Standard Model of Particle Physics

Lectures: Tilman Plehn, Ulrich Uwer
Exercises: James Barry, Christoph Englert, Dorival Gonçalves Netto, David López Val

Problem Sheet 4

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For some of the exercises suggested in this problem sheet, a visit to the **Particle Data Group** (PDG) website will be useful: *http://pdglive.lbl.gov/*

Problem 1: leptonic decays of the τ

Consider the leptonic τ decay mode into a muon, $\tau \to \mu \bar{\nu}_\mu \nu_\tau$. This process can be described by the effective Lagrangian:

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}}\, J_e^{\mu\dagger}\, J_{\mu\tau} + h.c. \qquad J_l^\mu = \bar{\nu}_l\, \gamma^\mu (1-\gamma_5)\, l, \tag{1}$$

1. Derive the Feynman rule corresponding to the effective $\tau\,\nu_\tau\,\mu\,\nu_\mu$ 4-fermion interaction.
2. Draw the Feynman diagram that describes $\tau \to \mu\,\bar{\nu}_\mu\,\nu_\tau$ at the leading-order.
3. Use the Feynman rule for the $\tau\,\nu_\tau\,\mu\,\nu_\mu$ vertex you derived above, alongside with those that describe the external fermions (cf. your theory lectures) to compute the tree-level matrix element for the muonic decay of a τ, $\mathcal{M}(\tau \to \mu\,\bar{\nu}_\mu\,\nu_\tau)$.
4. Again from the above Feynman rule, and simply by dimensional analysis, **justify** (no detailed calculations required !) that the total decay width for the τ fulfills the *Sargent rule*: namely, that for a weak $1 \to 3$ decay of a particle of mass M, the partial decay width behaves like $\Gamma \sim M^5$ (neglecting the mass of the final states).
5. Making use of the previous result, give an analytical estimate for the leptonic partial width $\Gamma(\tau \to \mu\bar{\nu}_\mu\nu_\tau)$ in terms of the muon lifetime, τ_μ (recall that $\tau = \Gamma^{-1}$). Use the experimental value τ_μ^{exp} quoted by the PDG to evaluate the former estimate numerically, and compare your result with the actual measurement $\Gamma^{exp}(\tau \to \mu\bar{\nu}_\mu\nu_\tau)$.

Problem 2: hadronic decays of the τ

Similarly to the leptonic modes that we studied in the previous problem, the τ may also decay into quarks – and hence into hadrons. An effective description of the Electroweak dynamics that controls these decay channels is provided by the Lagrangian:

$$\mathcal{L} = -\frac{G_F}{\sqrt{2}}\, J_q^{\mu\dagger}\, J_{\mu\tau} + h.c. \tag{2}$$

where the hadronic current corresponds to

$$J_q^\mu = \bar{u}\gamma^\mu(1-\gamma_5)(V_{ud}\, d + V_{us}\, s) \tag{3}$$

1. Draw the Leading-Order Feynman diagrams that describe the hadronic decay modes of the τ. Which type of hadrons can arise as decay products of the τ?.

   (*hints:* You should determine which hadronic states correspond to the possible quark combinations that may result from the τ decays. Limit yourself to three-body decays, with two quarks + ν_τ in the final state. You should also bear in mind how the mass of the τ compares to that of the possible final-state hadrons. Look up the **Particle Data Group** website to check all the data you may need).

2. Provide analytical estimates for the following branching ratios:

   (a) $\mathcal{B}(\tau \to e\bar{\nu}_e\nu_\tau)$

   (b) $\mathcal{B}(\tau \to \mu\bar{\nu}_\mu\nu_\tau)$

   (c) $\mathcal{B}(\tau \to hadrons + \nu_\tau)$,

   and evaluate them numerically. When needed you can take $V_{ud} = \cos\theta_c$ and $V_{us} = \sin\theta_c$, with the Cabbibo angle being $\theta_C \simeq 0.22$.

   (*hints*: again, no detailed computations are needed ! Just remember to take into account all possible decay channels of the τ (both to leptons and hadrons). As before, limit yourself to three-body decays and neglect the masses of the final-state products.

3. Justify that the ratio of hadron-to-lepton tree-level decay widths renders:

$$R_\tau = \frac{\Gamma(\tau \to hadrons + \nu_\tau)}{\Gamma(\tau \to \nu_\tau\, e^-\, \bar{\nu}_e)} = N_C\,(V_{ud}^2 + V_{us}^2). \tag{4}$$

   where N_C stands the number of colors. Evaluate this expression numerically and compare your result with the experimental measurement. What observations can you highlight?

   → R_τ is a very important quantity in the experimental study of the τ lepton physics. Notice its analogy to the R_{had} factor for $e^+e^- \to$ hadrons that you have considered in the lectures.

4. The Next-to-Leading (NLO) QCD corrections to R_τ modify the above expression as

$$R_\tau = \frac{\Gamma(\tau \to hadrons + \nu_\tau)}{\Gamma(\tau \to \nu_\tau\, e^-\, \bar{\nu}_e)} = N_C\,(V_{ud}^2 + V_{us}^2)\left[1 + \frac{\alpha_s}{\pi}\right]. \tag{5}$$

   Sketch the kind of Feynman diagrams that describe such NLO QCD corrections. How do you think these higher order effects may be relevant to conduct studies on perturbative QCD by analysing the decays of the τ?

Additional information: put contact information here:
Christoph (c.englert@thphys.uni-heidelberg.de)
David (d.lopez@thphys.uni-heidelberg.de),
Dorival (d.goncalves@thphys.uni-heidelberg.de)
James (james.barry@mpi-hd.mpg.de)
