3.4 V-A coupling of leptons and quarks

Reminder

$$\bar{u}_\ell \gamma^\mu (1-\gamma^5) u_\nu = \bar{u}_\ell \gamma^\mu u_\nu^L = (\bar{u}_\ell^L + \bar{u}_\ell^R)\gamma^\mu u_\nu^L = \bar{u}_\ell^L \gamma^\mu u_\nu^L$$

In V-A theory the weak interaction couples left-handed lepton/quark currents (right-handed anti-lepton/quark currents) with an universal coupling strength:

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

Weak transition appear only inside weak-isospin doublets:

Not equal to the mass eigenstate

Lepton currents:

1. $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ $\quad j^\mu_{e\nu} = \bar{u}_e \gamma^\mu (1-\gamma^5) u_\nu$
2. $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$ $\quad j^\mu_{\mu\nu} = \bar{u}_\mu \gamma^\mu (1-\gamma^5) u_\nu$
3. $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$ $\quad j^\mu_{\tau\nu} = \bar{u}_\tau \gamma^\mu (1-\gamma^5) u_\nu$

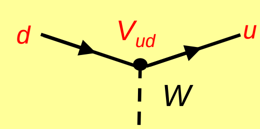
Quark currents:

1. $\begin{pmatrix} u \\ d' \end{pmatrix}$ $\quad j^\mu_{du} = \bar{u}_d \gamma^\mu (1-\gamma^5) u_u$
2. $\begin{pmatrix} c \\ s' \end{pmatrix}$ $\quad j^\mu_{sc} = \bar{u}_s \gamma^\mu (1-\gamma^5) u_c$
3. $\begin{pmatrix} t \\ b' \end{pmatrix}$ $\quad j^\mu_{bt} = \bar{u}_b \gamma^\mu (1-\gamma^5) u_t$

3.5 CKM matrix to describe the quark mixing

One finds that the weak eigenstates of the down type quarks entering the weak isospin doublets are not equal to the their mass/flavor eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \underbrace{\begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}}_{} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Cabibbo-**K**obayashi-**M**askawa mixing matrix

The quark mixing is the origin of the flavor number violation of the weak interaction.

Until the early 70s, only 3 quark flavor were known. The weak transition between quarks was described by a quark doublet:

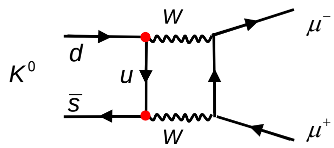
$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ \cos\theta_c \cdot d + \sin\theta_c \cdot s \end{pmatrix}$$

Mixing angle θ_c = Cabibbo-Angle

The mixing described automatically the suppression of $\Delta S=1$ transitions ($\sim \sin^2 \theta_c$)

Missing FCNC and GIM mechanism

FCNC in the 3 quark model: $K^0 \to \mu^+\mu^-$



$M \sim \sin\theta_c \cos\theta_c$

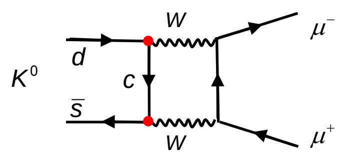
Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

$$\frac{BR(K_L \to \mu^+\mu^-)}{BR(K_L \to all)} = (7.2 \pm 0.5)\cdot 10^{-9}$$

Proposal by Glashow, Iliopoulos, Maiani, 1970:

There exists a fourth quark which builds together with the s quark a second doublet:

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\theta_c \cdot d + \cos\theta_c \cdot s \end{pmatrix}$$



$M \sim -\sin\theta_c \cos\theta_c$

→ Additional Feynman-Graph for $K^0 \to \mu\mu$ which compensates the first one:

Prediction of a fourth quark

Properties of the CKM-Matrix

$$\mathbf{V_{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Unitary N × N Matrix: → N^2 Parameters: N=3

$V_{CKM}V_{CKM}^{+} = 1$ — 9

	N=3
N^2 Parameters	9
N (N - 1)/2 Euler angles (rotation angles)	3
Remaining parameters are phases:	6
2N-1 are unmeasurable phase diff	5
Observable phases	1
$(N - 1)^2$ observable parameters	4

Parameterization of CKM Matrix: 3 Angles + 1 Phase

PDG choice where $c_{ij} = \cos\theta_{ij}\,,\quad s_{ij} = \sin\theta_{ij}$

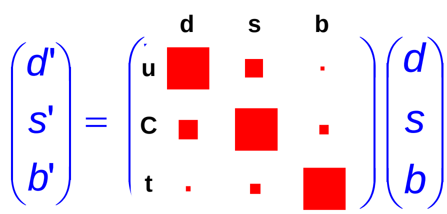
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Wolfenstein Parameterization $\lambda,\ A,\ \rho,\ \eta$

→ hierarchy expressed by orders of λ = **sinθ$_c$≈ 0.22**

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Modulus of the matrix elements: $|\mathbf{V}_{ij}|$



PDG 2006

$$\begin{pmatrix} 0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.0010} & (3.96^{+0.09}_{-0.09})\times 10^{-3} \\ 0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & (42.21^{+0.10}_{-0.80})\times 10^{-3} \\ (8.14^{+0.32}_{-0.64})\times 10^{-3} & (41.61^{+0.12}_{-0.78})\times 10^{-3} & 0.999100^{+0.000034}_{-0.000004} \end{pmatrix}$$

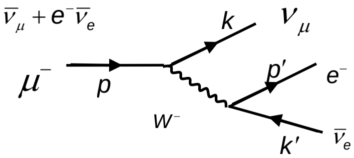
In leading order only the elements $\mathbf{V}_{ub}$ and $\mathbf{V}_{td}$ are complex.

3.6 Test of V-A structure in particle decays

a) Muon decay

Muon lifetime

$\mu^- \to \overline{\nu}_\mu + e^- \overline{\nu}_e$



$$M = \frac{G_F}{\sqrt{2}}\left[\overline{u}_\nu(k)\gamma_\alpha(1-\gamma^5)u_\mu(p)\right]\left[\overline{u}_e(p')\gamma^\alpha(1-\gamma^5)v_\nu(k')\right]$$

Analogous to the QED calculations of chapter III one finds after a lengthy calculation:

$$M = \frac{1}{2}\sum_{Spins}|M|^2 = 64G_F^2(k\cdot p')(k'\cdot p)$$

Using $d\Gamma = \frac{1}{2E}|M|^2 d\Phi$ one obtains the electron spectrum in the muon rest frame:

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3}m_\mu^2 E'^2\left(3-\frac{4E'}{m_\mu}\right)$$

with E' = electron energy

$$\frac{1}{\tau} = \Gamma = \int_0^{m_\mu/2}\frac{d\Gamma}{dE'}dE' = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

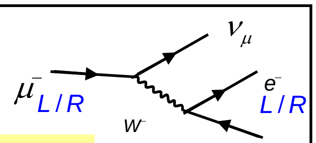
Measurement of the muon lifetime thus provides a determination of the fundamental coupling $\mathbf{G_F}$

$\tau_\mu = (2.19703 \pm 0.00004)\cdot 10^{-6}s$

$G_\mu = (1.16639 \pm 0.00001)\cdot 10^{-5}\text{GeV}^{-2}$

Fermi constant measured in muon decays is often called $\mathbf{G_\mu}$

Test of V-A structure in the muon decay



Most general form of the matrix element for

$$M = \frac{G_F}{\sqrt{2}}\cdot \sum_{\substack{i=S,V,T\\ \lambda=L,R\\ \lambda'=\begin{cases}\pm\lambda \text{ for S,V}\\ -\lambda \text{ for T}\end{cases}}} g^i_{\lambda'\lambda}\left(\overline{u}_{\lambda'}(e)\Gamma^i v_{\lambda'_i}(\nu_e)\right)\left(\overline{u}_{\lambda_i}(\nu_\mu)\Gamma^i u_\lambda(\mu)\right)$$

L/R … L/R

Chirality λ_i, λ'_i determined by Γ_i

$$\lambda'_i = \begin{cases}\lambda' & i = S,T\\ -\lambda' & i = V\end{cases}$$

$$\lambda_i = \begin{cases}\lambda & i = V\\ -\lambda & i = S,T\end{cases}$$

Possible current-current couplings:

i \ λ'λ	RR	RL	LR	LL
S	x	x	x	x
V	x	x	x	x
T		x	x	

There are in general 10 complex amplitudes $g^i_{\lambda'\lambda}$

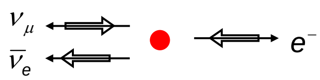
Pure V-A coupling: $g^V_{LL} = 1$

all other $g^i_{\lambda'\lambda} = 0$

Experimental determination of $g^i_{\lambda'\lambda}$ from energy spectra and spin correlation of the decay electrons from the polarized muons

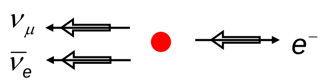
Idea:

V-A at μ vertex $\Rightarrow$ LH ν_μ



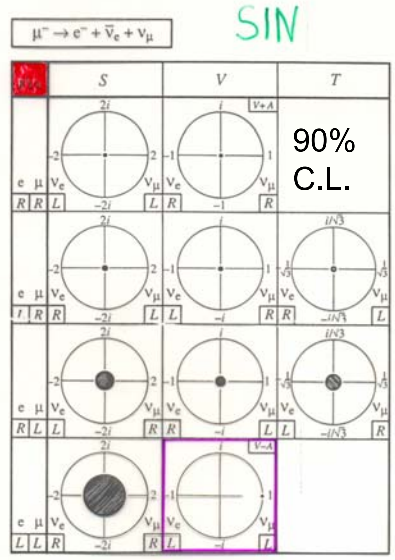
Configuration w/ max e- momentum possible

V+A at μ vertex $\Rightarrow$ RH ν_μ



Due to angular momentum conservation not possible

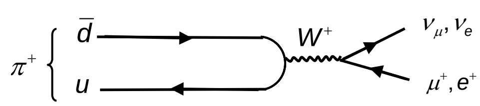
Couplings in muon decay



V-A theory is confirmed

b) Pion decay

$$\pi^+ \to \mu^+ \nu_\mu$$
$$\pi^+ \to e^+ \nu_e$$



Naïve expectation:

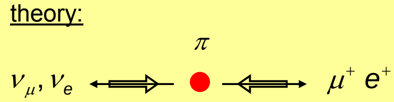
Assuming the same decay dynamics the decay rate to e^+ should be much larger than to μ^+ as the phase space is much bigger.

Measurement: (PDG)

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \cdot 10^{-4}$$

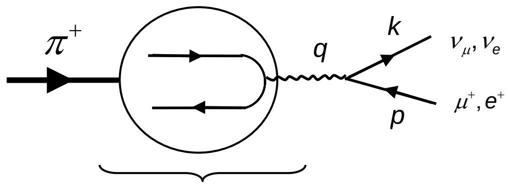
Large suppression due to a dynamic effect.

Qualitative explanation within V-A theory:



Angular momentum conservation forces the lepton into the "wrong" helicity state: suppressed ~ β=v/c i.e. for vanishing lepton masses the pion could not decay into leptons.

Determination of decay rates:



Quarks in pion are bound

$$M = \frac{G_F}{\sqrt{2}} \cdot (\ \pi\ )_\mu \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right] \Rightarrow M = \frac{G_F}{\sqrt{2}} \cdot \left(p_\mu + k_\mu\right) \cdot f_\pi \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

As the pion spin s_π=0, q is the only relevant 4-vector:

$$q^\mu = p^\mu + k^\mu$$

$$(\ \pi\ )_\mu = q_\mu \underbrace{f_\pi(q^2)}$$

Pion form factor:

$$q^2 = m_\pi^2 : \quad f_\pi(q^2) = f_\pi(m_\pi^2) = f_\pi$$

$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_\mu^2 (1 - \frac{m_\mu^2}{m_\pi^2})$$

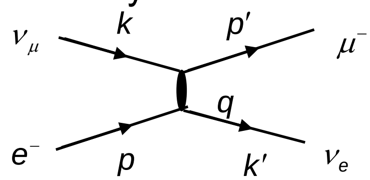
$$\Gamma(\pi^+ \to e^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_e^2 (1 - \frac{m_e^2}{m_\pi^2})$$

$$\frac{\Gamma(\pi^+ \to e^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) = 1.275 \cdot 10^{-4}$$

The prediction of the V-A theory is confirmed by the experimental observation. The pion decay rates, although in agreement with the V-A theory, are not a proof of the V-A coupling. V or A coupling together with LH neutrinos would result to the same rates.

3.7 Neutrino scattering in V-A theory

a) Neutrino-electron scattering



$$\nu_\mu e^- \rightarrow \mu^- \nu_e$$

$\nu_e \qquad e^-$

$$M = \frac{G_F}{\sqrt{2}}\left[\bar{u}_\nu(k')\gamma_\alpha(1-\gamma^5)u_e(p)\right]\left[\bar{u}_\mu(p')\gamma^\alpha(1-\gamma^5)u_\nu(k)\right]$$

$$\overline{|M|^2} = \frac{1}{2}\sum_{Spins}|M|^2 = \ldots = 64G_F^2(k\cdot p)(k'\cdot p') = 16G_F^2\cdot s^2$$

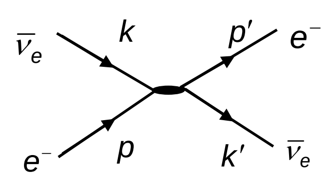
Limit $m_e \approx m_\mu \approx 0$ $\qquad s=(k+p)^2 = 2kp = 2k'p'$

Using the phase space factor of chapter II:

$$\frac{d\sigma}{d\Omega}(\nu_\mu e^-) = \frac{1}{64\pi^2 s}\overline{|M|^2} = \frac{G_F^2 s}{4\pi^2}$$

$$\sigma(\nu_\mu e^-) = \frac{G_F^2 s}{\pi}$$

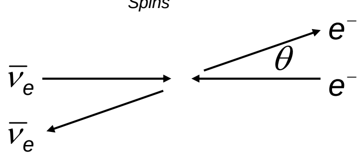
b) Anti-Neutrino-electron scattering (V-A)



$$\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$$

Crossing: s ⇔ t (u)

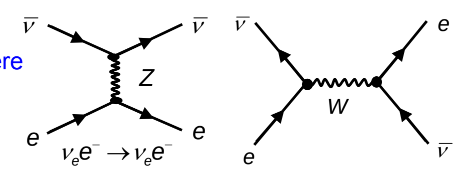
$$\overline{|M|^2} = \frac{1}{2}\sum_{Spins}|M|^2 = 16G_F^2\cdot t^2 = 4G_F^2\cdot s^2(1-\cos\theta)^2$$



$$\frac{d\sigma}{d\Omega}(\bar{\nu}e^-) = \frac{G_F^2 s}{16\pi^2}(1-\cos\theta)^2$$

$$\sigma(\bar{\nu}e^-) = \frac{G_F^2 s}{3\pi}$$

Beside the charged current contribution there is of course a neutral current contribution which is ignored in this reasoning.



Result of V-A structure

For the charged current (CC) contribution to the (anti) neutrino electron scattering one finds

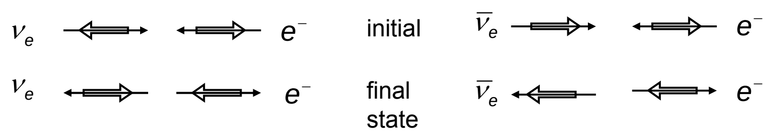
$$\frac{\sigma^{cc}_{\nu e}}{\sigma^{cc}_{\bar{\nu} e}} = 3$$

Different angular distribution of (anti) neutrino scattering can be understood from a helicity discussion

$$\frac{d\sigma}{d\Omega}(\nu_e e^-) = \frac{G_F^2 s}{4\pi^2}$$

$$\frac{d\sigma}{d\Omega}(\bar{\nu} e^-) = \frac{G_F^2 s}{16\pi^2}(1-\cos\theta)^2$$

$\nu_e e^- \to \nu_e e^-$ | $\bar{\nu}_e e^- \to \bar{\nu}_e e^-$

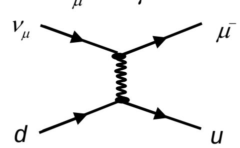


Allowed → isotropic

Forbidden (angular mom.) → (1-cosθ) suppression

c) Anti) neutrino-quark scattering

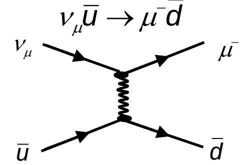
$\nu_\mu d \to \mu^- u$



$$\frac{d\sigma}{d\Omega}(\nu_\mu d) = \frac{G_F^2 s}{4\pi^2}$$

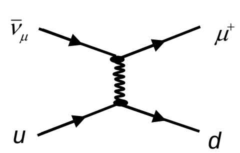
$$\sigma(\nu_\mu d) = \frac{G_F^2 s}{\pi}$$

$\nu_\mu \bar{u} \to \mu^- \bar{d}$



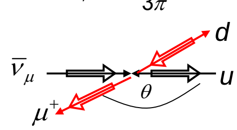
$$\frac{d\sigma}{d\Omega}(\nu_\mu \bar{u}) = \frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u)$$

$\bar{\nu}_\mu u \to \mu^+ d$



$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u) = \frac{G_F^2 s}{16\pi^2}(1+\cos\theta)^2$$

$$\sigma(\bar{\nu}_\mu u) = \frac{G_F^2 s}{3\pi}$$



$\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}$

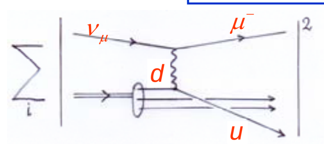
$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu \bar{d}) = \frac{d\sigma}{d\Omega}(\nu_\mu d)$$

Neutrinos only interact w/ d and anti-u quarks
Anti-neutrinos only interact w/ u and anti-d quarks

d) Neutrino-nucleon (iso-scalar) scattering

QPM: $x = \frac{Q^2}{2M\nu}$ $y = \frac{\nu}{E}$ $\nu = E - E'$

$$\frac{d\sigma(\nu_\mu N \to \mu^- X)}{dy} = \sum_i \left| \quad \right|^2$$



$$\frac{d\sigma(\nu d)}{dy} = \frac{G_F^2 xs}{\pi}$$

$$\frac{d\sigma(\bar{\nu} u)}{dy} = \frac{G_F^2 xs}{\pi}(1-y)^2$$

$$\frac{d^2\sigma(\nu N)}{dxdy} = \sum_i f_i(x) \left(\frac{d\sigma_i(\nu q_i)}{dy} \right)_{\hat{s}=xs}$$

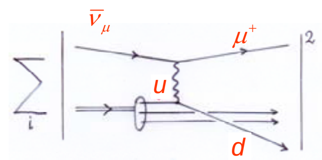
$$\frac{d^2\sigma(\nu N)}{dxdy} = \frac{G_F^2 xs}{2\pi} \cdot \left[Q(x) + \bar{Q}(x)(1-y)^2 \right]$$

Unter Vernachlässig-ung der Massen gilt:

$$y = \frac{1-\cos\theta}{2}$$

$$1 - y \approx \frac{1}{2}(1+\cos\theta)$$

$$\frac{d\sigma(\bar{\nu}_\mu N \to \mu^+ X)}{dy} = \sum_i \left| \quad \right|^2$$



$$\frac{d^2\sigma(\bar{\nu} N)}{dxdy} = \frac{G_F^2 xs}{2\pi} \cdot \left[\bar{Q}(x) + Q(x)(1-y)^2 \right]$$

Total cross section after integration over x and y (0…1):

$$\sigma(\nu N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[Q_I + \frac{1}{3}\bar{Q}_I \right]$$

$$\sigma(\bar{\nu} N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[\bar{Q}_I + \frac{1}{3} Q_I \right]$$

with $Q_I = \int xQ(x)dx$

$$R = \frac{\sigma_{\bar{\nu}N}}{\sigma_{\nu N}} = \frac{1 + 3\bar{Q}_I/Q_I}{3 + \bar{Q}_I/Q_I}$$

If nucleon consists only of valence quarks ($\bar{Q}$=0): R=1/3 , because of V-A structure

Measurement: $R = \frac{0.34}{0.67}$ $\Rightarrow$ $\bar{Q}_I/Q_I \approx 0.15$

$\Rightarrow$ There are sea quarks !

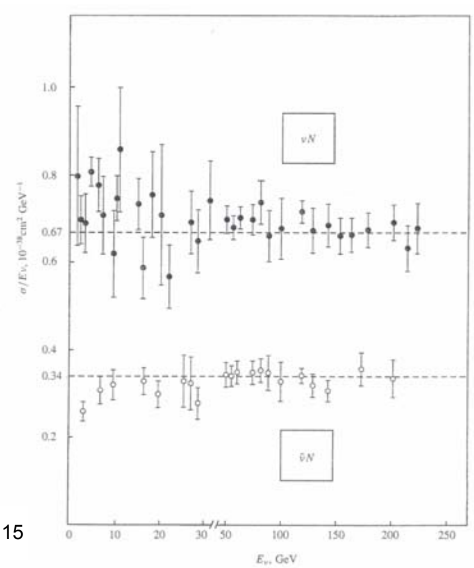


Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio σ/E_ν is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).

3.8 Problems with V-A theory

- Cross section for $\nu e^- \to e^- \nu_e$ in 4-fermion ansatz: $\sigma(\nu e^-) = \frac{G_F^2 s}{\pi}$
  i.e. cross section goes to infinity if s→∞: violates unitarity
- Lee and Wu (1965) introduced a massive exchange boson. Effect of propagator:

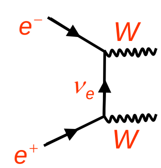
$$\frac{G_F}{\sqrt{2}} \mapsto \frac{G_F}{\sqrt{2}} \frac{1}{1-q^2/M_W^2} \qquad \sigma(\nu e^-) \mapsto \frac{G_F^2 M_w^2}{\pi}$$

Energy behavior of cross section becomes better but still violates unitarity at very high s.

- In addition there are boson production processes of the kind:

  In the V-A theory they also violate unitarity !!

W pair production:



Violates unitarity

We need a new theory: Standard Model