Universität Heidelberg **summer term '11**

Standard Model of Particle Physics

Lectures: Tilman Plehn, Ulrich Uwer
Exercises: James Barry, Christoph Englert, Dorival Gonçalves Netto, David López Val

Problem Sheet 3

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Problem 1: Total cross section for $e^+e^- \to \mu^+\mu^-$

The differential cross section for the $2 \to 2$ scattering process

$$a + b \to 1 + 2 \tag{1}$$

is given by (cf. Ulrich Uwer's lecture slides)

$$d\sigma = \frac{|\mathcal{M}|^2}{\Phi}(2\pi)^4\delta^{(4)}(p_a + p_b - p_1 - p_2)\frac{d^3\mathbf{p}_1}{(2\pi)^3 2E_1}\frac{d^3\mathbf{p}_2}{(2\pi)^3 2E_2}\,, \tag{2}$$

where $\Phi = 4\sqrt{(p_a \cdot p_b)^2 - m_a^2 m_b^2}$ is the flux factor.

1. Assuming that the initial particles travel on the beam axis (the x^3 direction, say), show that the flux factor Φ can be expressed as

   $$\Phi = 2\lambda(s, m_a^2, m_b^2)\,, \tag{3}$$

   where $\lambda(x, y, z) = \sqrt{x^2 + y^2 + z^2 - 2xy - 2xz - 2yz}$ and $s = (p_a + p_b)^2$.
   (*Hint:* It is simpler to work in a frame where one of the initial particles is at rest.)

2. Use the result of question (1.1) to show that in the high energy limit (neglect all particle masses), the formula in Eq. (2) can be simplified to

   $$d\sigma = \frac{1}{64\pi^2 s}|\mathcal{M}|^2 d\Omega \tag{4}$$

   in the centre-of-mass frame, where $d\Omega = d\phi\, d\cos\theta$ is the solid angle into which the particles are scattered.

3. In problem 2 of sheet 2 we computed the spin-averaged squared matrix element $|\mathcal{M}(e^-\mu^- \to e^-\mu^-)|^2$ in terms of the Mandelstam variables, s, t, u. Use this result to write down the matrix element $|\mathcal{M}(e^+e^- \to \mu^+\mu^-)|^2$, by making use of crossing symmetry.

4. Use Eq. (4) and the result from question (1.3) to compute the total QED cross section, σ_{total}, for this process. Express the result in terms of the fine-structure constant α and the centre-of-mass energy, s. You may work in the centre-of-mass frame, and neglect particle masses.

5. What do you expect would be the *qualitative* effect of Z boson exchange on the result calculated in question (3)? Write down any additional Feynman graph(s) that may arise in this case.

Problem 2: Bhabha Scattering ($e^+e^- \;\rightarrow\; e^+e^-$)

Show that the differential cross section for Bhabha scattering, $e^+e^- \rightarrow e^+e^-$, is given by

$$\frac{d\sigma}{d\cos\theta} = \frac{\pi\alpha^2}{s}\left[u^2\left(\frac{1}{s}+\frac{1}{t}\right)^2 + \left(\frac{t}{s}\right)^2 + \left(\frac{s}{t}\right)^2\right] \tag{5}$$

1. Write down the two first order Feynman diagrams that contribute to this process. Why is there a relative minus sign between the two diagrams?

2. Use the QED Feynman rules to show that the amplitude can be written as

   $$i\mathcal{M} = \frac{ie^2}{t}\mathcal{A}_1 - \frac{ie^2}{s}\mathcal{A}_2\,, \tag{6}$$

   where $\mathcal{A}_1$ and $\mathcal{A}_2$ are products of spinors and gamma matrices.

3. Calculate $|\mathcal{M}|^2$, in the high energy limit (neglect the electron mass)

   (a) Use trace identities to show that the first term is given by

   $$\frac{1}{4}\sum_{\text{spins}} |\mathcal{A}_1|^2 = 2(u^2+s^2) \tag{7}$$

   Note that this is the same expression obtained for the $e\mu$ scattering process in problem sheet 2, in the massless limit.

   (b) Calculate (or use crossing symmetry, cf. question 1.3) the second term, $\frac{1}{4}\sum_{\text{spins}}|\mathcal{A}_2|^2$.

   (c) Use the identities $\gamma^\mu \not{a}\not{b}\not{c}\gamma_\mu = -2\not{c}\not{b}\not{a}$ and $\gamma^\mu\not{a}\not{b}\gamma_\mu = 4(a\cdot b)$ to show that the two interference terms are given by

   $$\frac{1}{4}\sum_{\text{spins}} \mathcal{A}_1\mathcal{A}_2^* = \frac{1}{4}\sum_{\text{spins}} \mathcal{A}_2\mathcal{A}_1^* = -2u^2. \tag{8}$$

4. Add up the different contributions and use Eq. (4) to translate the matrix element into the differential cross section $d\sigma/d\cos\theta$.

Additional information: put contact information here:
Christoph (`c.englert@thphys.uni-heidelberg.de`)
David (`d.lopez@thphys.uni-heidelberg.de`),
Dorival (`d.goncalves@thphys.uni-heidelberg.de`)
James (`james.barry@mpi-hd.mpg.de`)
