Standard Model of Particle Physics

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Problem Sheet 6

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Problem 1: Symmetries of the Higgs Sector

Consider the Lagrangian of the Higgs sector in the Standard Model:

$$\mathcal{L}_{higgs} = (D^\mu \Phi)^\dagger (D_\mu \Phi) - \mu^2 \Phi^\dagger \Phi + \lambda(\Phi^\dagger \Phi)^2 \tag{1}$$

where Φ denotes the $SU_L(2)$ doublet describing the Higgs field and D_μ stands for the $SU_L(2) \otimes U_Y(1)$ covariant derivative,

$$\Phi = \begin{pmatrix} \phi^+ \\ \phi^0 \end{pmatrix} \qquad D^\mu \Phi = \left(\partial_\mu + i\,\frac{g}{2}\,\vec{\sigma}\cdot\vec{W}_\mu + i\frac{g'}{2}\,B_\mu\right)\Phi \tag{2}$$

Let us define following matrix (*bifield*) notation:

$$\Sigma \equiv \frac{1}{\sqrt{2}}\,(\epsilon\,\Phi^*, \Phi) = \frac{1}{\sqrt{2}}\begin{pmatrix} \phi^{0*} & \phi^+ \\ -\phi^- & \phi^0 \end{pmatrix}, \tag{3}$$

with $\phi^{+*} = \phi^-$.

1. Show that the Lagrangian (1) may be rewritten in terms of the above matrix notation as

$$\mathcal{L}_{higgs} = Tr\,(D^\mu \Sigma)^\dagger (D_\mu \Sigma) - \mu^2\, Tr\,(\Sigma^\dagger \Sigma) + \lambda\, Tr\,(\Sigma^\dagger \Sigma)^2 \tag{4}$$

2. Taking into account the transformation properties of Σ under the Electroweak gauge transformations,

$$\Sigma \xrightarrow{SU_L(2)} L\,\Sigma \qquad \Sigma \xrightarrow{SU_R(2)} \Sigma\,R^\dagger \qquad \Sigma \xrightarrow{U_Y(1)} \Sigma\,\exp\left(-\frac{i}{2}\,\sigma_3\,\theta\right), \tag{5}$$

   where L, R denote generic $SU(2)$ matrices, prove that $\mathcal{L}_{higgs}$ is invariant under $SU_L(2) \otimes U_Y(1)$ transformations.

3. Show that, in the absence of electromagnetic interactions ($g' = 0$), $\mathcal{L}_{higgs}$ is also invariant under $SU_R(2)$ transformations, and hence under a global $SU_L(2) \otimes SU_R(2)$ symmetry.

4. Prove that, once the Higgs field acquires a non-vanishing vacuum expectation value $\langle\Sigma\rangle \to v$, the resulting Lagrangian remains invariant under the so-called *Custodial symmetry* $SU_{L+R}(2)$,

$$\langle\Sigma\rangle \xrightarrow{SU_{L+R}(2)} L\langle\Sigma\rangle L^\dagger. \tag{6}$$

5. From the expressions analytical relation between M_W and M_Z (cf. e.g. your lecture notes), check that $M_W \to M_Z$ in the limit $g' \to 0$. Can you relate this observation to the previous result?

Similarly, let us now consider the Lagrangian of the Yukawa sector, which accounts for the masses of the quarks in the Standard Model,

$$\begin{aligned} \mathcal{L}_{Yuk} &= -Y_u \left(-\bar{u}_R \, \phi^+ \, d_L - \bar{d}_L \, \phi^- \, u_R + \bar{u}_R \, \phi^0 \, u_L + \bar{u}_L \phi^{0*} \, u_R\right) \\ & \quad -Y_d \left(\bar{u}_L \phi^+ \, d_R + \bar{d}_R \, \phi^- \, u_l + \bar{d}_L \, \phi^0 \, d_R + \bar{d}_R \, \phi^{0*} \, d_L\right). \end{aligned} \tag{7}$$

1. Using again the matrix notation of Eq. (3), show that $\mathcal{L}_{Yuk}$ can be rewritten as

$$\mathcal{L}_{Yuk} = -\bar{Q}_L \, \Sigma \, Y_Q \, Q_R + h.c., \tag{8}$$

with $Q_L = \begin{pmatrix} u \\ d \end{pmatrix}$ being a $SU_L(2)$ doublet and Q_R the corresponding singlet, while the Yukawa coupling takes on the matrix form $Y_Q = \begin{pmatrix} Y_u & \\ & Y_d \end{pmatrix}$.

2. Prove the invariance of the Yukawa terms $Y_Q \, \bar{Q}_L \, \Sigma \, Q_R$ under $SU_L(2) \otimes U_Y(1)$ gauge transformations.

3. Would it be possible to use the very same Yukawa structure to account for the masses of the neutrinos?

Problem 2: EWPO as indirect probes of New Physics

The generic features of New Physics may be analysed from a *bottom-top* approach. Let us assume that a generic form of New Physics at a certain energy scale Λ modifies the Standard Model through an effective Lagrangian of the sort

$$\mathcal{L}_{eff} = \frac{a}{\Lambda^2} \, |\mathrm{H}^\dagger \, \mathrm{D}_\mu \, \mathrm{H}|^2, \tag{9}$$

with a being a dimensionless coupling constant.

1. Prove that, after the Electroweak symmetry breaking, $\langle H \rangle = (0, v/\sqrt{2})$, the above effective Lagrangian encodes a bilinear term in the gauge fields of the sort:

$$\mathcal{L}_{BW} = \frac{a \, v^4}{8 \, \Lambda^2} \left(g' \, B_\mu + g \, W_\mu^3\right)^2. \tag{10}$$

2. Show that this Lagrangian provides an additional contribution to the mass of the Z^0 boson, while leaving M_W unaltered.

3. Show that these non-standard contributions induce a shift in the parameter $\rho \equiv M_W^2/(M_Z^2 \, \cos^2\theta_W)$, so that $\delta\rho = -\frac{av^2}{\Lambda^2}$.

4. Taking into account the Electroweak precision data, in particular recalling that $\delta\rho \lesssim 10^{-3}$, estimate a lower bound for the characteristic New Physics scale Λ in this model (as a function of the coupling a).

5. The LHC offers good prospects for accurate measurements of M_W. Precisions at the level of $\Delta M_W \simeq 5$ MeV could be attained upon dedicated analysis of $pp \to W^{\pm}$+jets. Estimate (as a function of the coupling a), up to which scale Λ would the measurement of M_W at the LHC be sensitive to indirect effects induced by this generic New Physics model.

   hints:

   Bear in mind that a correction to $\delta\rho$ translates into a shift in M_W as

$$\delta M_W \simeq \frac{M_W}{2} \frac{\cos^2\theta_w}{\cos^2\theta_w - \sin^2\theta_w} \delta\rho \,. \tag{11}$$

6. Precise theoretical calculations in the Standard Model, including up to Next-to-Next-to-leading order effects, predict a value for $M_W^{th} = 80.354$ GeV (cf. Freitas, Hollik, Wager, Weiglein 2002).

   Current experimental measurements, on the other hand, render $M_W^{exp} = 80.399 \pm 0.023$ GeV. Could the effects associated to $\mathcal{L}_{eff}$ help to bring both values closer ? How large could the coupling a be with respect to the characteristic scale Λ, if we wanted $\mathcal{L}_{eff}$ to account for the current mismatch between theory and experiments?

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