Standard Model of Particle Physics

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Problem Sheet 8

hand in until: 15.06.2011, 12.00 (Notice that Monday is public holiday !)

Problem 1: The Higgs sector with dimension-6 operators

Consider that we extend the Higgs sector of the Standard Model with the following dimension 6 operators:

$$\mathcal{O}_1^{d6} = \frac{1}{2}\,\partial_\mu(\Phi^\dagger\Phi)\,\partial^\mu(\Phi^\dagger\Phi), \qquad \mathcal{O}_2^{d6} = -\frac{1}{3}(\Phi^\dagger\Phi)^3 \tag{1}$$

in such a way that the Higgs Lagrangian now renders

$$\mathcal{L}_{higgs}^{SM+d6} = (D^\mu\,\Phi)^\dagger\,(D_\mu\,\Phi) - \mu^2\,\Phi^\dagger\,\Phi - \lambda(\Phi^\dagger\,\Phi)^2 + \frac{f_1}{2\,\Lambda^2}\,\partial_\mu(\Phi^\dagger\Phi)\,\partial^\mu(\Phi^\dagger\Phi) - \frac{f_2}{3\Lambda^2}\,(\Phi^\dagger\Phi)^3 \tag{2}$$

where Φ denotes the $SU_L(2)$ doublet describing the Higgs field, $\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix}$.

1. First of all, we shall study the impact of $\mathcal{O}_1^{d6}$ on the kinetic term of the Lagrangian. Prove that we can reabsorb it as a global rescaling of the Higgs field $H \to (1+\frac{f_1\,v^2}{\Lambda^2})^{-1/2}\,H$. Here we define H to represent the *physical* Higgs boson, namely $\phi_0 = (v+H)/\sqrt{2}$, with v being the Vacuum Expectation Value (VEV).

2. Next we consider the influence of $\mathcal{O}_2^{d6}$ on the Higgs potential, which now renders

$$V_{\Phi,\Phi^\dagger}^{SM+d6} = \mu^2\,\Phi^\dagger\,\Phi + \lambda(\Phi^\dagger\,\Phi)^2 + \frac{f_2}{3\,\Lambda^2}\,(\Phi^\dagger\Phi)^3. \tag{3}$$

   Minimize $V_{\Phi,\Phi^\dagger}^{SM+d6}$ to find the vacuum solutions and discuss your results (bear in mind that, in our notation, we define the VEV as $v^2/2 \equiv \langle\phi\rangle_{\text{min}}$.) Show explicitly that the (VEV) of the Standard Model $v_{sm}^2 \equiv -\frac{\mu^2}{\lambda}$ is shifted upon the addition of $\mathcal{O}_2^{d6}$ as $\delta\,v \equiv v - v_{sm} = \frac{f_2\,v_{sm}^4}{8\lambda\Lambda^2} + \mathcal{O}(\Lambda^{-4})$

3. Finally, analyse the combined effect of $\mathcal{O}_1^{d6}$ and $\mathcal{O}_2^{d6}$ on the relation between the Higgs boson mass and the quartic Higgs self-coupling. Prove that the following relation holds:

$$m_H^2 = \frac{\lambda\,v^2}{2}\,\left[1 - \frac{f_1\,v^2}{\Lambda^2} + \frac{f_2\,v^2}{2\Lambda^2\,\lambda}\right] + \mathcal{O}(\Lambda^{-4}) \tag{4}$$

   (*Hint: Do not forget to include the shift* $H \to (1+\frac{f_1\,v^2}{\Lambda^2})^{-1/2}\,H$ *! – you can think why . . .*)

4. Making use of the results you have derived so far, write down an explicit expression for the modified triviality bound on the Higgs mass, once the dimension-6 operators are taken into account. Would these additional operators allow heavier/lighter Higgs boson masses ? Discuss your results qualitatively.

   (*Note: there is no need to rederive the formulae you may need here, just look them up (carefully !) in your lecture notes.*).

Problem 2: Neutral and Charged current Interactions

Considering just the first family of quarks and leptons and starting from $\mathcal{L}_{fermion} = \sum \bar{\psi}(x)\gamma^\mu D_\mu \psi(x)$, show that the interactions between fermions and gauge bosons from $SU(2)_L \otimes U(1)_Y$ are given by:

1. $\mathcal{L}_{cc} = -\frac{g}{\sqrt{2}}\left[W_\mu^+\left(\bar{u}_L\gamma^\mu d_L + \bar{\nu}_{eL}\gamma^\mu e_L\right) + W_\mu^-\left(\bar{d}_L\gamma^\mu u_L + \bar{e}_L\gamma^\mu \nu_{eL}\right)\right]$

2. $\mathcal{L}_{NC} = -\sum_f \bar{f}\gamma^\mu\left[eQ_f A_\mu + \frac{e}{sin\theta_W cos\theta_W}\left(g_V^f - g_A^f\gamma^5\right)Z_\mu\right]f$

   where f stands for a generic fermion with charge Q_f, weak isospin (3rd component) I_3^f and the following vectorial and axial couplings with the Z boson: $g_V^f = I_3^f$ and $g_A^f = I_3^f\left(1 - 4|Q_f| sin^2\theta_W\right)$. Notice that one of the predictions from these expressions is that the neutrinos don't couple with the photons, explain the reason.

Problem 3: Disallowed Lagrangian Terms

Explain as briefly as possible why each of the following terms is not a permissible term in the Lagrangian density $\mathcal{L}(\mathrm{x})$ of the Standard Model. Fields are evaluated at point x unless indicated otherwise.

1. $\bar{L}P_L L + h.c.$

2. $\phi^\dagger \phi G_{\mu\nu}^a G_a^{\mu\nu}$

3. $\bar{E}\gamma^\mu P_R E$

4. $\int d^3y e^{-(x-y)^2/\sigma^2}\phi^\dagger\phi(x)\phi^\dagger\phi(y)$

5. $\mathrm{i}\left(\phi^\dagger\phi\right)^2$

6. $\phi^\dagger\tilde{\phi}$

Some remarks on the notation: L stands for a $SU_L(2)$ doublet; E accounts for the corresponding (right-handed) singlet; $G_{\mu\nu}^a$ represents the gluon field-strength; ϕ denotes a scalar field and $\tilde{\phi}$ its charge-conjugated $\tilde{\phi} \equiv i\,\sigma_2\,\phi^*$.

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