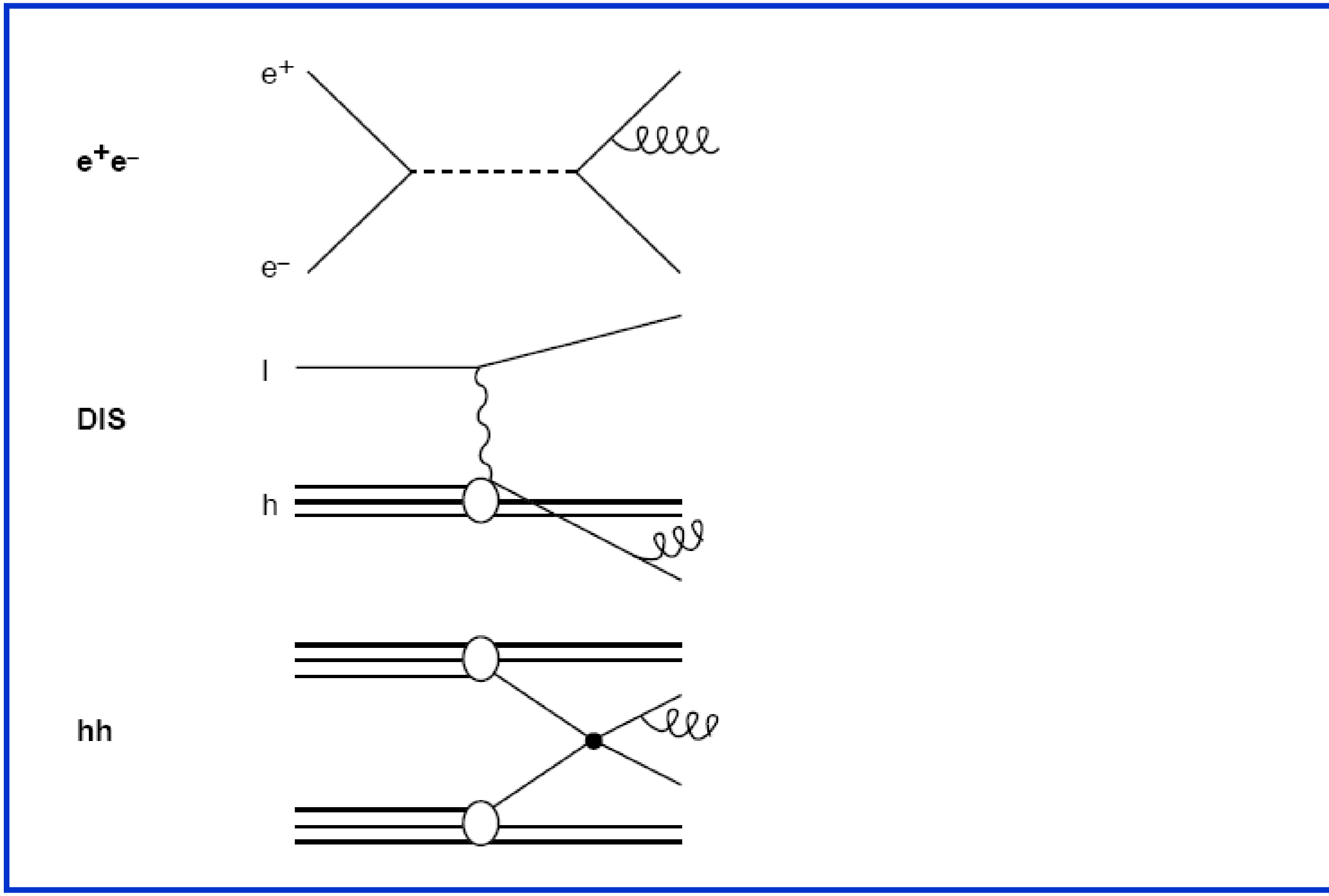
Experimental Tests of QCD

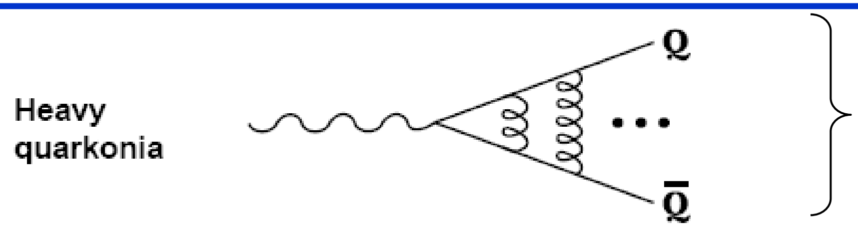
1. Test of QCD in e+e- annihilation
2. Running of the strong coupling constant
3. Study of QCD in deep inelastic scattering
4. Hadron-hadron collisions

Excellent lecture notes:

*www.lpthe.jussieu.fr/~**salam**/repository/talks/2010-MariaLaach-**lecture**1...4.pdf*

Test of QCD in different processes

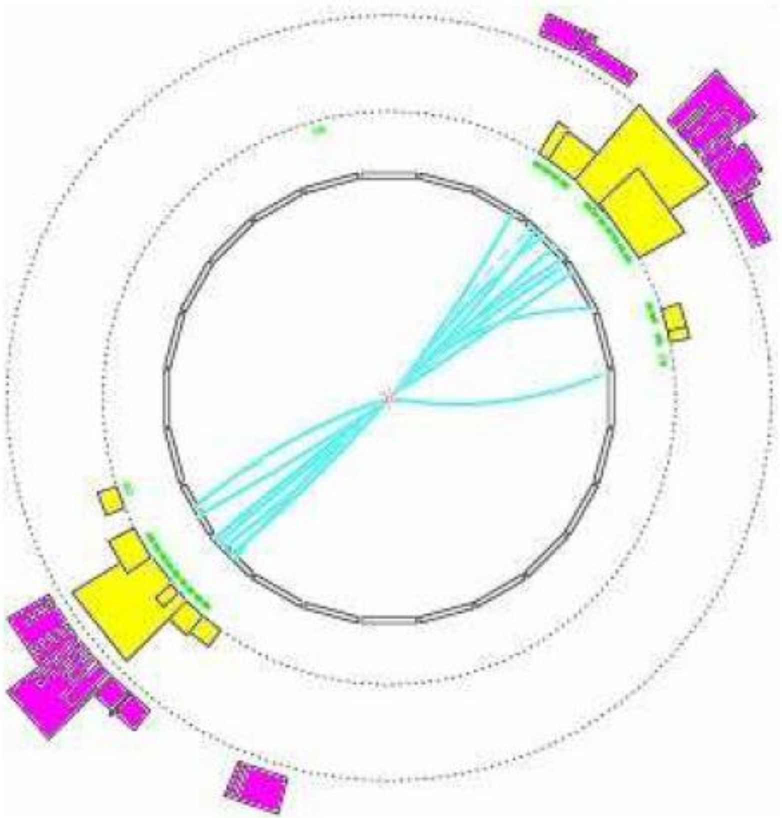


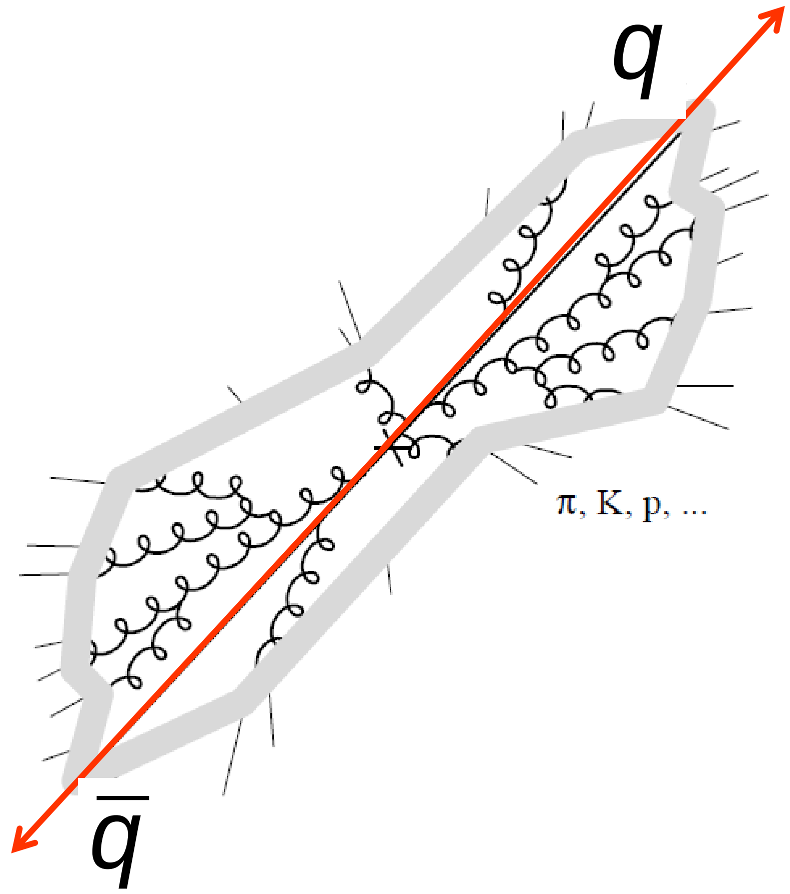


not discussed

1. Test of QCD in e^+e^- annihilation

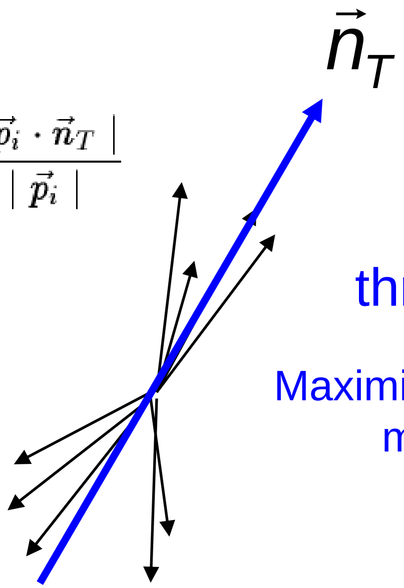
$e^+ e^- \to q\, \bar{q}$ and hadron jets





2-Jet-likeness: Thrust

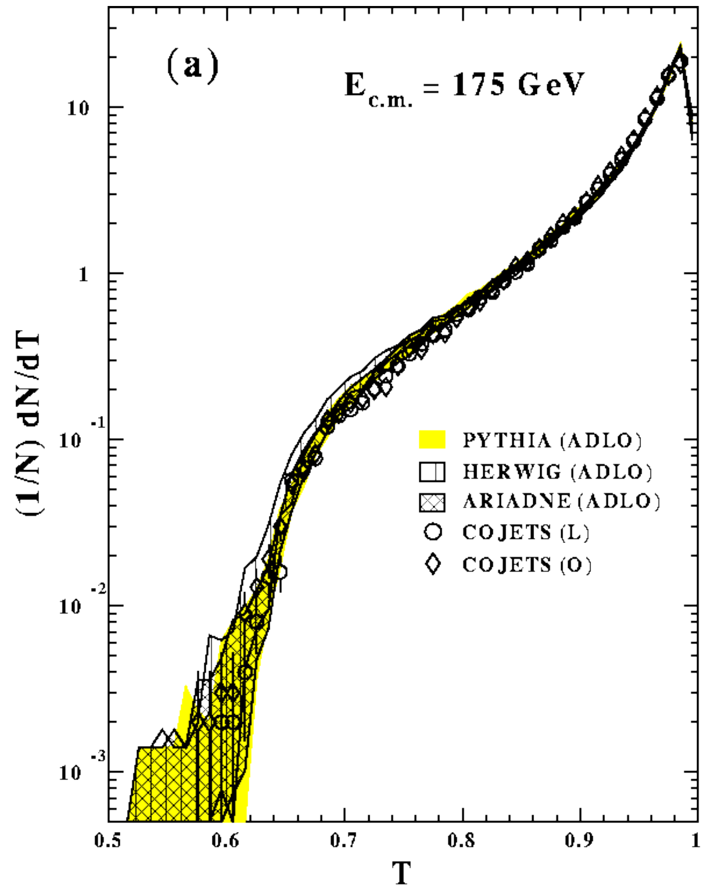
$$T = \max \frac{\sum_i | \vec{p}_i \cdot \vec{n}_T |}{\sum_i | \vec{p}_i |}$$



thrust axis

Maximizes longitudinal momentum

Example: →



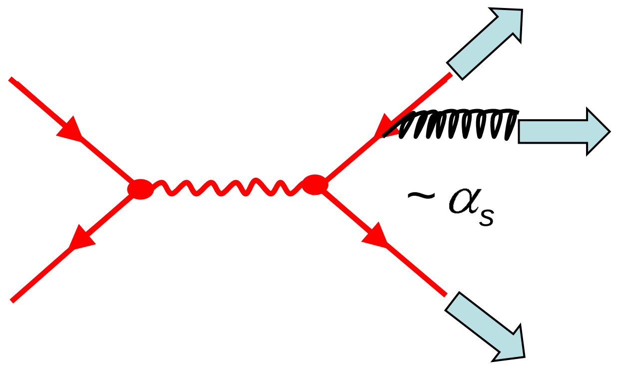
thrust distribution (LEP) in e^+e^- -events at $\sqrt{s}$ = 175 GeV: very much 2-jet like.

At early PETRA energies not that pronounced (boost effect).

1.1 Discovery of the gluon

P. Söding, On the discovery of the gluon
www.springerlink.com/content/124362w3075v6042/

Discovery of 3-jet events by the TASSO collaboration (PETRA, DESY) in 1977:



JADE

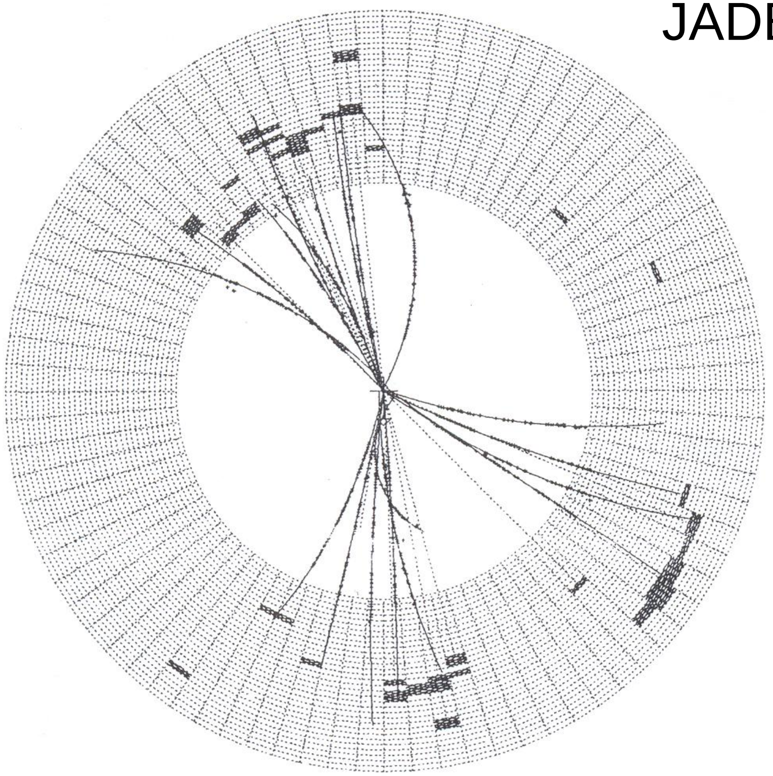


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

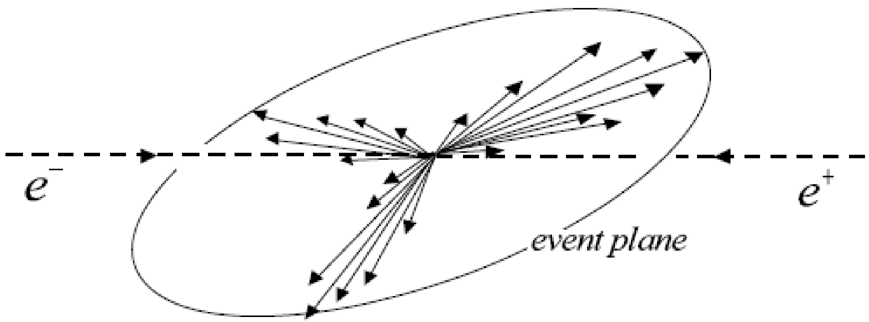
3-jet events are interpreted as quark pairs with an additional hard gluon.

$$\frac{\#3-\text{jet events}}{\#2-\text{jet events}} \approx 0.15 \sim \alpha_s$$

at $\sqrt{s}$=20 GeV

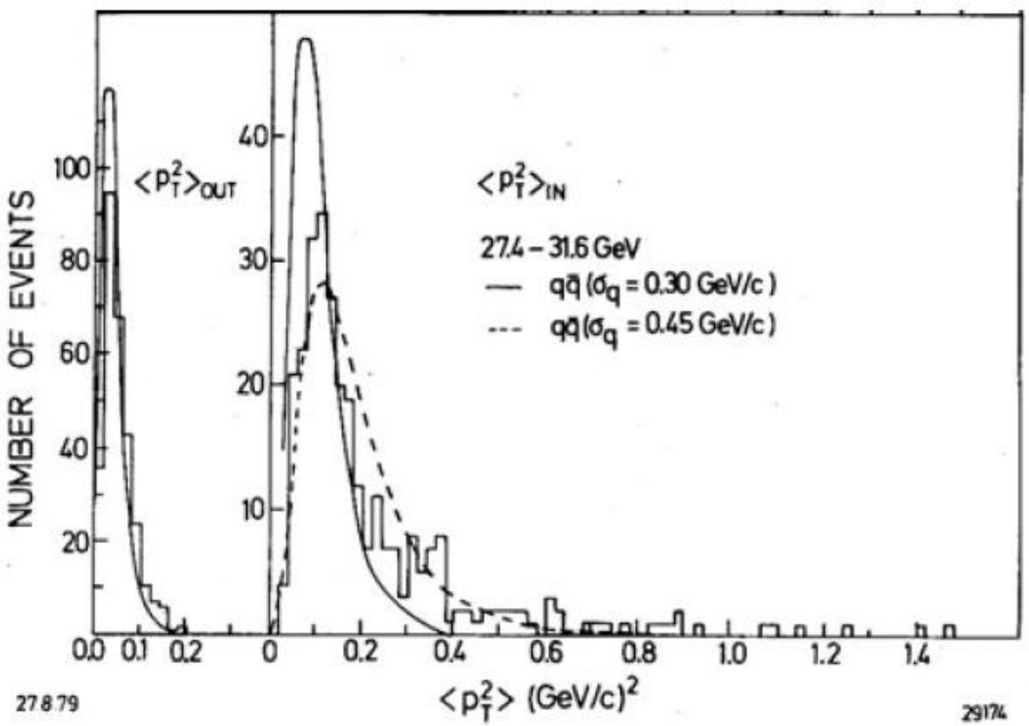
⇨ α_s is large

Fluctuation or real signature?



qqg three jet events should be planar.

Look for $<p_T^2>$ in-side and out-side the event plane.



$<p_T^2>_{in}$ cannot be described by qq-assumption (curves), even when assuming different mean transverse p_T in the jet.

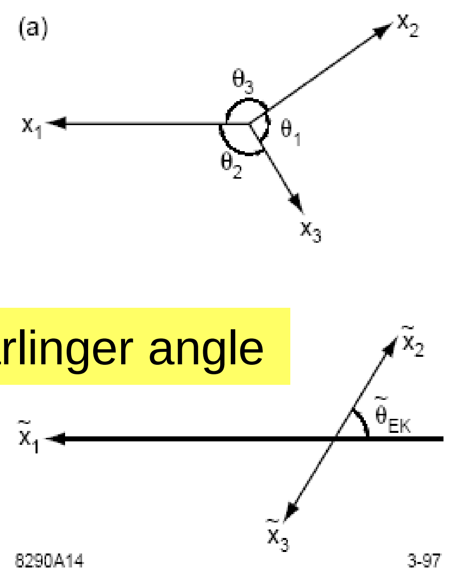
$<p_T^2>_{out}$ however is well described.

Observed signatures consistent with 3 jet-picture.

1.2 Spin of the gluon

Angular distribution of jets depend on gluon spin:

Ordering of 3 jets: $E_1>E_2>E_3$



Ellis-Karlinger angle

Figure 8: (a) Representation of the momentum vectors in a three-jet event, an (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: θ_{EK}

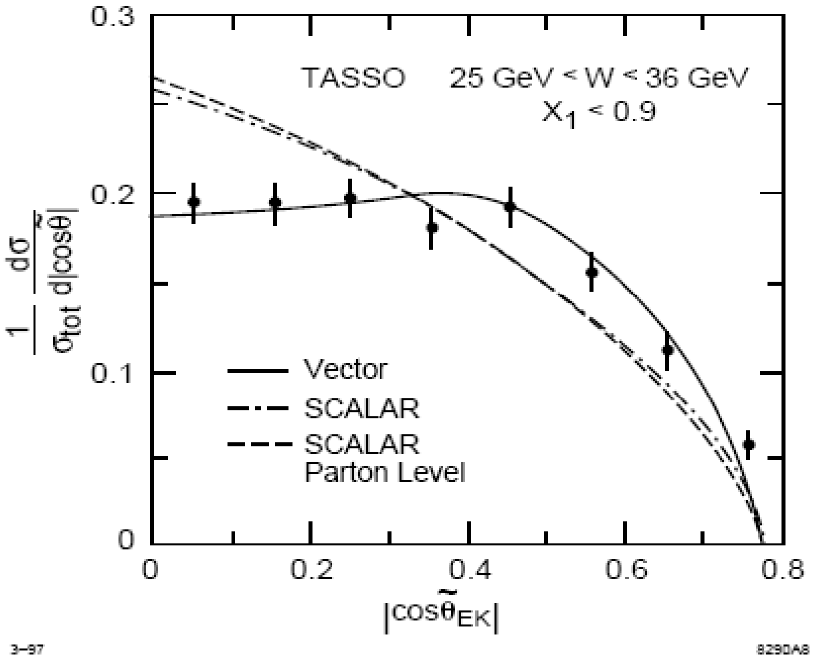


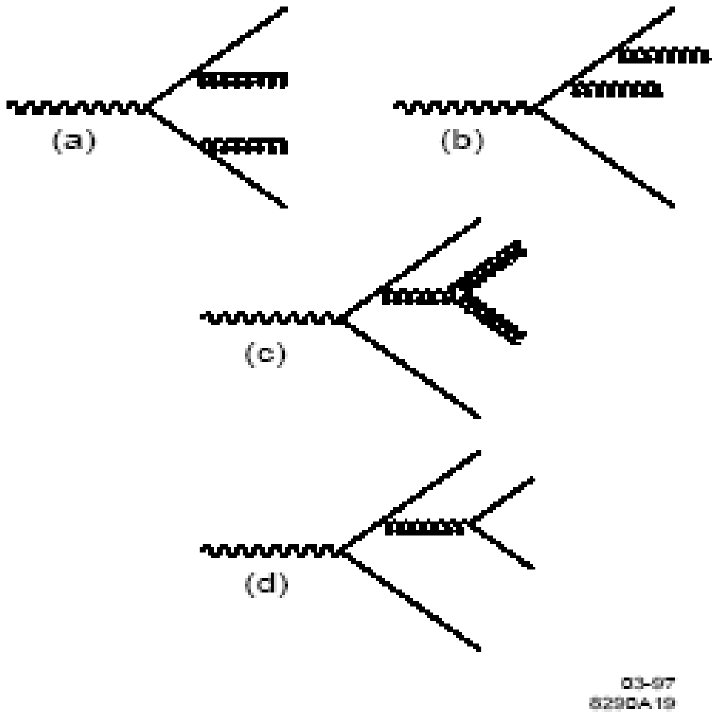
Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

Gluon spin J=1

1.3 Multi-jet events and gluon self coupling

Non-abelian gauge theory (SU(3))

4-jet events



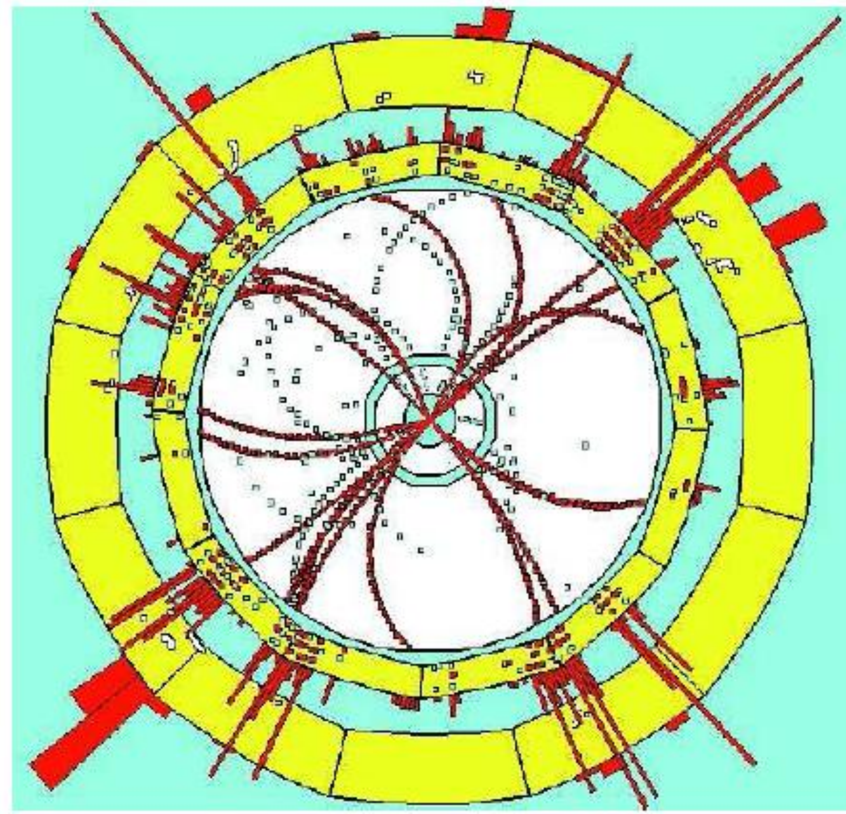
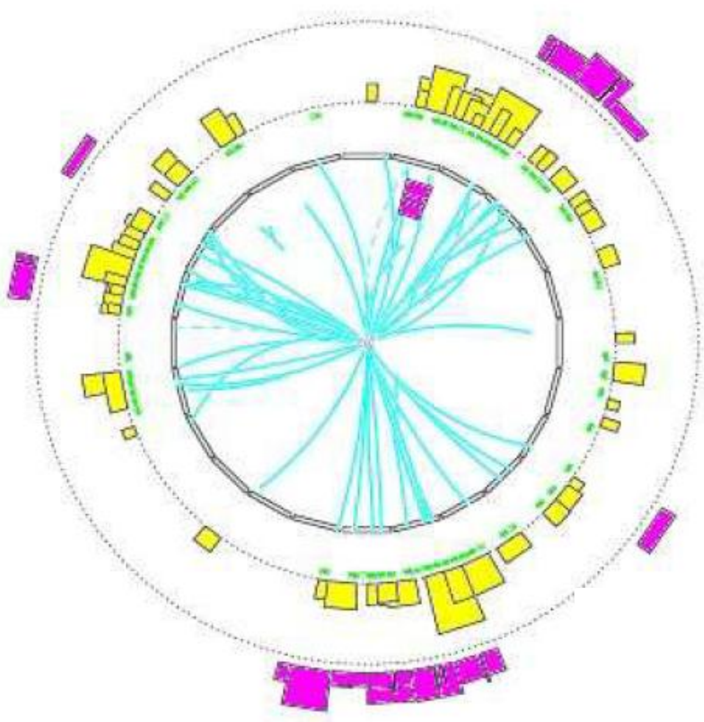


Figure 1: Hadronic event of the type $e^+e^- \to 4$ jets recorded with the ALEPH detector at LEP-I.

Multi-jet event ALEPH exp (LEP)

➡ 4 jet events allow to test the existence of gluon self coupling.

How many jets?

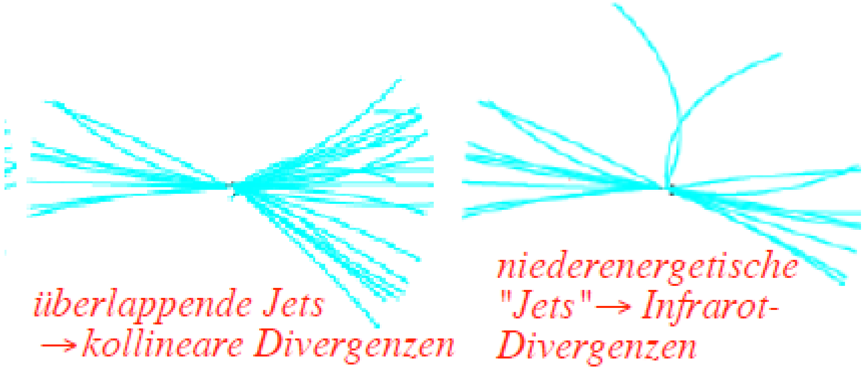


Problem:

There is no “natural” definition of jets.

Need well defined algorithms which are also applicable to “theoretical calculations”, i.e. at parton level.

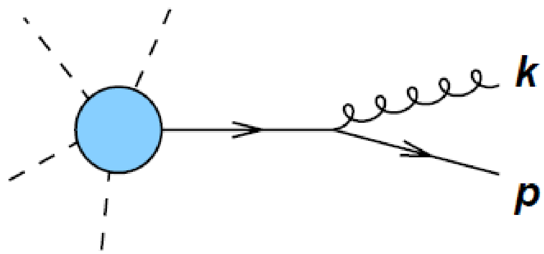
In addition jet-algorithms should be “collinear” and “infra-red” safe:



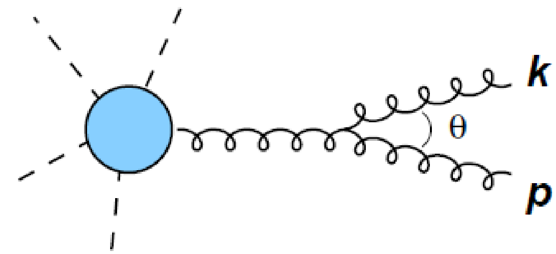
Reminder: Gluon emmission

divergent

Gluon emission from quark: $\frac{2\alpha_s C_F}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$



from gluon: $\frac{2\alpha_s C_A}{\pi} \frac{dE}{E} \frac{d\theta}{\theta}$



Both expressions valid only for $\theta \ll 1$.

$C_F = 4/3$ v. $C_A = 3$

Gluons have two color charges
→ probability to emit gluon is higher.
Associated color factor C_A is larger

Infrared (1/E) and collinear (1/θ) divergent.

Interlude: Jet algorithms

2 principle classes of jet-algorithms exist:

- Define "distance measure" between particles and do a successive recombination of closest pairs until resulting pseudo particle are too "far way". Remaining pseudo particles are jets. Every particle belongs to exactly one jet.

Durham Jet-Algorithm

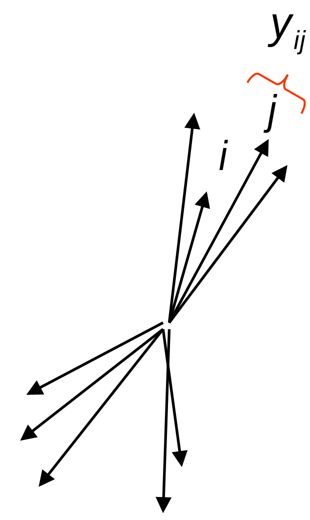
- Most frequent algorithm in e^+e^- exp.
- Particles (group of particles) i and j are not resolved and grouped to a single pseudo particle k if the **resolution parameter y_{ij}**

$$y_{ij} = \frac{2 \cdot \min(E_i^2, E_j^2) \cdot (1 - \cos\theta_{ij})}{s}$$

is smaller than the resolution $y_{cut.}$ The new pseudo particle k is obtained by:

$$E_k = E_i + E_j \quad \vec{p}_k = \vec{p}_i + \vec{p}_j$$

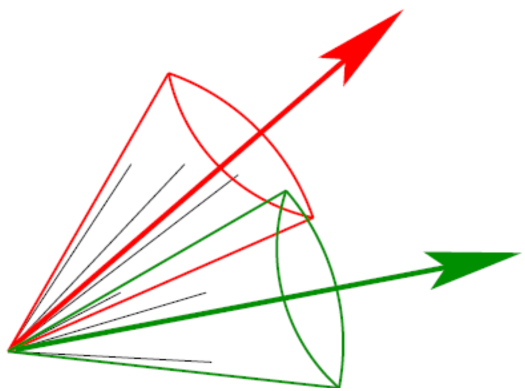
Recursive iteration until all $y_{ij} \geq y_{cut}$.
Remaining pseudo-particle are the final jets.



Beside the Durham algorithm there are other algorithms with slightly different definition of y_{ij} and "joining scheme": JADE, Aachen, Cambridge

- In cone algorithms jets are defined as the dominant direction of energy flow. Introduce concept of stable cone as a circle of fixed radius R in the plane such that the sum of all the momenta of the particles within the cone points in the direction of the cone-center. Cone algorithms attempt to identify all the stable cones.

  Most implementations use a seeded approach to do so: starting from one seed for the centre of the cone, one iterates until the cone is found stab

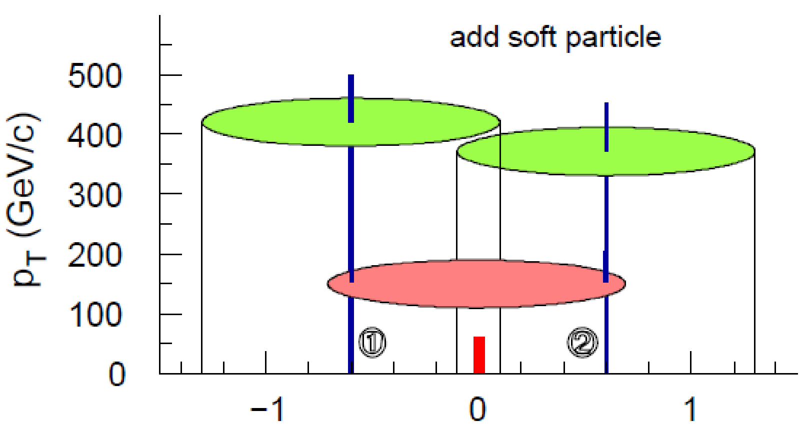


How to find the stable cones?

How to avoid that particles belong to two or no cone?

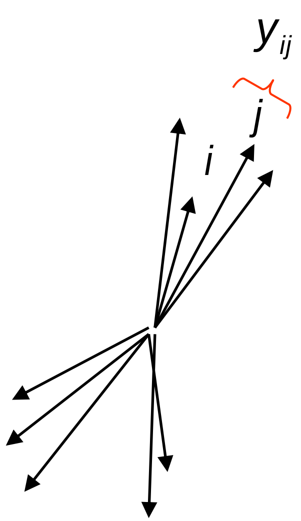
Different implementations of cone algorithms are used at hadron colliders: They differ in the start-point definition, how they find stable cones and how they deal with overlapping jets (split/merge).

Not all cone algorithms are infra-red and collinear safe.

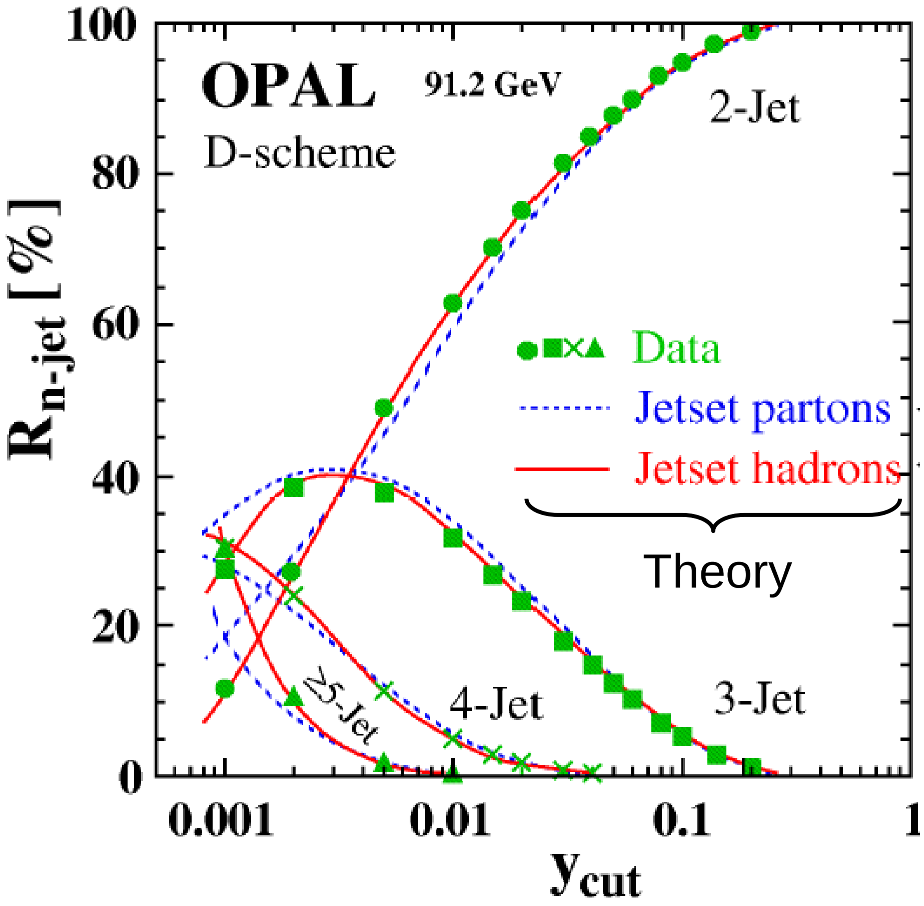


← Infra-red problem

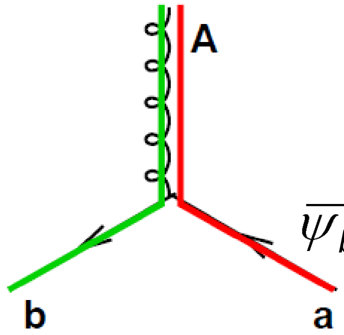
Multiple-jet events in e^+e^- annihilations



$y_{ij} \geq y_{cut}$



Gauge group structure and color factors

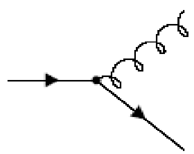


vertex factor

$$\bar{\psi}_b \left(-i g_s T^A_{ba} \gamma^\mu\right) \psi_a$$

Guon emmission “repaints” the quarks.

The generators of the gauge group describe the gluons and appear in the vertex functions.



$$\left| \cdots \right|^2 \sim \sum_{k,c} T^k_{ac} T^k_{cb} = \delta_{ab} C_F = \delta_{ab} \frac{4}{3}$$

In perturbative calculations the average and sum over all possible color configuartions lead to combinatoric factors: color factors.

Color factors related to the structure constants of the SU(3) gauge group.

SU(3) gauge group

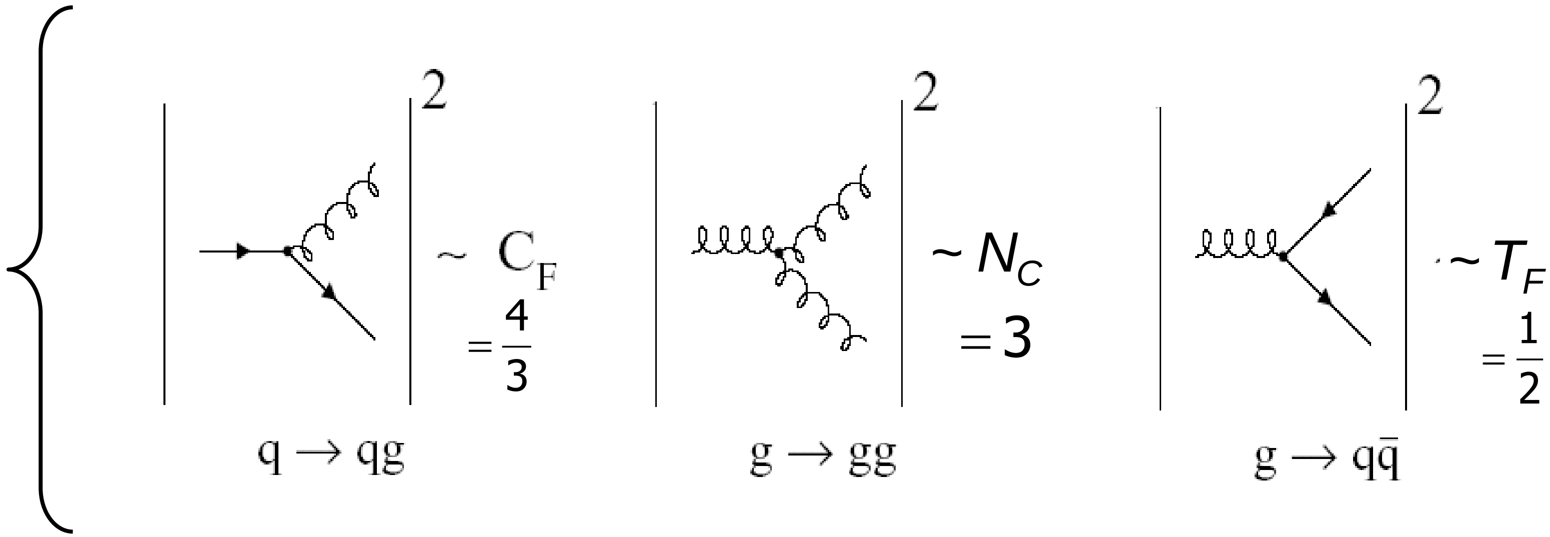
$$\left[T^a, T^b\right] = i \sum_c f^{abc} \cdot T^c \qquad T^a = \frac{\lambda^a}{2}$$

λ^a Gell-Mann matrices

f_{abc} are the structure constants

Color factors relevant for 4-jet events

Different relative angular distribution



$$\sum_{k,\eta} T^k_{\alpha\eta} T^k_{\eta\beta} = \delta_{\alpha\beta} C_F = \frac{N_C^{\,2}-1}{2N_C} \cdot \mathbf{1} = \frac{4}{3} \cdot \mathbf{1}$$

Casimir operator of SU(N_C), here: SU(N_C=3).

$$\sum_{a,b} f_{abc} f_{abd} = \delta_{cd} C_A \qquad C_A = N_C$$

Casimir operator of adjoint representation of gluons.

$$\sum_{\alpha,\beta} T^a_{\alpha\beta} T^b_{\beta\alpha} = \delta_{ab} T_F \qquad T_F = \frac{1}{2}$$

$$\frac{T_F}{C_F} = \frac{N_C}{N_A} = \frac{\#\,colors}{\#\,gluons}$$

C_F, C_A describe the effective color charge of quark/gluon.

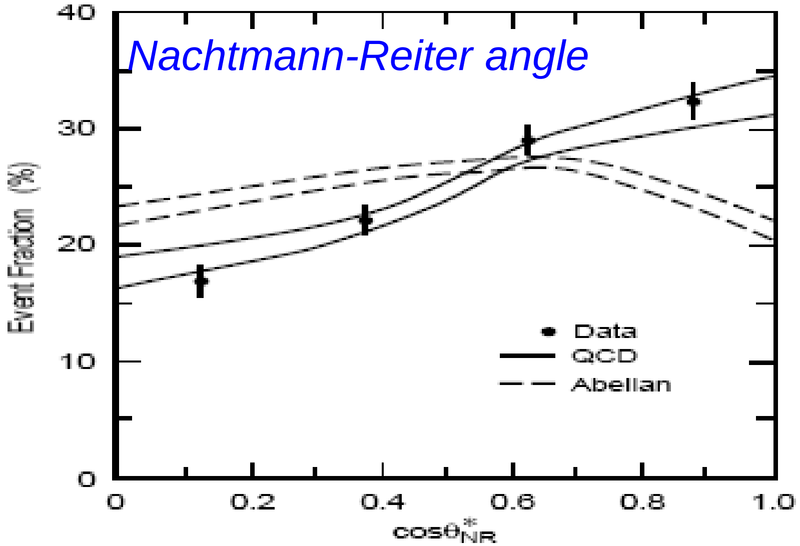
Angular correlation of jets in 4-jet events

4-jet cross section:

$$\frac{1}{\sigma_0} d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[F_A + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_B + \frac{N_C}{C_F} F_C\right]$$

$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[\frac{T_F}{C_F} N_f F_D + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions



Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle

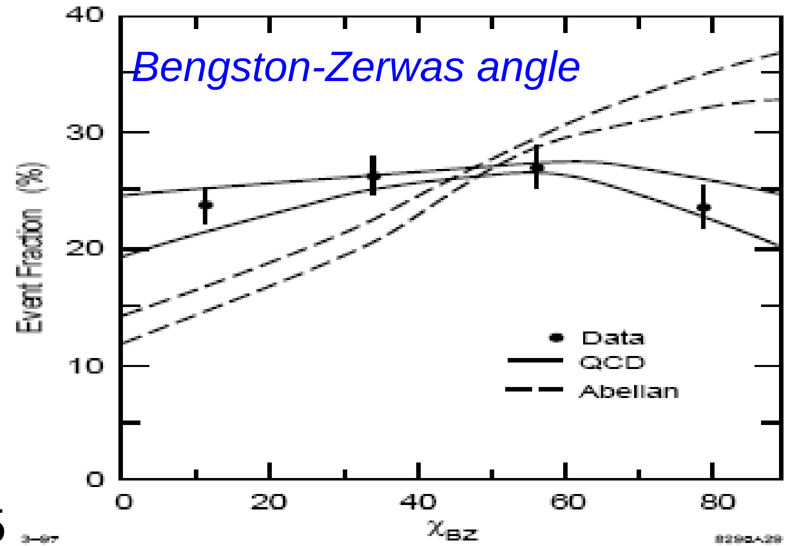
$$\cos\chi_{BZ} \propto (\vec{p}_1 \times \vec{p}_2)\cdot(\vec{p}_3 \times \vec{p}_4)$$

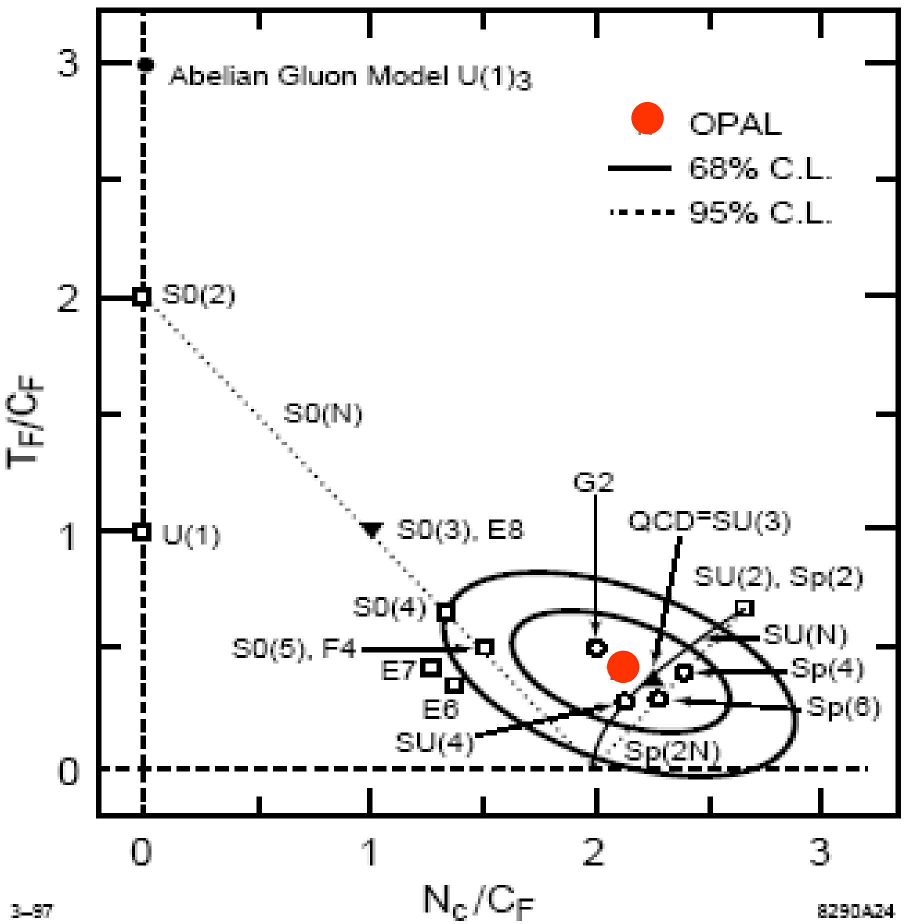
- Nachtmann-Reiter angle

$$\cos\theta_{NR} \propto (\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)$$

Allows to measure the ratios T_F/C_F and N_C/C_F

SU(3) predicts: $T_F/C_F = 0.375$ and $N_C/C_F = 2.25$



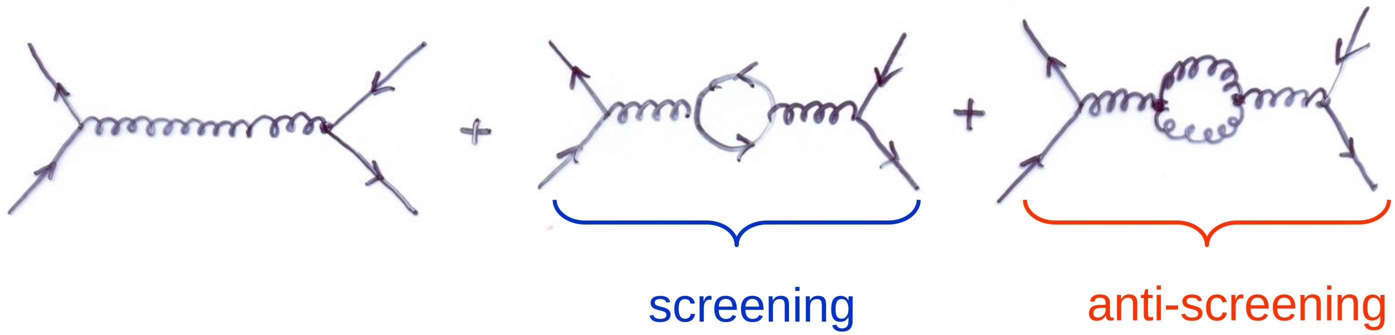


Confirms QCD prediction (SU(3)) and gluon self-coupling:

T_F/C_F = 0.375 and N_C/C_F =2.25

2. "Running" of the strong coupling α_s

Propagator corrections:



Strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{1}{12\pi}(33 - 2n_f)\log\frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{1}{12\pi}(33 - 2n_f)$$

n_f = active quark flavors
μ^2 = renormalization scale
conventionally $\mu^2 = M_Z^2$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$

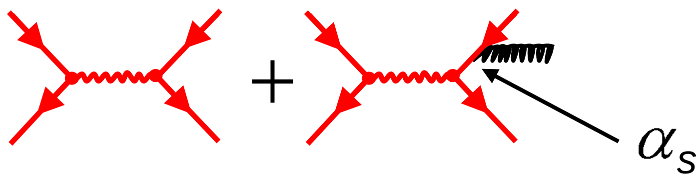
with $\Lambda_{QCD} \approx 200\text{MeV}$
scale at which perturbation theory diverges

Measurement of Q^2 dependence of α_s

α_s measurements are done at given scale Q^2: $\alpha_s(Q^2)$

a) α_s from total hadronic cross section

$$e^+e^- \to q\bar{q}(g)$$



Final state gluon radiation.

$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = 3\sum Q_q^2 \left\{1 + \frac{\alpha_s}{\pi} + 1.411\frac{\alpha_s^2}{\pi^2} + \ldots\right\}$$

$\alpha_s(s)$

b) α_s from hadronic event shape variables

3-jet rate: $R_3 \equiv \frac{\sigma_{3-jet}}{\sigma_{had}}$ depends on α_s

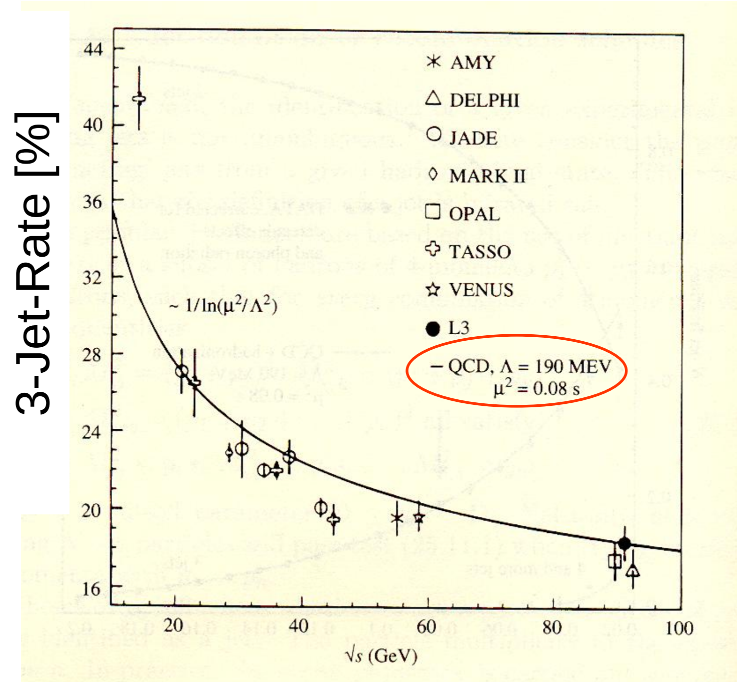
3-jet rate is measured as function of a jet resolution parameter y_{cut}

QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s , y_{cut})$

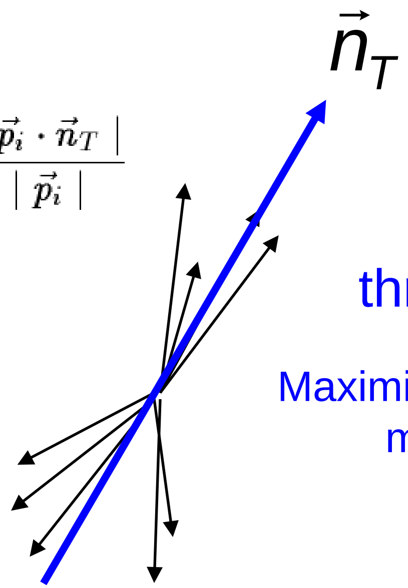
$\rightarrow$ fit $R_3^{theo}(\alpha_s , y_{cut})$ to the data to determine α_s

Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for α_s

$\Rightarrow \alpha_s(s)$

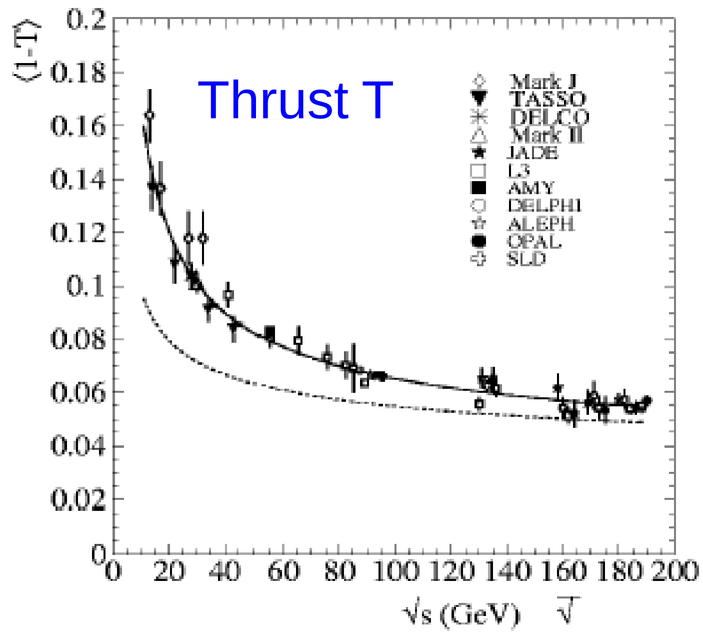


$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



thrust axis

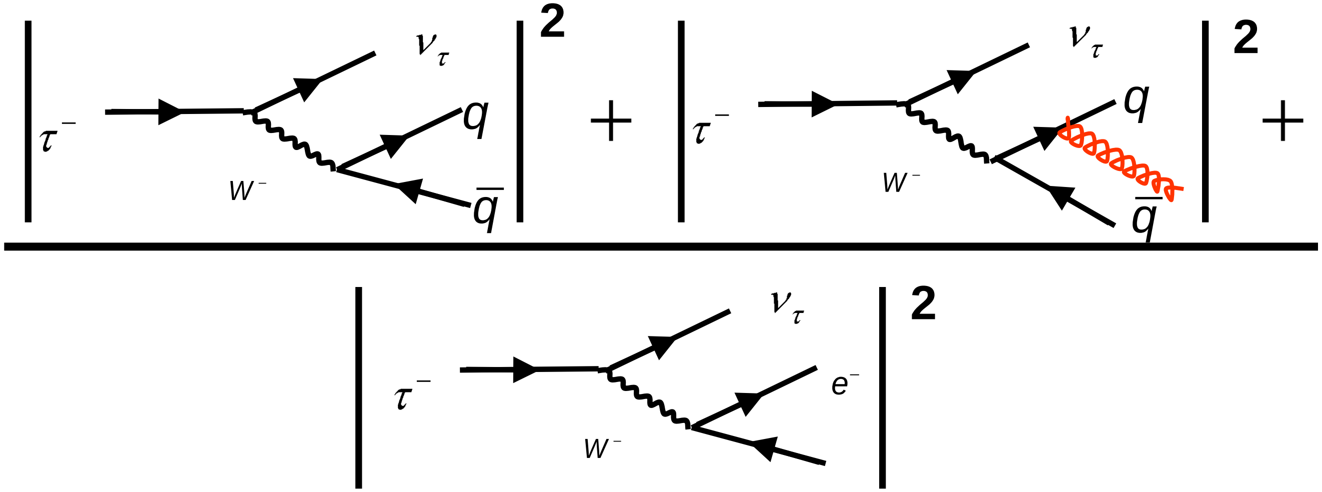
Maximizes longitudinal momentum



c) α_s from hadronic τ decays

$$R^{\tau}_{had} = \frac{\Gamma(\tau \rightarrow \nu_{\tau} + Hadrons)}{\Gamma(\tau \rightarrow \nu_{\tau} + e\bar{\upsilon}_e)} \sim f(\alpha_s)$$

$R^{\tau}_{had} =$



$$R^{\tau}_{had} = R^{\tau,0}_{had}\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots\right)$$

d) α_s from DIS (deep inelastic scattering)

Running of α_s and asymptotic freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$