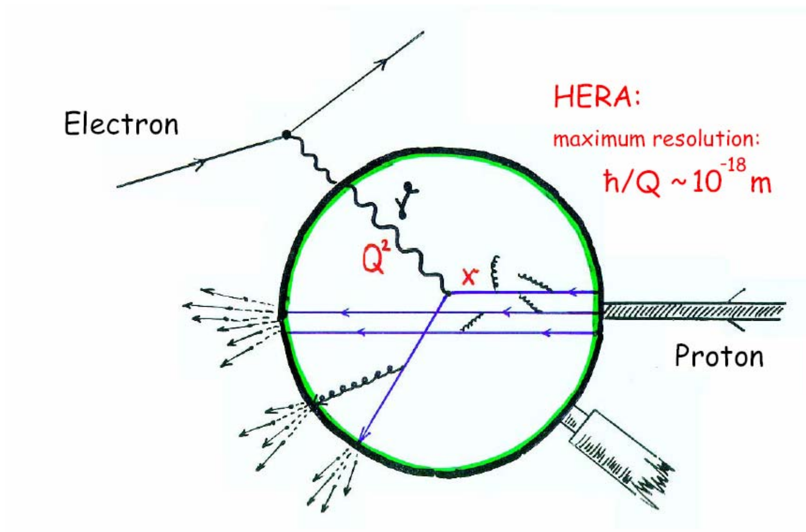
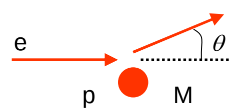
3. Study of QCD in deep inelastic scattering (DIS)



Courtesy: H.C. Schultz-Coulon

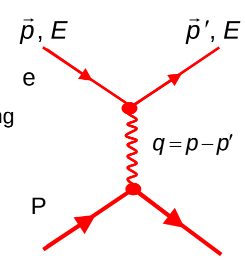
3.1 Elastic electron-proton scattering



General form of differential cross section

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4EE'\sin^4\frac{\theta}{2}}}_{\text{Rutherford}} \{\ldots\ldots\}$$

$\{\ldots\ldots\}$ ← non pointlike scattering partners w/ spin



$$-q^2 = 4EE'\sin^4\frac{\theta}{2}$$

Spin ½ electron +

Pointlike target w/o spin
Mott scattering

$$\{\ldots\ldots\}^{elastic}_{Mott} = \left(\cos^2\frac{\theta}{2}\right)$$

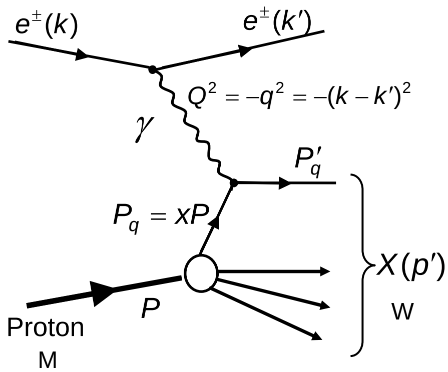
Pointlike target w/ spin
and mass M

$$\{\ldots\ldots\}^{elastic}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

Extended proton w/ spin

$$\{\ldots\ldots\}^{elastic}_{ep\to ep} = \left(\frac{G_E^2 + \tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2 \sin^2\frac{\theta}{2}\right) \quad \text{mit } \tau = \frac{Q^2}{4M^2}$$

3.2 DIS in the quark parton model (QPM)



- Elastic scattering: W = M

  ⇒ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W ≠ M

  ⇒scattering described by 2 independent variables

$$(E,\nu),\ (Q^2,x),\ (x,y),\ \ldots$$

x = fractional momentum of struck quark

y = Pq/Pk = fractional energy transfer in proton rest frame

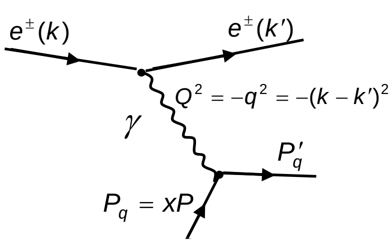
ν = E - E′ = energy transfer in lab

$$Q^2 = sxy \qquad \text{s = CMS energy}$$

$$x = \frac{Q^2}{2M\nu} \qquad \text{(Bjorken x)}$$

Cross section in quark parton model (QPM)

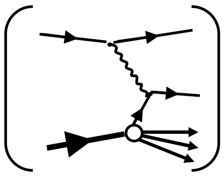
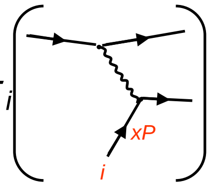
Elastic scattering on single quark



$$\{\ldots\ldots\}^{elastic}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

charge ↓

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot e_i^2\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

σ  $= \sum_i q_i(x)\sigma_i$ 

Parton density $q_i(x)dx$: Probability to find parton i in momentum interval [x, x+dx]

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\sum_i\int_0^1 e_i^2\cdot q_i(\xi)\cdot\delta(x-\xi)d\xi\left(\cos^2\frac{\theta}{2}+\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

Structure functions

$$F_2(x) = x\sum_i\int_0^1 e_i^2 q_i(\xi)\cdot\delta(x-\xi)d\xi = x\sum_i e_i^2 q_i(x)$$

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\left(\frac{F_2(x)}{x}\cos^2\frac{\theta}{2}+2F_1(x)\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

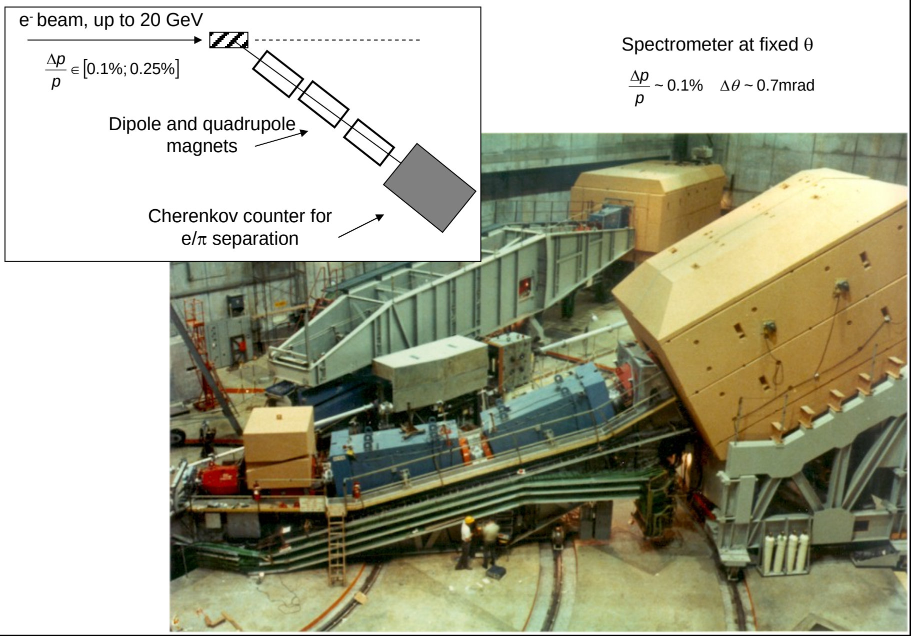
Kinematical relations

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4x}\right)\cdot\left((1-y)F_2(x)+xy^2F_1(x)\right)$$

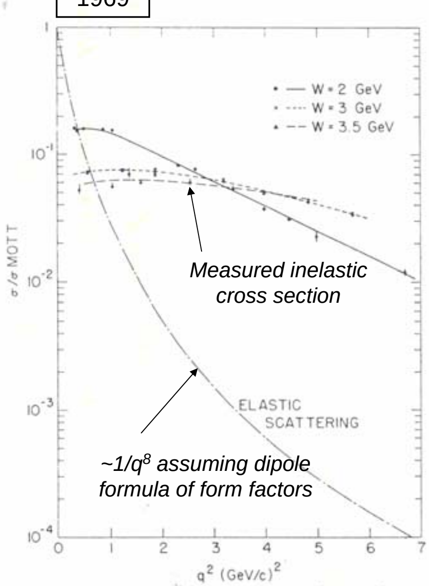
Deep inelastic electron-proton scattering:

- Free partons: $F_2=F_2(x)$ ⇔ “scaling”
- Spin ½ partons: $2xF_1(x) = F_2(x)$ (Callan-Gross relation)

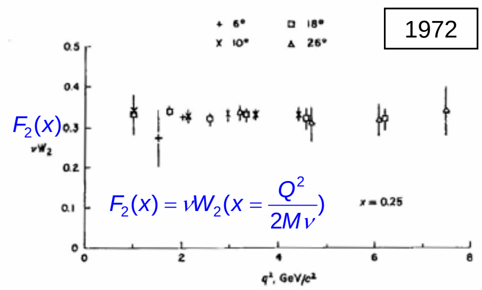
SLAC/MIT Experiment (1969)



1969



1972



Structure function F_2 (=νW_2) depends only on the dimensionless variable x:

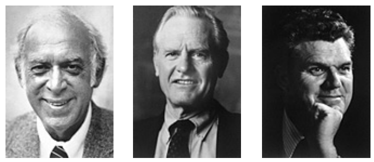
$$x = \frac{Q^2}{2M\nu}$$

→ **Scale invariance: "scaling"**

Indicates elastic scattering at point-like free constituents of the proton: partons

U. Uwer

The Nobel Prize in Physics 1990

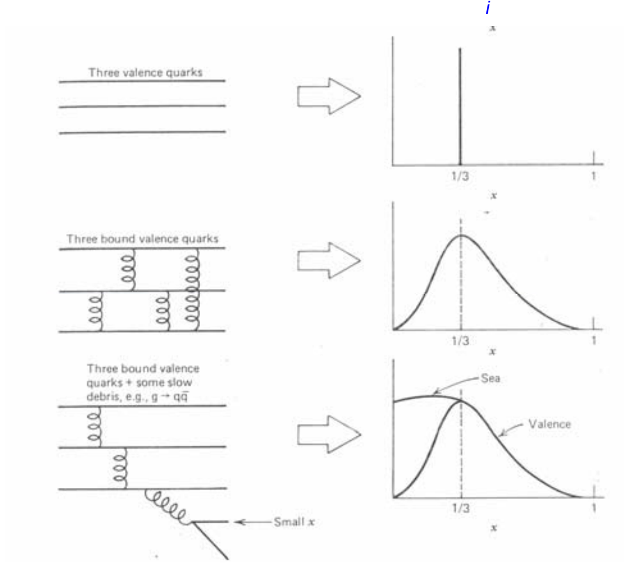


Jerome I. Friedman	**Henry W. Kendall**	**Richard E. Taylor**

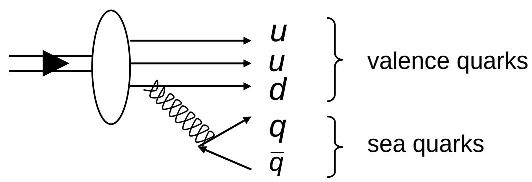
"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

3.3 Structure functions – Nucleon structure

Proton model $$F_2(x) = x\sum_i e_i^2 q_i(x)$$



Sea and valence quarks in the proton



Quark composition of the proton

$$u_v + u_v + d_v + \underbrace{(u_s + \bar{u}_s) + (d_s + \bar{d}_s) + (s_s + \bar{s}_s)}$$

Sea: Heavy quark contribution strongly suppressed

$$\frac{F_2^{ep}(x)}{x} = \sum_i e_i^2 \cdot q_i(x)$$

$$= \frac{4}{9}(u^p(x) + \bar{u}^p(x)) + \frac{1}{9}(d^p(x) + \bar{d}^p(x)) + \frac{1}{9}(s^p(x) + \bar{s}^p(x))$$

Quark composition of the neutron

$$\frac{F_2^{en}(x)}{x} = \sum_i e_i^2 \cdot q_i(x)$$

$$= \frac{4}{9}(u^n(x) + \bar{u}^n(x)) + \frac{1}{9}(d^n(x) + \bar{d}^n(x)) + \frac{1}{9}(s^n(x) + \bar{s}^n(x))$$

$$\frac{F_2^{en}(x)}{x} = \frac{4}{9}(d(x) + \bar{d}(x)) + \frac{1}{9}\left(u(x) + \bar{u}(x) + s(x) + \bar{s}(x)\right)$$

Iso-spin symmetry

$$u^n(x) = d^p(x) = d(x)$$
$$d^n(x) = u^p(x) = u(x)$$
$$s^n(x) = s^p(x) = s(x)$$
$$\bar{q}^n(x) = \bar{q}^p(x) = \bar{q}(x)$$

In total 6 unknown quark distributions ⬅

Sum rules

$$q^i(x) = q_v^i(x) + q_s^i(x)$$
$$\bar{q}^i(x) = \bar{q}_s^i(x)$$

$$\int_0^1 u(x) - \bar{u}(x)dx = \int_0^1 u_v(x)dx = 2$$

$$\int_0^1 d(x) - \bar{d}(x)dx = \int_0^1 d_v(x)dx = 1$$

valence

$$\int_0^1 (q_s(x) - \bar{q}_s(x))dx = 0$$

sea

Sum of quark momentum

Scattering at an iso-scalar target N: #p = #n (e.g. C, Ca)

$$F_2^{eN} = \frac{1}{2}\left[F_2^{ep} + F_2^{en}\right] = \frac{5}{18} x \cdot [u + \bar{u} + d + \bar{d}] + \frac{1}{9} x \cdot [s + \bar{s}]$$

$$\approx \underbrace{\frac{5}{18} x \cdot [u + \bar{u} + d + \bar{d}]}_{\text{Small s quark distribution neglected}} = \frac{5}{18}[\text{Sum of all quark momenta}]$$

Naively one expects: $\frac{18}{5} \cdot \int_0^1 F_2^{eN}(x)dx \approx 1$

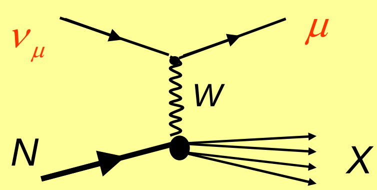
Experimental observation: $\frac{18}{5} \cdot \int_0^1 F_2^{eN}(x)dx \approx 0.5$

- probed quarks and anti-quarks carry only 50% of nucleon momentum
- Remaining momentum carried by gluons (see later)

3.4 Neutrino nucleon scattering

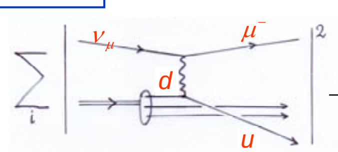
$$\nu_\mu N \rightarrow \mu^\pm X$$

- More information on quark distribution
- Separation between quarks / anti-quarks



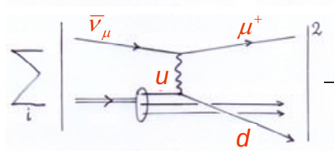
QPM: $x = \frac{Q^2}{2M\nu}$ $y = \frac{\nu}{E}$ $\nu = E - E'$

$$\frac{d\sigma(\nu_\mu N \rightarrow \mu^- X)}{dy} =$$



$\rightarrow F_i^{\nu p}(x)$ and $F_i^{\nu n}(x)$

$$\frac{d\sigma(\bar{\nu}_\mu N \rightarrow \mu^+ X)}{dy} =$$



$\rightarrow F_i^{\bar{\nu} p}(x)$ and $F_i^{\bar{\nu} n}(x)$

Structure functions for neutrino scattering

$$F_i^{\nu n} = F_i^{\bar\nu p}, \quad F_i^{\nu p} = F_i^{\bar\nu n}$$

Equal because of Charge symmetry

$$F_i^{\nu N} = \frac{1}{2}(F_i^{\nu p} + F_i^{\nu n}) = \frac{1}{2}(F_i^{\bar\nu n} + F_i^{\bar\nu p}) = F_i^{\bar\nu N} \quad \text{for i = 1,2}$$

$$F_3^{\bar\nu N} = -F_3^{\nu N}$$

Additional structure function to account for parity violation

Double differential cross section: Scattering at iso-scalar target

$$\frac{d^2\sigma(\nu N, \bar\nu N)}{dxdy} = 2ME\frac{G_F^2}{2\pi}\left[(1-y)F_2^{\nu N}(x) + \frac{y^2}{2}2xF_1^{\nu N}(x) \pm y(1-\frac{y}{2})xF_3^{\nu N}(x)\right]$$

$$\frac{4\pi\alpha^2}{Q^4} \mapsto \frac{G_F^2}{2\pi}$$

$\nu = +$

$\bar\nu = -$

$y(1-\frac{y}{2})xF_3^{\nu N}(x)$: to account for parity violation

Structure functions in QPM

$$F_1^{\nu N} = \frac{1}{2x}F_2^{\nu N}$$

$$F_2^{\nu p} = 2x[d + \bar u] \qquad xF_3^{\nu p} = 2x[d - \bar u]$$

$$F_2^{\nu n} = 2x[d^n + \bar u^n] = 2x[u + \bar d] \qquad xF_3^{\nu n} = 2x[d^n - \bar u^n] = 2x[u - \bar d]$$

Iso-scalar target:

$$F_2^{\nu N} = x[u + \bar u + d + \bar d] \qquad xF_3^{\nu N} = x[(u + d) - (\bar u + \bar d)]$$

$$F_2^{\nu N} = x[Q(x) + \bar Q(x)] \qquad xF_3^{\nu N} = x[Q(x) - \bar Q(x)]$$

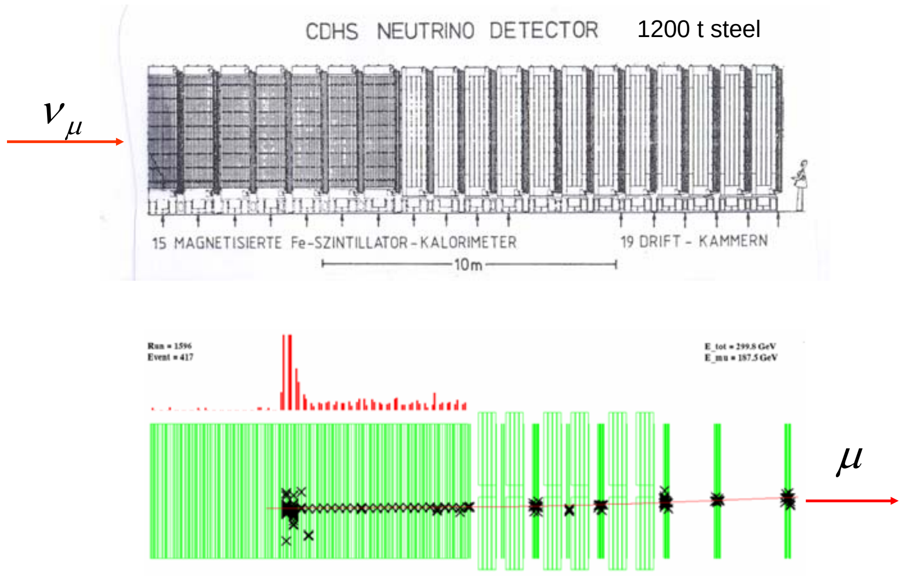
$F_2^{\nu N}$: Measures sum of quarks and anti-quarks

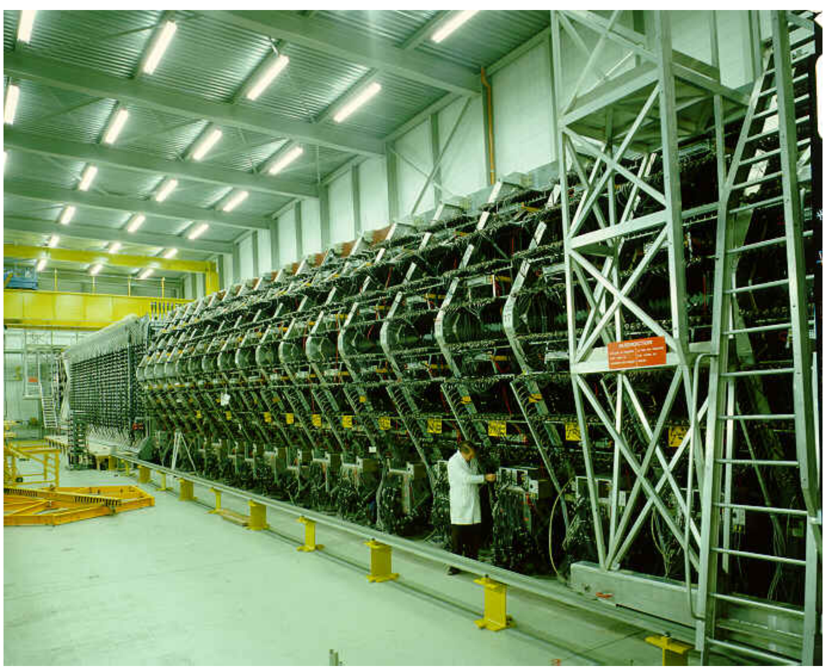
$xF_3^{\nu N}$: Measures valence quarks

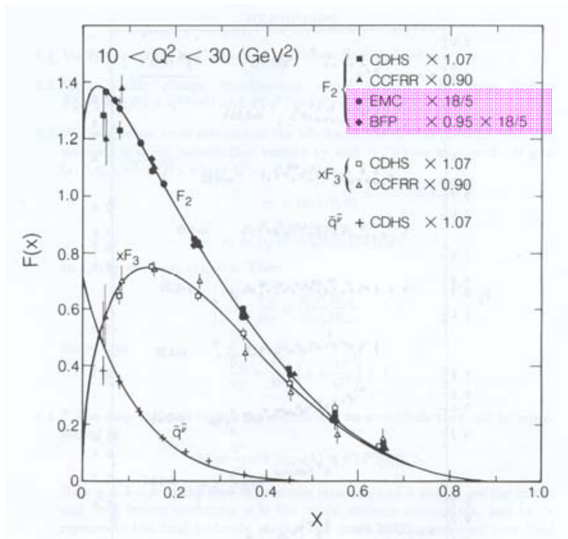
Measurement: $F_2^{\nu N} + xF_3^{\nu N} = 2xQ(x)$ ➡ Sea and valence quarks

$F_2^{\nu N} - xF_3^{\nu N} = 2x\bar Q(x)$ ➡ Sea quarks

CHDS (CERN-Dortmund-Heidelberg-Saclay) Experiment







Parton distribution in eN and νN scattering

Question:

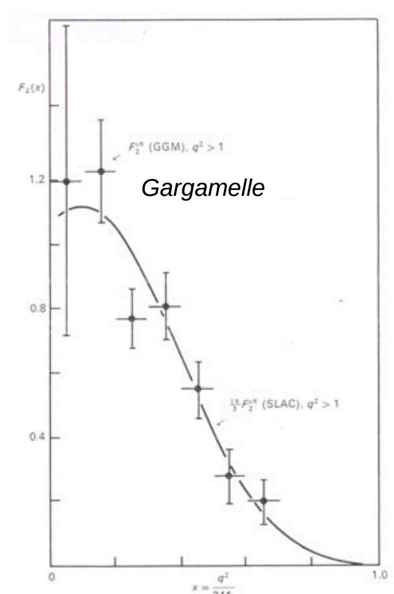
Do the parton distribution seen in electro-magnetic (F_2^{eN}) and in weak interaction (F_2^{eN}) agree ?

$$\frac{F_2^{\nu N}(x)}{F_2^{eN}(x)} = \frac{x[Q(x)+\overline{Q}(x)]}{\frac{5}{18}\cdot x[Q(x)+\overline{Q}(x)]} = \frac{18}{5}$$

Factor from fractional charge

Answer:

- e.m. and weak quark structure is the same
- Factor 18/5 → fractional quark charge



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Summary: eN and νN scattering (N=iso-scalar target)

eN scattering

$$\frac{d^2\sigma^{eN}}{dxdy} = \frac{2\pi\alpha^2}{Q^4} xs\left[1+(1-y)^2\right]\cdot\frac{5}{18}\left[Q(x)+\overline{Q}(x)\right]$$

$$F_2^{eN}(x) = \frac{5}{18} x\left[Q(x)+\overline{Q}(x)\right]$$

$\nu N + \overline{\nu} N$ scattering

$$\frac{d^2\sigma^{\nu N}}{dxdy} = \frac{G_F^2}{2\pi} xs\left[Q(x)+(1-y)^2\cdot\overline{Q}(x)\right]$$

$$\frac{d^2\sigma^{\overline{\nu} N}}{dxdy} = \frac{G_F^2}{2\pi} xs\left[(1-y)^2\cdot Q(x)+\overline{Q}(x)\right]$$

$$F_2^{\nu N}(x) = x\left[Q(x)+\overline{Q}(x)\right] \quad F_3^{\nu N}(x) = x\left[Q(x)-\overline{Q}(x)\right]$$

$$F_2^{\overline{\nu} N}(x) = x\left[Q(x)+\overline{Q}(x)\right] \quad F_3^{\overline{\nu} N}(x) = x\left[\overline{Q}(x)-Q(x)\right]$$