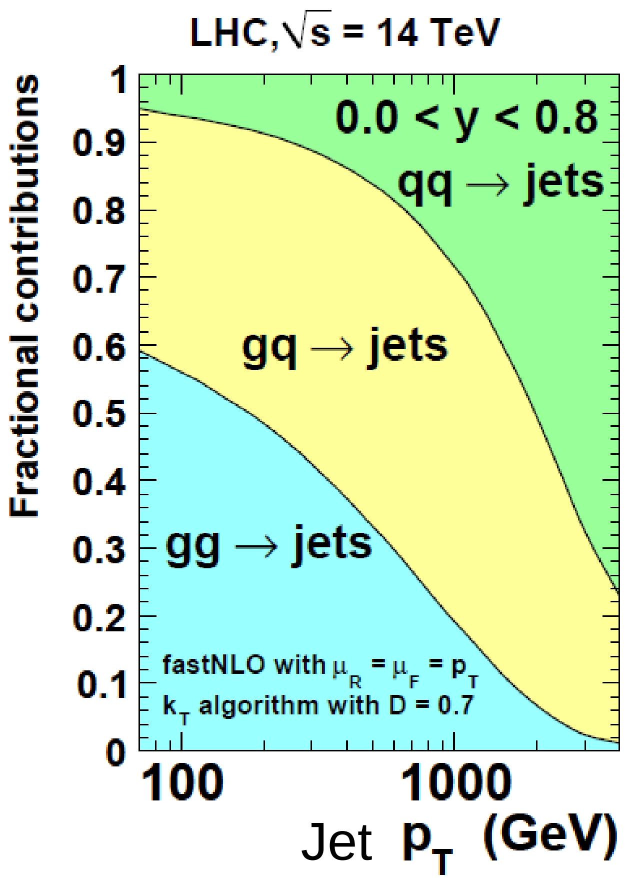
factorization scale

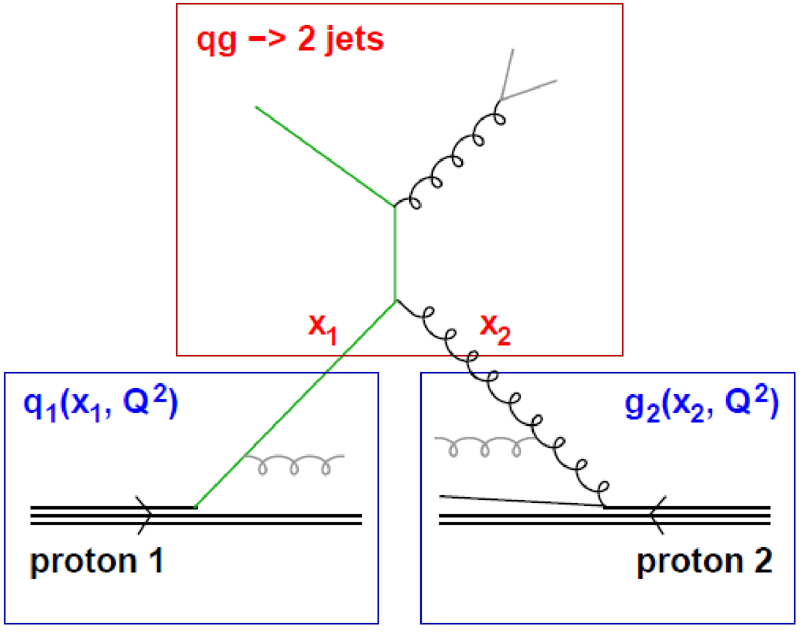
Total cross section is factorized into a “hard part” and into a “normalization” from process independent parton distribution functions.

For all cross section estimation the knowledge of the PDF is necessary.

Example process: 2-jet production

Jet production in proton-proton collision is an excellent test of PDFs, in particular of gluon PDF, since there are large direct contributions from $gg \to gg$ and $qg \to qg$:





$$\sigma_{pp\to 2\,jets} = \sigma_{qg\to 2\,jets} \otimes q_1 \otimes g_2 + \cdots$$

Inclusive jet production well described with known PDF.