Quark mixing in the Standard Model:

- Quark masses and CKM matrix
- Mixing of neutral mesons
- CP violation in meson decays

Literature:
G.Hiller & U.Uwer, Quark Flavor Physics,
in “Physics at the Terascale”, ed. I.Brock & T.Schoerner-Sadenius,
Wiley (2011)

1. Quark masses and CKM matrix

Quark mass terms in Lagrangian (after spontaneous symmetry breaking:

Yukawa coupling

$$\mathcal{L}_Y^{quarks} = -\frac{\upsilon}{\sqrt{2}}\left\{\bar{d}_L Y_d d_R + \bar{u}_L Y_u u_R + h.c.\right\}$$

Short-hand notation for

Mass matrix: $\tilde{M}^{U,D} = Y^{U,D}\frac{\upsilon}{\sqrt{2}}$

$$\mathcal{L}_Y^{quarks} = -\frac{\upsilon}{\sqrt{2}}\sum_{j,k}\left\{\underbrace{\bar{d}_L^j Y_d^{jk} d_R^k + \bar{u}_L^j Y_u^{jk} u_R^k}_{\bar{d}_L^j \tilde{M}^D_{jk} d_R^k + \bar{u}_L^j \tilde{M}^U_{jk} u_R^k} + h.c.\right\}$$

Yukawa matrices and thus the mass matrices are in general not diagonal in “generation space”! In fact for the Standard Model they are not!

Diagonalization

Diagonalization using unitary transformations to obtain mass eigenstates $\tilde{q}_A$

$$\tilde{q}_A = V_{A,q} q_A \quad \text{with} \quad \left. \begin{array}{l} q = u,d \quad A = R,L \\ V_{A,q} V_{A,q}^{+} = 1 \end{array} \right\} \quad \text{and}$$

Set of 4 matrices!

Matrices $V_{A,q}$ are determined by:

$$M_u = \begin{pmatrix} m_u & 0 & 0 \\ 0 & m_c & 0 \\ 0 & 0 & m_t \end{pmatrix} = \mathrm{diag}(m_u, m_c, m_t) = \frac{v}{\sqrt{2}} V_{L,u} Y_u V_{R,u}^{\dagger} ,$$

$$M_d = \begin{pmatrix} m_d & 0 & 0 \\ 0 & m_s & 0 \\ 0 & 0 & m_b \end{pmatrix} = \mathrm{diag}(m_d, m_s, m_b) = \frac{v}{\sqrt{2}} V_{L,d} Y_d V_{R,d}^{\dagger} .$$

With usual Dirac masses m_q:

$$\mathcal{L}_Y^{\mathrm{quarks}} = -\bar{\tilde{d}}_L M_d \tilde{d}_R - \bar{\tilde{u}}_L M_u \tilde{u}_R + \mathrm{h.c.}$$

CKM Matrix

If up-type and down-type Yukawa matrices cannot be diagonalised simultaneously, there is an net effect of the basis change on the charged current interaction (which connects u/d-type) :

The charged-current interaction gets a flavor structure which is encoded in the Cabibbo-Kobayashi-Maskawa matrix (CKM):

$$V_{CKM} = V_{L,u} V^{+}_{L,d}$$

Why don’t we see no quark mixing for NC?

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g_2}{\sqrt{2}} \left(\bar{\tilde{u}}_L \gamma^\mu W^+_\mu V_{\mathrm{CKM}} \tilde{d}_L + \bar{\tilde{d}}_L \gamma^\mu W^-_\mu V^\dagger_{\mathrm{CKM}} \tilde{u}_L \right)$$

The element $(V_{CKM})_{ij}$ connects the LH u-type quark of the ith generation with the LH d-type quark of the jth generation. We label the matrix element according to quark flavor instead to the generation index.

$$V_{\mathrm{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

CP violation

$$\mathcal{L}_{\mathrm{CC}} = -\frac{g_2}{\sqrt{2}} \left(\bar{\tilde{u}}_L \gamma^\mu W^+_\mu V_{\mathrm{CKM}} \tilde{d}_L + \bar{\tilde{d}}_L \gamma^\mu W^-_\mu V^\dagger_{\mathrm{CKM}} \tilde{u}_L \right)$$

⬇ CP

$$\mathcal{L}^{CP}_{\mathrm{CC}} = -\frac{g_2}{\sqrt{2}} \left(\bar{\tilde{d}}_L \gamma^\mu W^-_\mu V^T_{\mathrm{CKM}} \tilde{u}_L + \bar{\tilde{u}}_L \gamma^\mu W^+_\mu V^*_{\mathrm{CKM}} \tilde{d}_L \right)$$

CP – is only conserved if $V_{\mathrm{CKM}} = (V_{\mathrm{CKM}})^*$

Parameters of CKM matrix

$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Independent parameters:

18 parameter (9 complex elements)
-5 relative quark phases
(unobservable, see next slide)
-9 Unitarity conditions
=4 independent parameters: 3 rotation angles + phase

Unobservable Quark Phases

Phases of left-handed quark fields are unobservable: possible redefinition

$$u_L \to e^{i\phi(u)}u_L \qquad c_L \to e^{i\phi(c)}c_L \qquad t_L \to e^{i\phi(t)}t_L$$

$$d_L \to e^{i\phi(d)}d_L \qquad s_L \to e^{i\underset{\uparrow}{\phi}(s)}s_L \qquad b_L \to e^{i\phi(b)}b_L$$

Real numbers

RH quark fields are rotated simultaneously to keep mass terms real.

Under phase transformation:

$$V \to \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

$$\boxed{V\alpha j \to \exp[i(\phi(j) - \phi(\alpha))]V\alpha j}$$

$L^{phys}(f,G)$ invariant

$L(f,H)$ affected, rephasing q_R

Parametrization

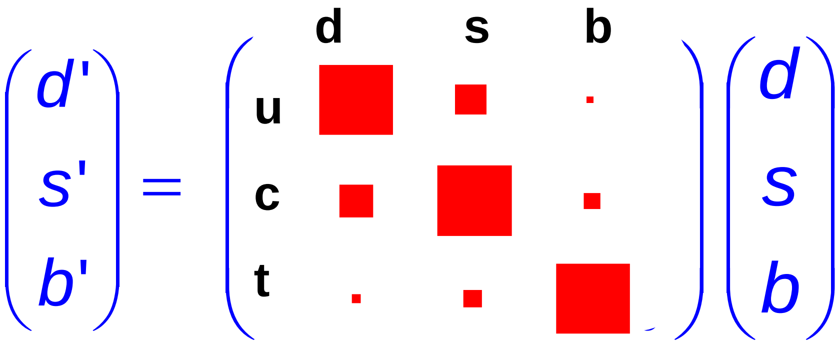
PDG parametrization: 3 Euler angles $\theta_{23}, \theta_{13}, \theta_{12}$ and 1 Phase δ

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}, \quad s_{ij} = \sin\theta_{ij}$

Wolfenstein Parametrization

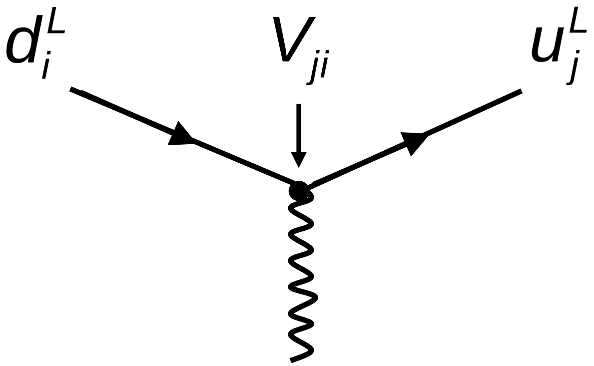


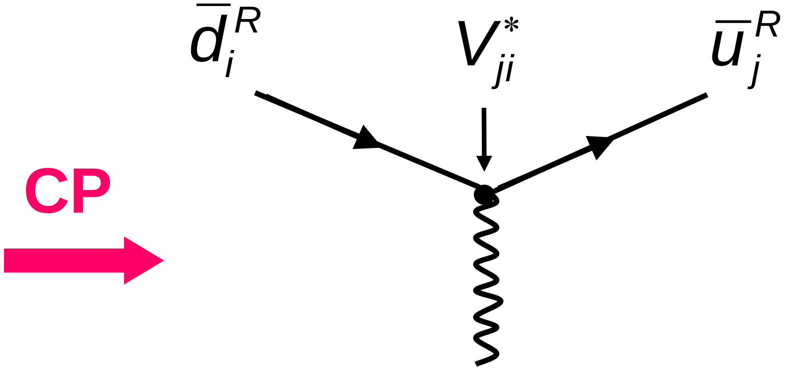
λ, A, ρ, η with $\lambda = 0.22$

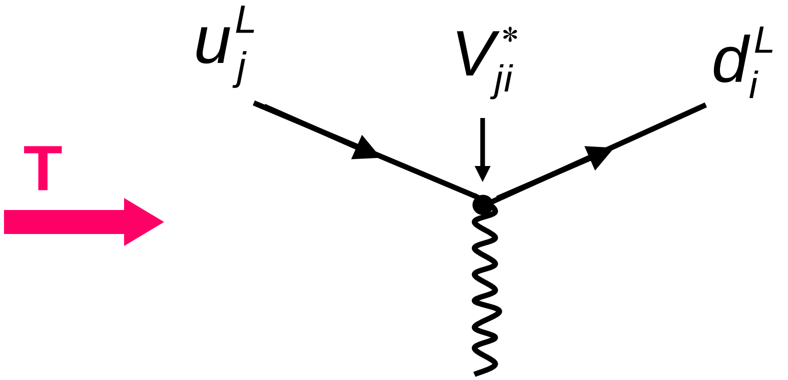
$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & \overset{|V_{ub}|\times e^{-i\gamma}}{A\lambda^3(\rho-i\eta)} \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ \underset{|V_{td}|\times e^{-i\beta}}{A\lambda^3(1-\rho-i\eta)} & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Reflects the “hierarchical structure” of the CKM matrix.

Complex CKM elements and CP violation







CP (T) violation ⇔ $V_{ji} \neq V_{ji}^*$

i.e. Complex elements

Remark: For 2 quark generations the mixing is described by the **real 2x2** Cabbibo matrix → **no CP violation**! To explain **CPV** in the SM Kobayashi and Maskawa have predicted a **third quark generation**.

CP Violation in the Standard Model

Requirements for CP violation

$$\boxed{\begin{aligned}&\left(m_t^2-m_c^2\right)\left(m_t^2-m_u^2\right)\left(m_c^2-m_u^2\right)\\ \times&\left(m_b^2-m_s^2\right)\left(m_b^2-m_d^2\right)\left(m_s^2-m_d^2\right)\times J_{CP}\neq 0\end{aligned}}$$

where

$$\boxed{J_{CP}=\left|\mathrm{Im}\left\{V_{i\alpha}V_{j\beta}V_{i\beta}^{*}V_{j\alpha}^{*}\right\}\right|\left(i\neq j,\alpha\neq\beta\right)}$$

Jarlskog determinant

Using above parameterizations

$$\boxed{J_{CP}=s_{12}s_{13}s_{23}c_{12}c_{23}c_{13}\sin\delta=\lambda^6A^2\eta=O\left(10^{-5}\right)}$$

➡ CP violation is small in the Standard Model

Unitarity Triangles

Unitarity condition of the CKM matrix can be described by an triangle relations in the complex plane:

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0 \quad \text{(db)}$$

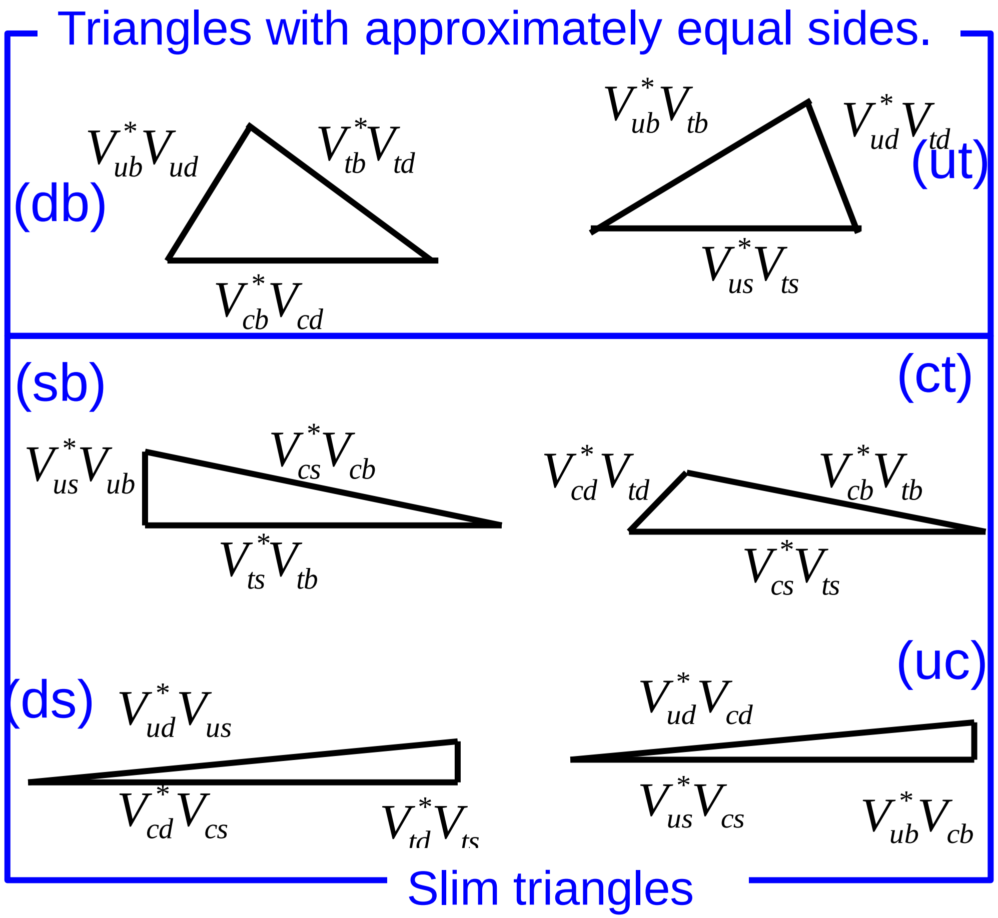
$$V_{us}V_{ub}^* + V_{cs}V_{cb}^* + V_{ts}V_{tb}^* = 0 \quad \text{(sb)}$$

$$V_{ud}V_{us}^* + V_{cd}V_{cs}^* + V_{td}V_{ts}^* = 0 \quad \text{(ds)}$$

$$V_{ud}V_{td}^* + V_{us}V_{ts}^* + V_{ub}V_{tb}^* = 0 \quad \text{(ut)}$$

$$V_{cd}V_{td}^* + V_{cs}V_{ts}^* + V_{cb}V_{tb}^* = 0 \quad \text{(ct)}$$

$$V_{ud}V_{cd}^* + V_{us}V_{cs}^* + V_{ub}V_{cb}^* = 0 \quad \text{(uc)}$$


Triangles with approximately equal sides.

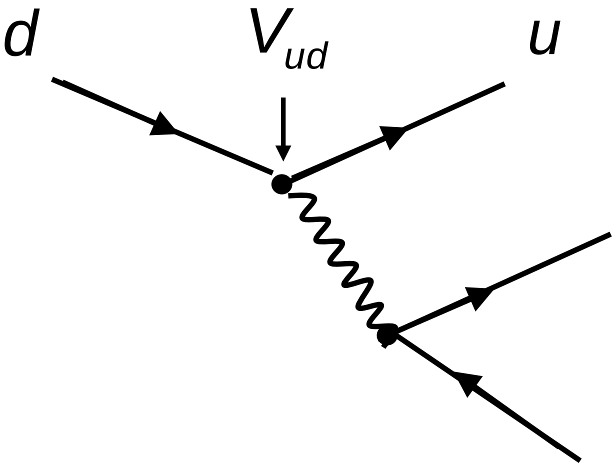
Slim triangles

All 6 triangles have the same area (= $J_{CP}/2$): A measure of CP violation.

Determination of CKM matrix elements

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$|V_{ud}|$ Nuclear beta-decays ($0^+ \rightarrow 0^+$ beta decays, neutron decay)

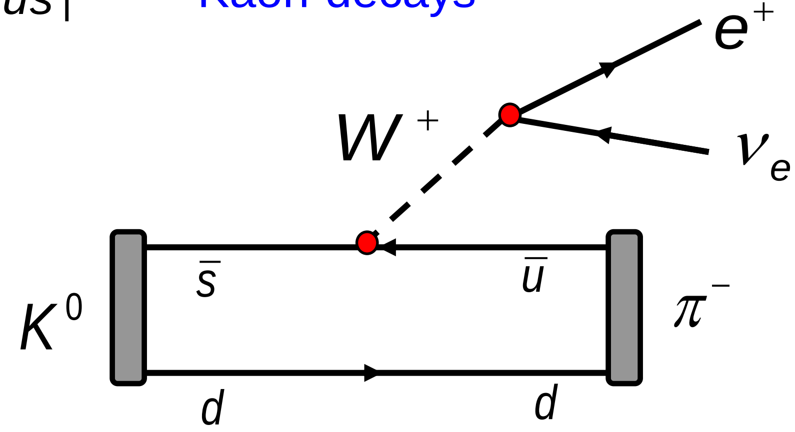


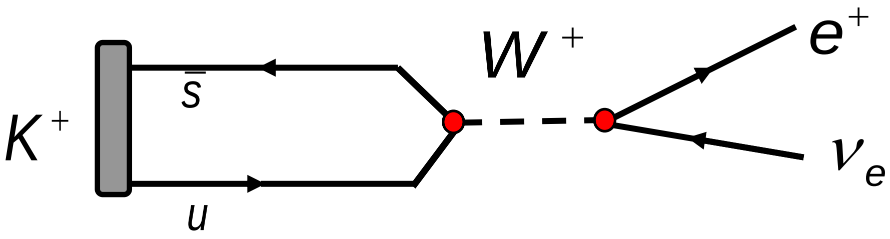
$$|V_{ud}| = 0.97425 \pm 0.00022.$$

Determination of CKM matrix elements

$\left|V_{us}\right|$ Kaon-decays

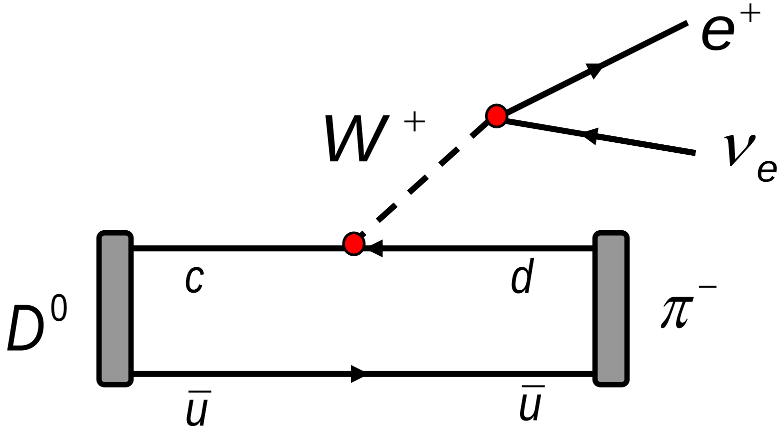
$$|V_{us}| = 0.2252 \pm 0.0009$$





Problem: Kaon and pion form factors (see also the section on “pion decay”)

$\left|V_{cd}\right|$ D-meson decays



Form faktor!

Determination of CKM matrix elements

$|V_{cd}|$

More precise: Double muon production in neutrino scattering

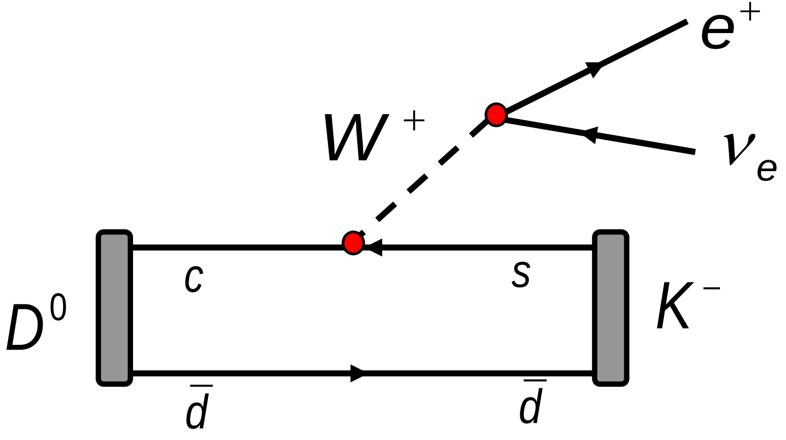
$$\sigma(\nu_\mu + d \to \mu + D + X \to \mu + \mu + Y) \sim |V_{cd}|^2$$

$$|V_{cd}| = 0.230 \pm 0.011$$

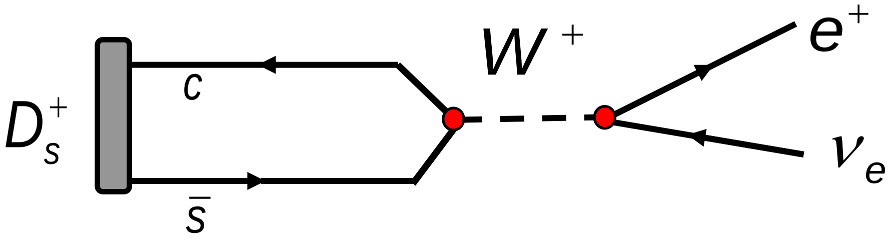
$|V_{cs}|$

Tagged on-shell W→cs decays at LEP II: $|V_{cs}| = 0.94^{+0.32}_{-0.26} \pm 0.13$

D-decays



Form factors!



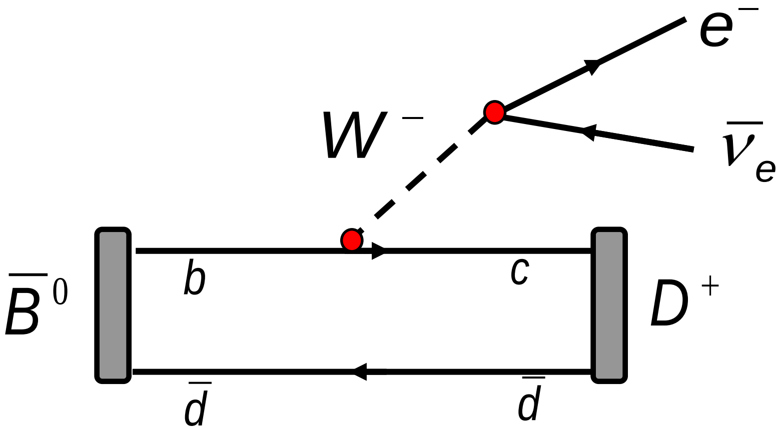
$$|V_{cs}| = 1.023 \pm 0.036$$

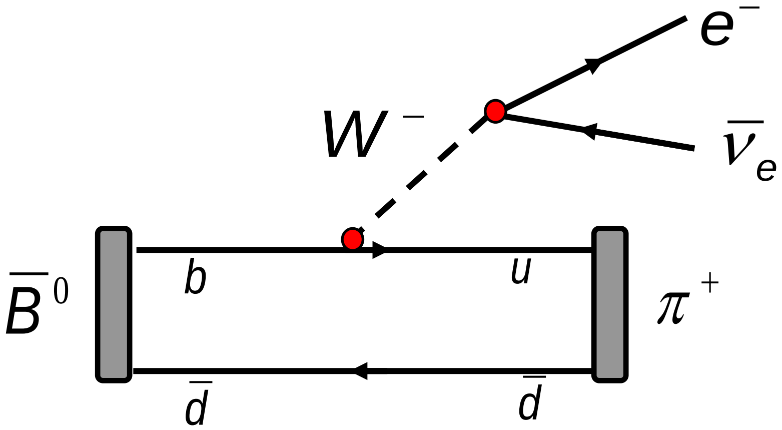
Determination of CKM matrix elements

$|V_{cb}|$

$|V_{ub}|$

Semi-leptonic B decays





$$|V_{cb}| = (40.6 \pm 1.3) \times 10^{-3}$$

$$|V_{ub}| = (3.89 \pm 0.44) \times 10^{-3}$$

$|V_{tb}|$

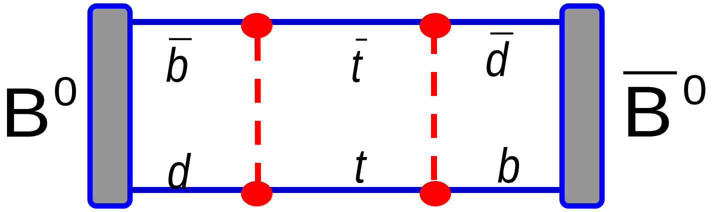
Single top-quark production at hadron colliders: W→ tb → Wb + b

Determination of CKM matrix elements

$|V_{td}|$

$|V_{ts}|$

Can be measured only via virtual effects:
top quark decays nearly entirely to b-quarks.



B_d and B_s oscillation:
Next section.

Determination of CKM Phases:

In the Wolfenstein parametrization at order $O(\lambda^4)$ (λ^6) only 2 (3) of the CKM matrix elements have non-trivial phases: V_{td}, V_{ub} (V_{ts}).

The CKM phases are measured via CP violation in B decays.

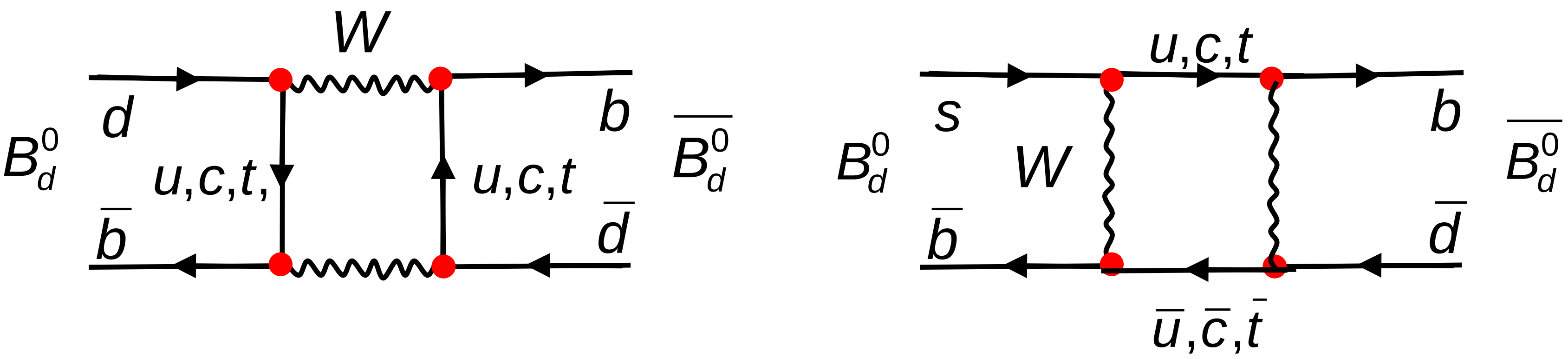
General remark:
B decays provide access to the modulus of 4 CKM elements and of two CKM phases. That’s the reason B decays are studied very intensively.

2. Mixing of neutral mesons

The quark mixing results into several interesting “loop” effects:
Standard Model predicts at loop-level: Flavor Changing Neutral Currents

(forbidden at tree-level)

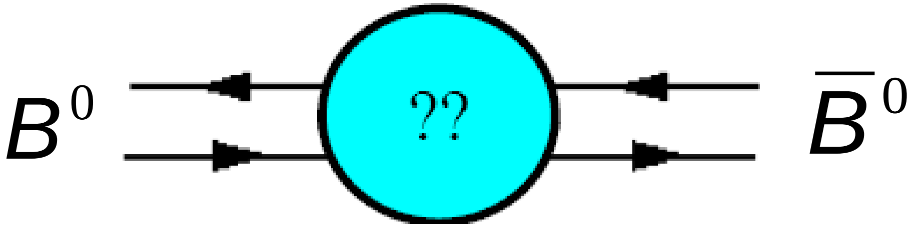
Mixing of neutral mesons, e.g.: $B_d^0 \Leftrightarrow \overline{B_d^0}$



Neutral mesons:	$\left\|P^0\right\rangle$:	$K^0=\left\|d\bar{s}\right\rangle$	$D^0=\left\|\bar{u}c\right\rangle$	$B_d^0=\left\|d\bar{b}\right\rangle$	$B_s^0=\left\|s\bar{b}\right\rangle$
	$\left\|\overline{P^0}\right\rangle$:	$\overline{K^0}=\left\|\bar{d}s\right\rangle$	$\overline{D^0}=\left\|\bar{u}c\right\rangle$	$\overline{B_d^0}=\left\|d\bar{b}\right\rangle$	$\overline{B_s^0}=\left\|s\bar{b}\right\rangle$
discovery of mixing		1960	2007	1987	2006

Mixing Phenomenology

Applies to all neutral mesons!



$$i\frac{d}{dt}\begin{pmatrix} B^0(t) \\ \overline{B}^0(t) \end{pmatrix} = \underbrace{\left(\mathbf{M} - \frac{i}{2}\mathbf{\Gamma}\right)}_{\textbf{H}}\begin{pmatrix} B^0(t) \\ \overline{B}^0(t) \end{pmatrix}$$

Flavor states
= No mass
eigenstates

Diagonalizing H:

Mass eigenstates: $|B_L\rangle = p|B^0\rangle + q|\overline{B^0}\rangle$ with $m_{L,}\Gamma_L$ light

$|B_H\rangle = p|B^0\rangle - q|\overline{B^0}\rangle$ with $m_{H,}\Gamma_H$ heavy

complex coefficients
$|p|^2 + |q|^2 = 1$

$$|B_{H,L}(t)\rangle = |B_{H,L}(0)\rangle \cdot e^{-im_{H,L}t} \cdot e^{-\frac{1}{2}\Gamma_{H,L}t}$$

Flavor eigenstates: $|B^0\rangle = \frac{1}{2p}(|B_L\rangle + |B_H\rangle)$ $|\overline{B}^0\rangle = \frac{1}{2q}(|B_L\rangle - |B_H\rangle)$

Mixing of neutral mesons

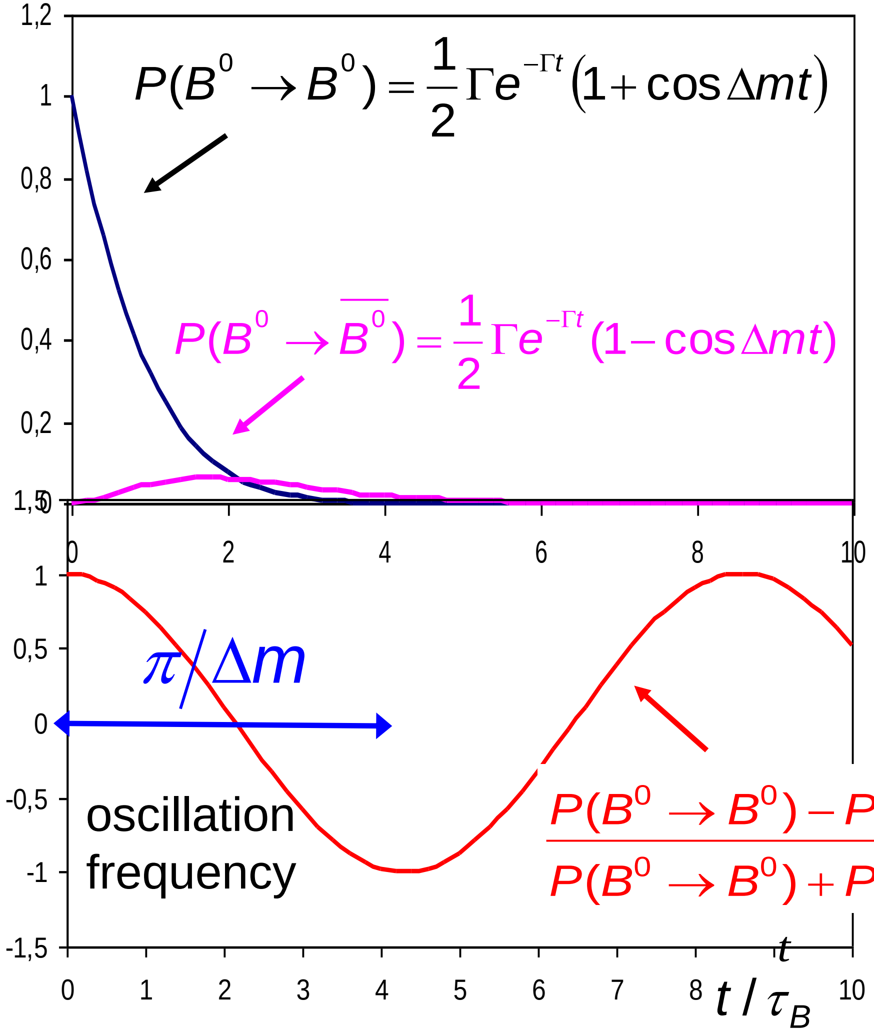
$$\underbrace{P(B^0 \to B^0) = P(\overline{B^0} \to \overline{B^0})}_{\text{CPT}} = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{4}\left|\frac{q}{p}\right|^2\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right] \qquad \Delta m = m_H - m_L$$

$$P(\overline{B^0} \to B^0) = \frac{1}{4}\left|\frac{p}{q}\right|^2\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos\Delta m t\right]$$

CP - violation in mixing: $P(B^0 \to \overline{B^0}) \neq P(\overline{B^0} \to B^0) \Rightarrow \left|\frac{q}{p}\right| \neq 1$

B^0-B^0 Mixing



$$P(B^0 \to B^0) = \frac{1}{2}\Gamma e^{-\Gamma t}(1+\cos\Delta m t)$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{2}\Gamma e^{-\Gamma t}(1-\cos\Delta m t)$$

$$\Delta m = m_H - m_L$$

Simplification for

$$\Gamma_H \approx \Gamma_L \approx \Gamma$$

$\pi / \Delta m$

Mixing asymmetry

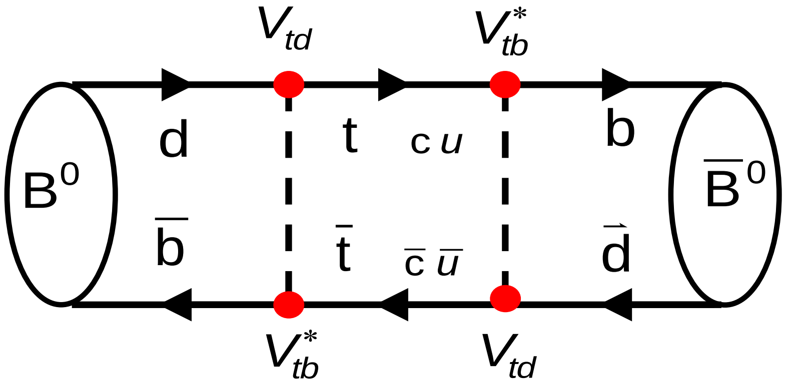
oscillation frequency

$$\frac{P(B^0 \to B^0) - P(B^0 \to \overline{B^0})}{P(B^0 \to B^0) + P(B^0 \to \overline{B^0})}$$

t / τ_B

Standard Model Prediction

$B_d^0 - \overline{B}_d^0$



$\Delta m_d \sim m_t^2 \cdot O(\lambda^6)$

Dominant contribution from top-loop:

$\eta_B = 0.55 \pm 0.01$

NLO QCD

$$\Delta m_d = \frac{G_F^2}{6\pi^2} m_B f_B^2 B_B (V_{td}^* V_{tb})^2 m_W^2 \eta_B F\left(\frac{m_t^2}{m_W^2}\right)$$

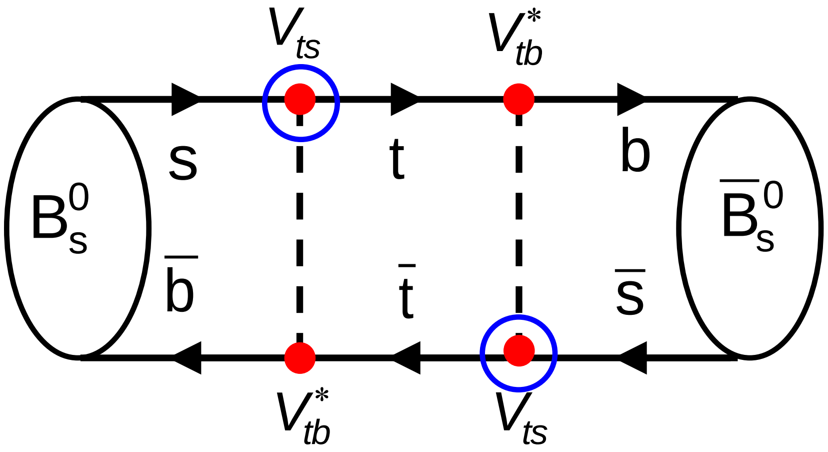
e.w. correction

$f_B^2 B_B = (235 \pm 33 \pm 12)^2 MeV^2$ from lattice QCD

Describes the binding of the quarks to a meson

Prediction for B_s mixing

$B_s^0 - \overline{B}_s^0$



$$\Delta m_s \sim m_t^2 \cdot O(\lambda^4)$$

$$\boxed{\Delta m_s \sim (V_{ts}^* V_{tb})^2}$$

Oscillation is about 35 times stronger than in the case of B_d
(V_{ts} much larger than V_{td})

B oscillation:

Deactivation of GIM(*) suppression because of large top mass:

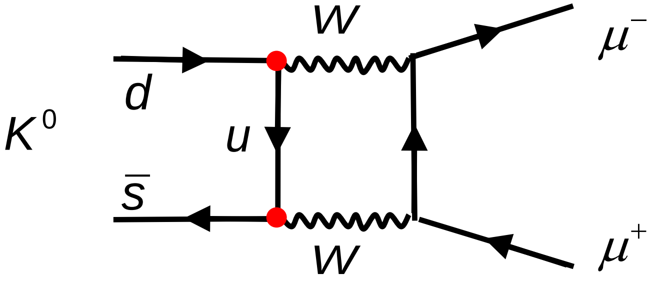
What would be the mixing if all quarks had the same masses?

(*) Glashow, Iliopoulos, Maiani, 1970, see next page.

Missing FCNC and GIM mechanism

Historical retrospect

FCNC in the 3 quark model: $K^0 \rightarrow \mu^+\mu^-$



$$M \sim \sin\theta_c \cos\theta_c$$

Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

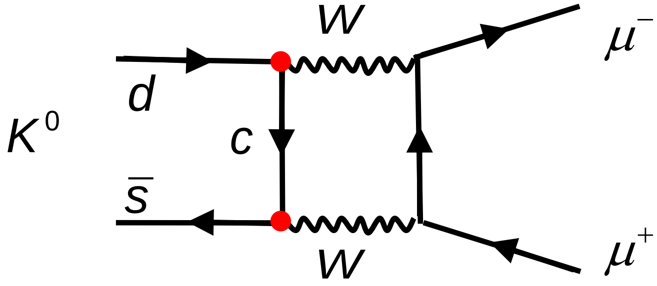
$$\frac{BR(K_L \rightarrow \mu^+\mu^-)}{BR(K_L \rightarrow all)} = (7.2 \pm 0.5) \cdot 10^{-9}$$

Proposal by Glashow, Iliopoulos, Maiani, 1970:

GIM

There exists a fourth quark which builds together with the s quark a second doublet:

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\theta_c \cdot d + \cos\theta_c \cdot s \end{pmatrix}$$



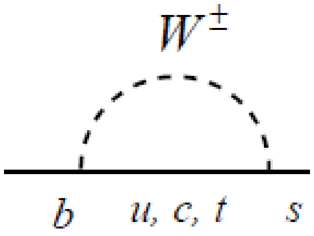
$$M \sim -\sin\theta_c \cos\theta_c$$

Additional Feynman-Graph for $K^0 \rightarrow \mu\mu$ which compensates the first one:

Prediction of a fourth quark:
Mass prediction BR=f(m_c,…)

GIM Suppression

Example: FCNC process b→s ("penguin process" as in B→K*γ)



$$\mathcal{A}(b \to s)_{\rm SM} = V_{ub}V_{us}^* A_u + V_{cb}V_{cs}^* A_c + V_{tb}V_{ts}^* A_t$$

where A_q denote the sub-amplitudes for the 3 possible internal quark. A_q depend on the quark masses only:

$$A_q \; = \; A(m_q^2/M_W^2)$$

Using the unitarity of the CKM matrix, especially: $\sum_i V_{ib}V_{is}^* = 0$

the total amplitude can be rewritten:

$$\mathcal{A}(b \to s)_{\rm SM} = V_{tb}V_{ts}^*(A_t - A_c) + V_{ub}V_{us}^*(A_u - A_c)$$

In case of approx. equal quark masses, total amplitude vanishes: GIM suppression.

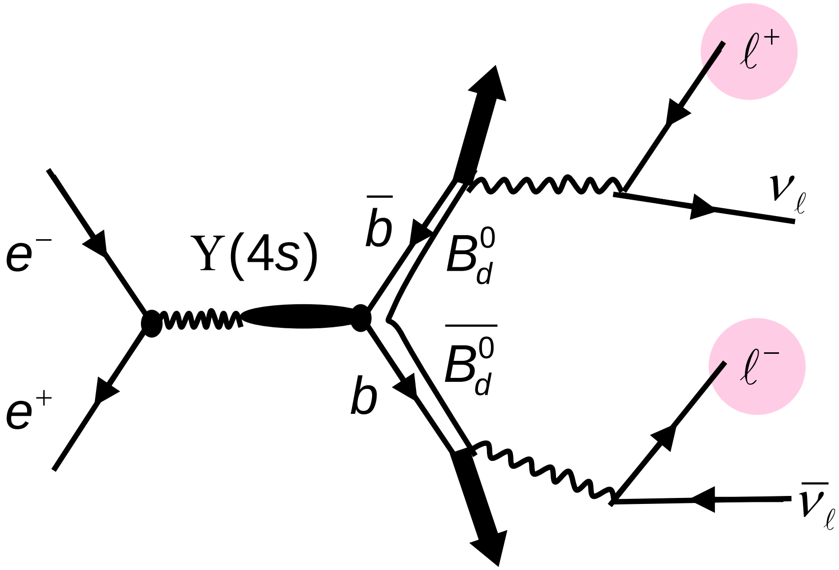
For large top quark mass: $A(b \to s)_{SM} = V_{tb}V_{ts}^* \cdot \frac{m_t^2}{m_W^2}$ GIM suppression inactive

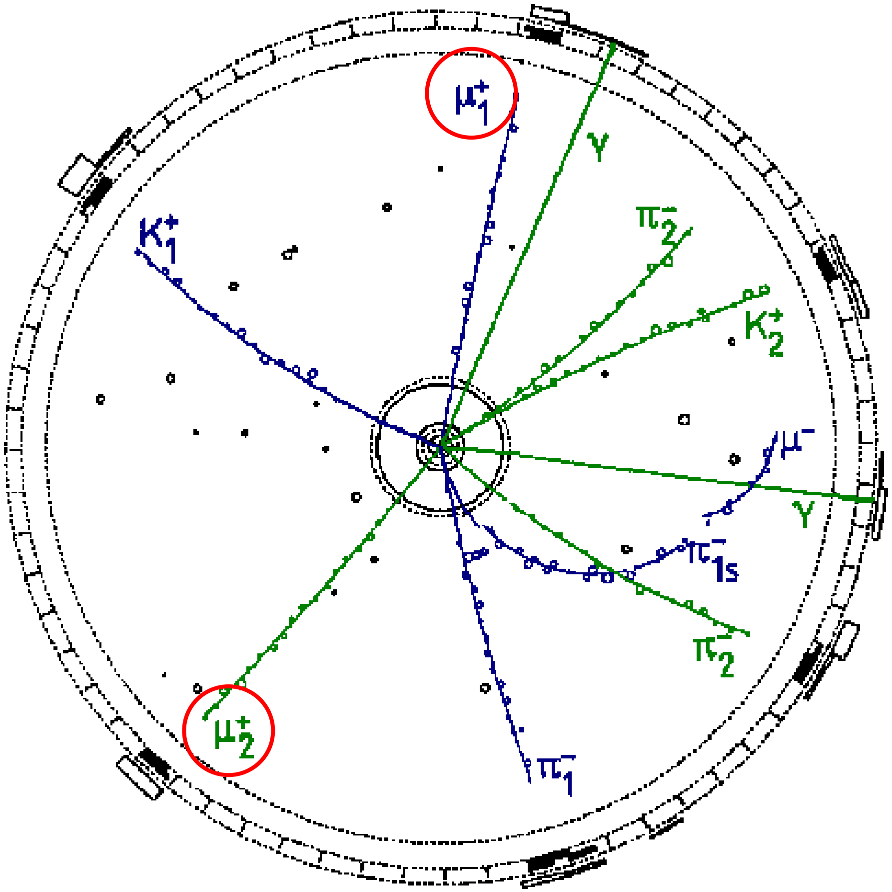
Discovery of B^0 mixing

ARGUS 1987

First e^+e^- B factory at DESY:

$$\left.\begin{array}{l}\text{at } \sqrt{s} = 10.58 \text{ GeV}: \\ e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0\end{array}\right\} \sigma(B\overline{B}) \approx 1\text{nb}$$





Unmixed: $B^0\overline{B}^0 \to \ell^+\ell^-$

Mixed:

$$\left.\begin{array}{l}B^0B^0 \to \ell^+\ell^+ \\ \overline{B}^0\overline{B}^0 \to \ell^-\ell^-\end{array}\right\} \text{Same charge}$$

$B^0 \to D^{*-}\mu^+\nu_\mu$

$\hookrightarrow \overline{D}^0\pi_S^-$

$\hookrightarrow K^+\pi^-$

$B^0 \to D^{*-}\mu^+\nu_\mu$

$\hookrightarrow D^-\pi^0$

$\hookrightarrow \gamma\gamma$

$\hookrightarrow K^+\pi^-\pi^-$

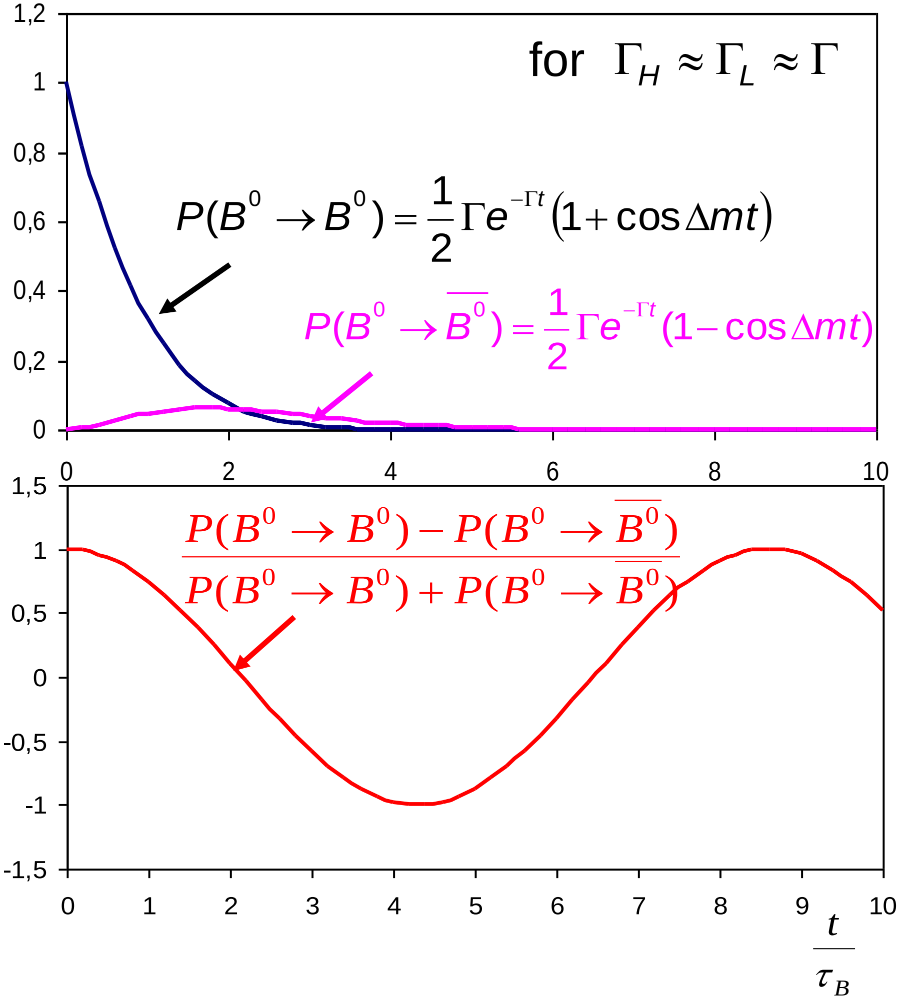
Historical remark:

The observation of the B_d meson mixing put the first lower limit on the top mass: $m_{top} > 50$ GeV. (GIM suppresison is inactive)

If the top mass was lower the GIM mechanism would lead to a small Δm, i.e. the B would oscillate very slowly and would decay before mixing.

The GIM mechanism is a result of the unitarity of the CKM matrix. Only different quark masses lead to a non-perfect cancellation and are the soruces of observable FCNCs at loop level.

Experimental Status of B_d meson mixing



for $\Gamma_H \approx \Gamma_L \approx \Gamma$

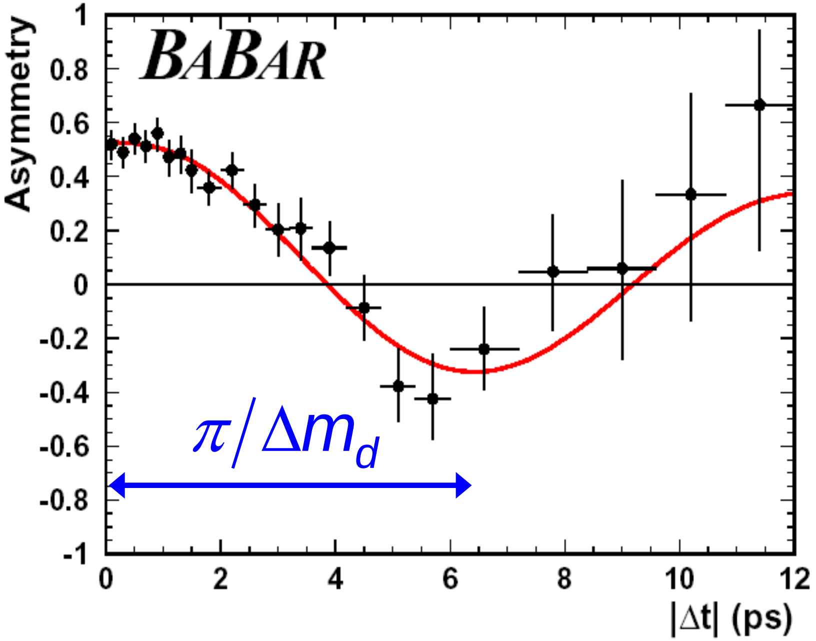
$$P(B^0 \to B^0) = \frac{1}{2}\Gamma e^{-\Gamma t}\left(1 + \cos\Delta m t\right)$$

$$P(B^0 \to \overline{B^0}) = \frac{1}{2}\Gamma e^{-\Gamma t}(1 - \cos\Delta m t)$$

$$\frac{P(B^0 \to B^0) - P(B^0 \to \overline{B^0})}{P(B^0 \to B^0) + P(B^0 \to \overline{B^0})}$$

$$\frac{t}{\tau_B}$$

$$A = \frac{unmixed - mixed}{unmixed + mixed}$$



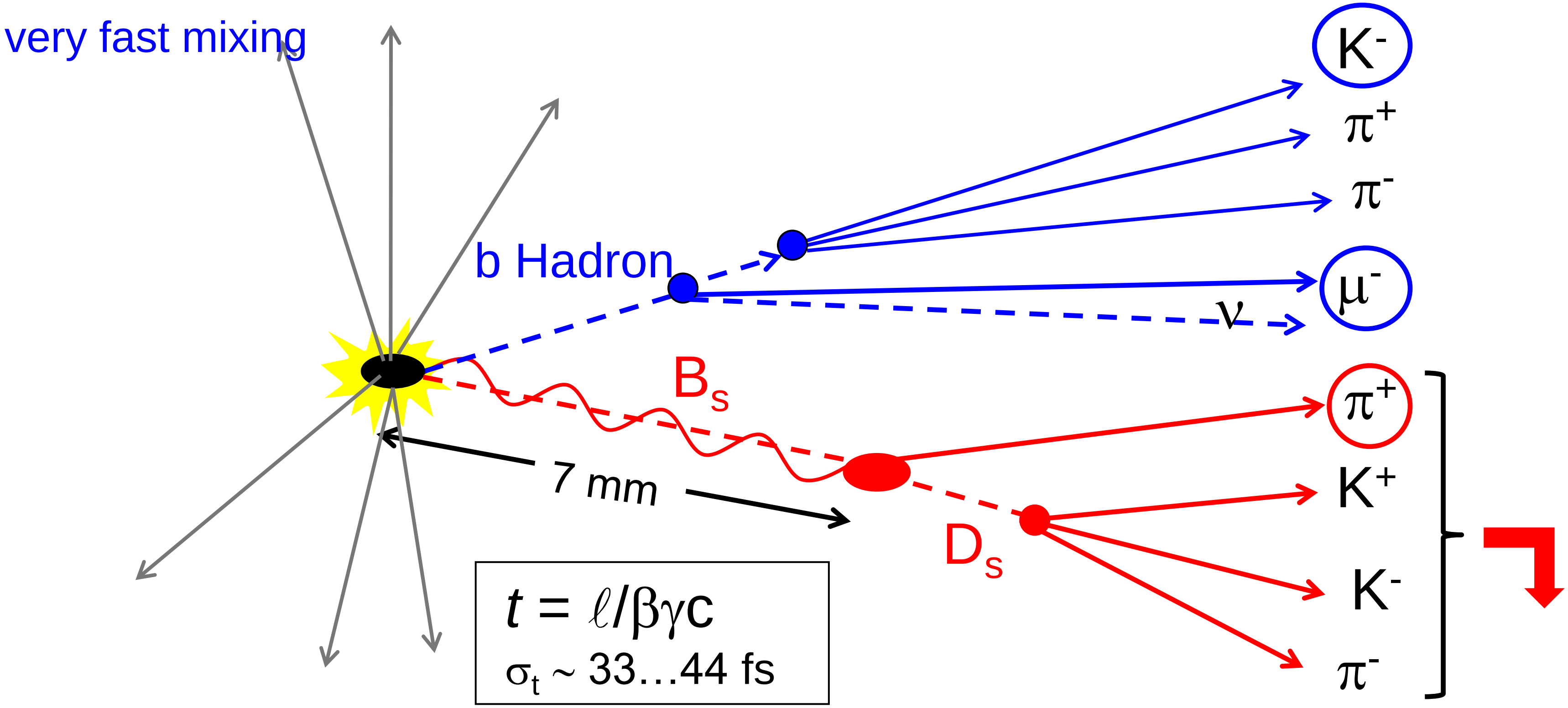
$\pi/\Delta m_d$

$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004\,\mathrm{ps}^{-1}$$

$$\approx \frac{0.774}{\tau_B}$$

B_s – Mixing measurement at LHC

very fast mixing

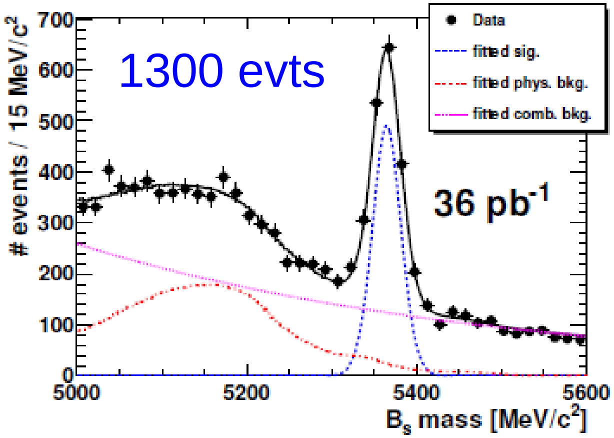


$t = \ell/\beta\gamma c$

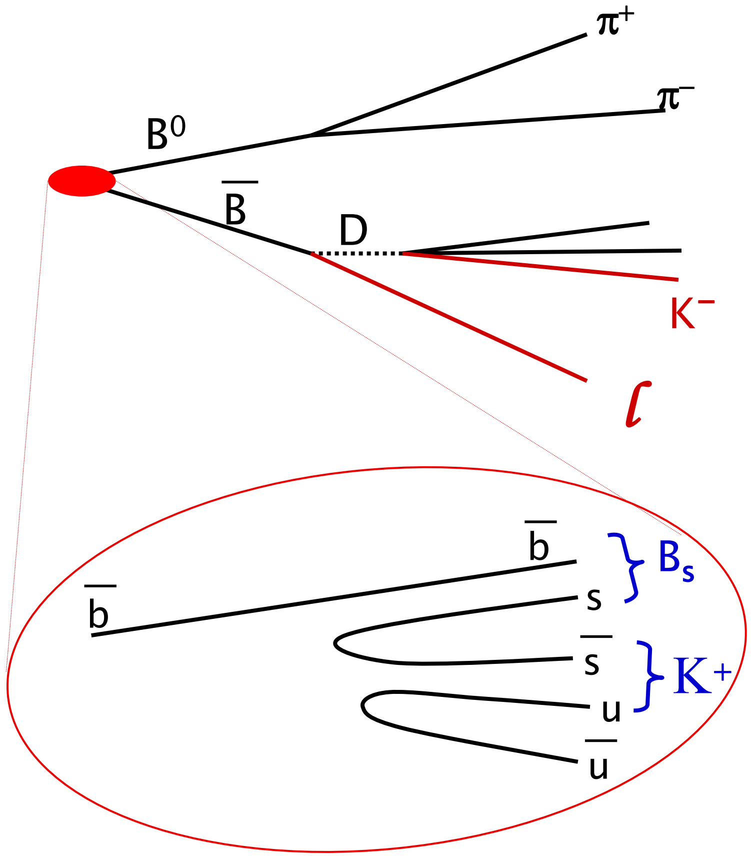
$\sigma_t \sim 33\ldots44$ fs

Analysis steps:

- B_s reconstruction: $B_s \rightarrow D_s\pi$ (self-tagging)
- Measurement of proper decay time
- Tagging of production flavor



Flavor Tagging & B_d Mixing



- Lepton
- Kaon
- Vertex charge

} Other B: “opposite”

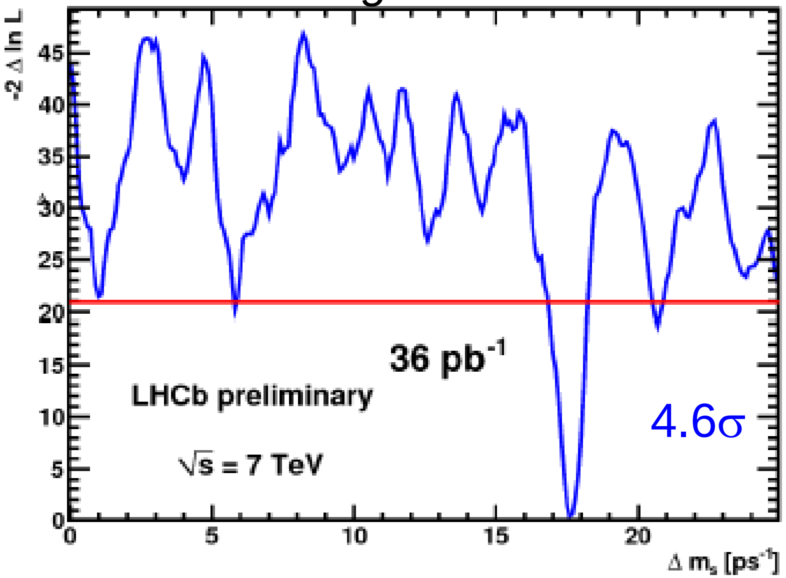
- Fragmentation hadron “same side”

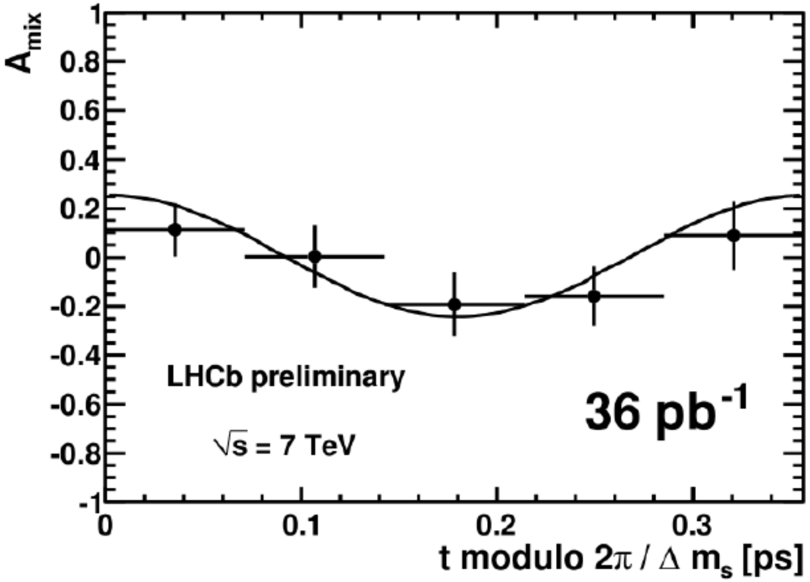
Figure of merit: $\varepsilon D^2 \sim 4.3\%$

- Tagging efficiency ε ~34%
- Dilution $D = (1 - 2\omega) \sim 32\%$
  ω = mistag probability

LHC*b* B_s Mixing Result

Unbinned Log-likelihood fit





- 1350 B_s candidates in 4 $B_s \to D_s \pi$ decay modes
- Proper time resolution: σ_t = 33-44 fs, OST

$$\boxed{\Delta m_s = 17.63 \pm 0.11 \pm 0.04 \text{ ps}^{-1}}$$

World best measurement