Standard Model of Particle Physics

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Problem Sheet 2

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Problem 1: Kinematical Invariants of $2 \to 2$ Scattering

Consider the scattering process

$$a + b \to 1 + 2\,, \tag{1}$$

with four momenta p_a, p_b, p_1, and p_2.

1. How many independent lorentz-invariant quantities can one construct from the incoming and outgoing four momenta? Take into account energy-momentum conservation of the scattering process.

2. Show that the *Mandelstam variables*

$$s = (p_a + p_b)^2\,, \tag{2a}$$
$$t = (p_a - p_1)^2\,, \tag{2b}$$
$$u = (p_a - p_2)^2\,, \tag{2c}$$

   suffice to fully describe the $2 \to 2$ scattering's kinematics.

3. The lorentz-invariant 2-body phase space, which shows up in the computation of the $2 \to 2$ scattering cross section reads

$$\mathrm{dLIPS}_2 = \frac{\mathrm{d}^3\mathbf{p}_1}{(2\pi)^3\, 2E_1}\frac{\mathrm{d}^3\mathbf{p}_2}{(2\pi)^3\, 2E_2}\delta^{(4)}(p_a + p_b - p_1 - p_2)\,, \tag{3}$$

   where $E_i^2 = p_i^2 + \mathbf{p}_i^2,\ i = 1, 2$. What is the phase space integration's dimensionality? How is this reflected in the Mandelstam variables? (*Hint:* Find a relation between $p_1^2, p_2^2, p_a^2, p_b^2$ and s, t, u.)

Problem 2: $e^-\mu^-$ Scattering

Compute the S matrix element for genuine $e^-(p_a) + \mu^-(p_b) \to e^-(p_1) + \mu^-(p_2)$ scattering in QED, i.e. neglect the case when no scattering takes place, $p_a = p_1$.

1. Write down the contributing Feynman graph(s) to $|\mathcal{M}|^2$ in the leading order approximation.

2. Apply the QED Feynman rules (cf. lecture notes on Tilman's webpage) and translate the Feynman diagrams into an algebraic expression for the amplitude $i\mathcal{M}$.

3. Compute the unpolarized matrix element $|\mathcal{M}|^2$, i.e. sum over final state spins, and average over initial state spins. Neglect the electron and muon masses for convenience. You will need the trace identity

$$\mathrm{Tr}\,(\gamma^\mu\gamma^\nu\gamma^\rho\gamma^\sigma) = 4\,(g^{\mu\nu}g^{\rho\sigma} - g^{\mu\rho}g^{\nu\sigma} + g^{\mu\sigma}g^{\nu\rho})\,, \tag{4}$$

and the rule for complex conjugation of the spinor product

$$[\bar{u}(p)\gamma^\mu u(k)]^\star = \bar{u}(k)\gamma^\mu u(p) \tag{5}$$

(no proofs needed). Express the unpolarized $|\mathcal{M}|^2$ in terms of the Mandelstam variables Eq. (2). What happens for forward scattering

$$\cos\theta_{a1} = \mathbf{p}_a \cdot \mathbf{p}_1/(|\mathbf{p}_a|\,|\mathbf{p}_1|) \to 0\,? \tag{6}$$

Is this result familiar to you?

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