Standard Model of Particle Physics

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Problem Sheet 7

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Problem 1: Electroweak symmetry breaking from a triplet Higgs

1. Verify that

$$t^1 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & 1 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{pmatrix}, \quad t^2 = \frac{1}{\sqrt{2}}\begin{pmatrix}0 & -i & 0\\ i & 0 & -i\\ 0 & i & 0\end{pmatrix}, \quad t^3 = \begin{pmatrix}1 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & -1\end{pmatrix} \tag{1}$$

form a triplet representation of the $\mathfrak{su}(2)$ algebra: $[t^a, t^b] = i\,\epsilon_{abc}\,t^c$.

2. Consider the (complex) triplet Higgs field

$$\Phi(x) = (\Phi_1(x), \Phi_2(x), \Phi_3(x))^{\mathrm{T}}, \tag{2}$$

coupled covariantly to a $SU(2)_L \times U(1)_Y$ gauge group:

$$D_\mu\Phi = \partial_\mu\Phi - ig\,t^a W^a(x)\Phi - ig'\mathbb{1}\,B(x)\Phi\,. \tag{3}$$

We introduce a potential $V(\Phi)$ such that the $SU(2)_L \times U(1)$ gauge symmetry breaks spontaneously. The precise form of the potential is not interesting for the moment. As a result, the Higgs field acquires a vacuum expectation value, which can be written

$$\Phi_0 = (0, 0, v)^{\mathrm{T}}\,. \tag{4}$$

Derive the gauge boson mass eigenstates of the broken $SU(2)_L \times U(1)_Y$ symmetry and the electroweak mixing angle for the symmetry breaking pattern induced by Eq. (**??**). What are the consequences for the $\rho = m_W^2/(m_Z^2\cos^2\theta_w)$ parameter compared to the Standard Model?

3. How does the scalar particle content differ from the Standard Model (*hint*: count the degrees of freedom)? How can this in principle be observed at an experiment?

Problem 2: $SU(2)$ and Doublet vs. Triplet Higgses

Problem 1 points us to symmetry breaking of $SU(2)_L$ with a Higgs field in the doublet representation being special. The reason for this is a larger *custodial* symmetry of the doublet Higgs sector as an accidental consequence of renormalizability and spontaneous symmetry breaking. Actually, $1 + \alpha T = \rho \approx 1$ is experimentally highly constrained, see Fig. **??**.

1. Show that ε_{kl}, the two-dimensional Levi-Civita tensor ($\varepsilon_{12} = +1$), is invariant under $SU(2)$ transformations. What is the implication for the transformation property of

   a) $\varepsilon_{lk}\, \phi_k\, \psi_l$, b) $\varepsilon_{lk}\, \phi_k^*\, \psi_l$, c) $\varepsilon_{lj}\varepsilon_{jk}\, \phi_k^*\, \psi_l$,

   for $\phi, \psi \to U\phi, U\psi$, $SU(2) \ni U = \begin{pmatrix} a & b \\ -b^* & a^* \end{pmatrix}$, $|a|^2 + |b|^2 = 1$? *Hint*: consider the transformation of $\varepsilon\phi^*$ under U.

2. Consider the charge-conjugated doublet Higgs field

$$\Phi_k^{\mathrm{C}} = \varepsilon_{kl}\Phi_l^* \quad (k, l = 1, 2)\,. \tag{5}$$

   One can construct a bi-doublet from the components of Φ, Φ^{C}:

$$\mathcal{H} = \begin{pmatrix} \Phi_1^{\mathrm{C}} & \Phi_1 \\ \Phi_2^{\mathrm{C}} & \Phi_2 \end{pmatrix}\,. \tag{6}$$

   Rewrite the Higgs potential $V(\Phi) = \mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2$ in terms of $\mathcal{H}$ (*hint*: $\mathrm{Tr}(\mathcal{H}^\dagger\mathcal{H})$). Under how many *global* $SU(2)$s is $\mathcal{L}(\Phi)$ invariant, apparently ?

3. What is the residual global symmetry of the vacuum *after* symmetry breaking $\mathcal{H} \to v/\sqrt{2}\,\mathbb{1}$?

4. *Discuss qualitatively*: What is the consequence for the gauge boson mass terms $\sim m_a^2 W_\mu^a W^{\mu,a}$ if $g' = 0$. And why is this different for the case of a triplet Higgs ? Is there a cure for the triplet Higgs ?

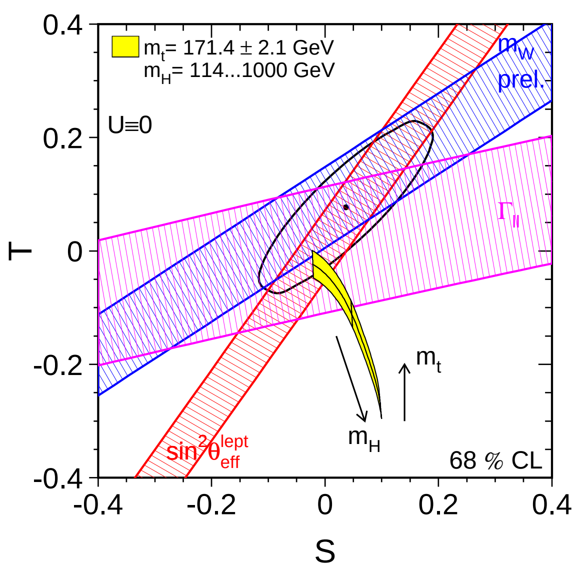

Figure 1: 95% confidence level bounds on T (and other precision observables) by the LEP electroweak working group by summer 2006. `http://lepewwg.web.cern.ch/LEPEWWG/plots/summer2006/`.

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