Standard Model of Particle Physics

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Problem Sheet 10

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Problem 1: The structure of the CKM matrix

Unitarity: Consider the Wolfenstein parametrization of the CKM matrix,

$$V_{CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3\,(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \quad (1)$$

1. Check that for any choice of i, j, l, k (between 1 and 3) the following relation holds:

$$\mathrm{Im}\,[V_{ij}\,V_{ik}\,V_{lk}\,V_{lj}^*] = \mathcal{J} \sum_{m,n=1}^{3} \epsilon_{ilm}\,\epsilon_{jkn} + \mathcal{O}(\lambda^4) \quad (2)$$

   Determine the form of the so-called Jarlskog Determinant $\mathcal{J}$ as a function of the Wolfenstein parameters.

2. Show that the off-diagonal unitarity condition

$$V_{ub}^*\,V_{ud} + V_{cb}^*\,V_{cd} + V_{tb}^*\,V_{td} = 0 \quad (3)$$

   can be visualized as a triangle in a complex plane, with area $|\mathcal{J}|/2$. What are the size of the three sides ? What are the angles ? As you can see in Figure 1, this geometrical constructions is indeed used in practice to experimentally constrain the CKM matrix elements.

CP-violation: Consider how the Parity $\mathcal{P}$ and Charge Conjugation $\mathcal{C}$ operators are implemented on vector fields and spinor bilinears:

	$\bar{\psi}_1\gamma^\mu\,\psi_2$	$\bar{\psi}_1\gamma^\mu\gamma_5\,\psi_2$	$V^\mu(x,t)$
$\mathcal{P}$	$\bar{\psi}_1\gamma_\mu\,\psi_2$	$-\bar{\psi}_1\gamma_\mu\gamma_5\,\psi_2$	$V_\mu(-x,t)$
$\mathcal{C}$	$-\bar{\psi}_2\gamma^\mu\,\psi_1$	$\bar{\psi}_2\gamma^\mu\,\gamma_5\psi_1$	$-V^{\mu\,\dagger}(x,t)$

With this information in mind, study how the charged-current interactions,

$$\mathcal{L}_{CC} = -\frac{g}{2\sqrt{2}}\,W_\mu^+ \sum_{i,j=1}^{3} V_{ij}\,(\bar{u}_i\gamma^\mu\,(1-\gamma_5)d_j) + h.c., \quad (4)$$

transform under successive applications of $\mathcal{C}$ and $\mathcal{P}$ and deduce that the presence of the CKM matrix V_{ij} leads to CP violation.

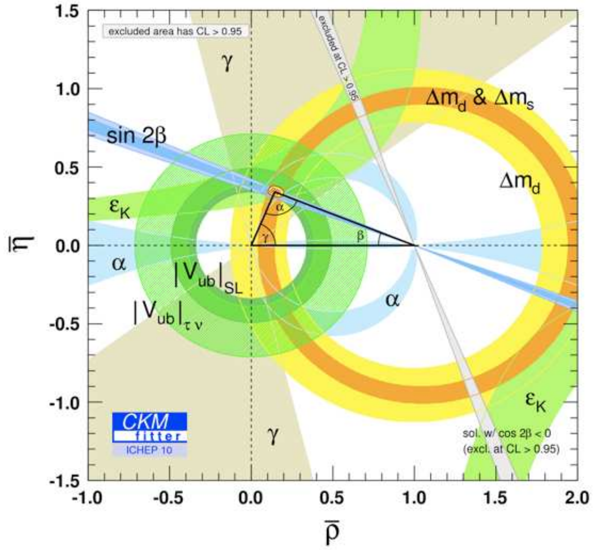


Figure 1: Geometrical representation of the CKM matrix, superimposing all the available experimental measurements that settle constraints on the determination of the different matrix elements. Figure taken from `http://ckmfitter.in2p3.fr`.

Problem 2: The Glashow-Iliopoulos-Maiani (GIM) mechanism

2a. GIM mechanism at the tree level:

1. Consider the SM with just the 3 lightest flavors u, d and s quarks. Their EW interactions can be described by means of the Lagrangian

$$\mathcal{L} = -\frac{g}{2\sqrt{2}} W_\mu^+ J_{cc}^\mu - \frac{g}{2\cos\theta_W} Z_\mu J_{NC}^\mu \tag{5}$$

$$J_{CC}^\mu = \bar{u}\gamma^\mu (1-\gamma_5) d', \quad J_{NC}^\mu = \bar{u}\gamma^\mu (g_v - g_a\gamma_5)\, u + \bar{d}'\gamma^\mu (g_v - g_a\gamma_5)\, d', \tag{6}$$

   where d' $= d\cos\theta_C + s\sin\theta_C$; s, d being the physical quark eigenstates and θ_C denoting the Cabbibo angle. Show that, under this assumption, both the charged and the neutral current interactions would allow flavor changing (in particular, $\Delta S = 1$ strangeness-changing) interactions.

2. This was the original motivation by Glashow, Iliopoulos and Maiani to predict (1973) the existence of the charm quark, in trying to avoid flavor-changing neutral currents from the theoretical description of the EW interactions. Check their argument by yourself and prove that, once a complete second generation of quarks is included, namely if we assume the existence of two quark doublets

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ d\cos\theta_C + s\sin\theta_C \end{pmatrix}; \qquad \begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ dX + sY \end{pmatrix}, \tag{7}$$

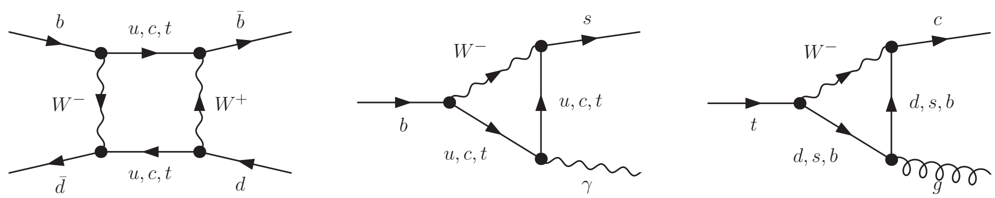
   with suitable values X, Y, then no flavor-changing neutral currents are permitted at the tree-level. Guess the form of X and Y and check that the mixing matrix $V_{dd',ss'}$,

$$\begin{pmatrix} d' \\ s' \end{pmatrix} = \begin{pmatrix} \cos\theta_C & \sin\theta_C \\ X & Y \end{pmatrix} \begin{pmatrix} d \\ s \end{pmatrix}, \tag{8}$$

   is indeed a unitary matrix.

3. Could you think of an example of a (tree-level) flavor-changing charged-current process involving the first and second generations ? Describe it in terms of the corresponding Feynman diagram(s).

2b. GIM mechanism at one-loop: neutral meson mixing



(a) (b) (c)

The above Feynman diagram (a) describes the flavor mixing in the $B^0_d - \overline{B^0_d}$ system, which is a paradigmatic example of a flavor-changing ($\Delta B = 2$) neutral-current process. The corresponding $B^0_d - \overline{B^0_d}$ oscillation frequency is given by the following expression:

$$\langle \overline{B^0_d} | i\mathcal{L}^{\Delta B=2} | B^0_d \rangle = \frac{G_F^2}{16\,\pi^2} \sum_{i,j=u,c,t} V^*_{id}\, V_{is}\, V^*_{jd}\, V_{js}\, f(m_i^2, m_j^2, m_b^2, m_d^2, M_W^2)\, \langle \overline{B^0_d} | \bar{d}\gamma^\mu P_L b\, \bar{d}\gamma_\mu P_L b | B^0_d \rangle \tag{9}$$

where $\langle \overline{B^0_d} | \bar{d}\gamma^\mu P_L b\, \bar{d}\gamma_\mu P_L b | B^0_d \rangle$ encodes the non-perturbative nature of the B^0_d-meson bound state (which needs to be related either to experimental measurements or to lattice simulations), while f denotes a certain function of the different quark masses and M_W.

1. From the Feynman rules of the EW Theory, **justify** the form of the above analytic expression for the $B^0_d - \overline{B^0_d}$ oscillation frequency. In particular, comment on the dependence with the CKM matrix elements and the reason why this process takes place at $\mathcal{O}(G_F^2)$.

2. Show that the mixing probability vanishes in the limit of equal quark masses. How could you link this observation to the GIM mechanism? (*hint*: exploit the unitarity relations of the CKM matrix).

2c. GIM mechanism at one-loop: radiative decays of heavy quarks

The Feynman diagrams (b) and (c) in the above Figure describe two different loop-induced decays involving heavy quarks. Although the dynamics underlying both processes looks very similar, they exhibit a markedly different phenomenological behavior. While radiative decays of the b-quark give rise to small, but yet measurable effects – e.g. $\mathcal{B}(B \to K_s\, \gamma) \simeq 3 \times 10^{-4}$ – the ones related to the top-quark are by far unobservable (for instance $\mathcal{B}(t \to cg) \lesssim 10^{-10}$). Could you explain why?

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