Problem sheet 4 - Physics V - WS 2006/2007

Due: November 16/17, 2006

Problem 4.1: Partial decay width (30 Points)

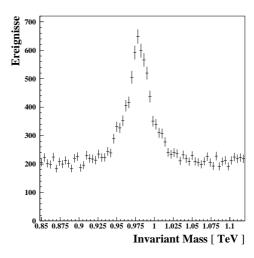
The π^+ meson decays into a charged lepton and the corresponding neutrino, $\pi^+ \to \ell^+ \nu_l$, where $\ell = \{e, \mu\}$.

- (a) Draw the Feynman diagrams of both decays.
- (b) Assume that the decay amplitudes are the same for the decay to muons and electrons. Calculate the ratio of the partial decay width of the two decay channels and compare your result with the measurements published in the PDG.

Problem 4.2: Discovery of a new resonance (40 Points)

In the year 2027 scientist finally succeeded to build and to operate a circular $\mu^+\mu^-$ collider with a center-of-mass energy of $\sqrt{s}=10$ TeV. Shortly afterwards, a new resonance was discovered among the decay products of the $\mu^+\mu^-$ annihilation. A preliminary invariant mass distribution is shown in the figure.

- (a) Estimate mass, decay width and lifetime of the new state.
- (b) The data shown in the mass distribution corresponds to a data-taking of one month (10^6 s). For this period the machine people report an average luminosity of $10^{32}~cm^{-2}s^{-1}$. Give an estimate of the cross section (in barn) to produce the new state in $\mu^+\mu^-$ annihilations. Assume that the detection of the new events is fully efficient.



Problem 4.3: Bethe-Bloch-Equation (30 points)

The energy loss of particles with nuclear charge z in matter (nuclear charge Z and nuclear number A) is given by the Bethe-Bloch equation

$$-\frac{dE_{\text{kin}}}{dx} = Kz^2 \frac{Z}{A} \frac{1}{\beta^2} \left[\ln(\frac{2m_e v^2 \gamma^2}{I}) - \beta^2 \right]. \tag{1}$$

where dx is measured in units of $[g/cm^2]$. The constant K has the value 0.307 MeV g^{-1} cm². I ist the average excitation energy (in eV) which is material dependent.

Assume that the particles are non-relativistic ($\beta \ll 1$) and use the approximation

$$-\frac{dE_{\rm kin}}{dx} = \alpha K z^2 \frac{Z}{A} \frac{1}{\beta^2},\tag{2}$$

with $\alpha = 6.53$. (α accounts for the fact that for $\beta \ll 1$ the Bethe-Bloch equation does not go exactly as $1/\beta^2$ and summarizes material - dependent quantities). The density of Si is $2.33 \,\mathrm{g/cm^3}$.

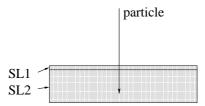


Abbildung 1: A schematic view of the Si - detector

Consider a particle detector constructed from a thin silicon semi-conductor layer (SL1 = $130 \,\mu\text{m}$) to measure ΔE and a thick silicon semi-conductor layer (SL2 = $1000 \,\mu\text{m}$) for the total energy (E) measurement (Fig. 1). With this detector it is possible to identify the type of a particle (see Fig. 2).

To calculate the path length x which a particle can travel inside the material, you need to integrate the Bethe-Bloch equation from the initial energy $E_{\rm kin}^0$ down to 0. However, since the equation does not hold for $\beta \to 0$, the equation is usually only integrated down to a given $E_{\rm kin}^{\rm min}$ and then assumed to be constant. The constant path length for $E_{\rm kin}^{\rm min} = 0.5\,{\rm MeV}$ is about $1.5\,{\rm mg/cm^2}$ for protons and $0.6\,{\rm mg/cm^2}$ for α -particles.

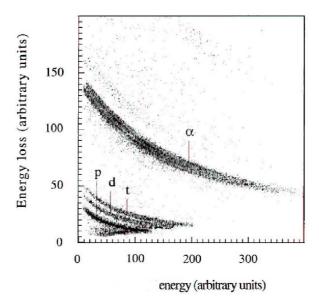


Abbildung 2: Energy loss $\Delta(E)$ vs total energy

- (a) What is the maximum energy a proton (resp. α -particle) may have to be completely stopped in the detector?
- (b) What is the minimum energy a proton (resp. α -particle) may have to reach the thick second silicon layer?