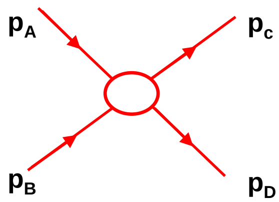
4. Cross section and phase space

$$A + B \rightarrow C + D$$



$$W_{fi} = \frac{(2\pi)^4}{V^4} \delta^4(p_A + p_B - p_C - p_D) \cdot |M_{fi}|^2$$

$$\text{cross section} = \frac{W_{fi}}{\text{(initial flux)}} \text{(number of final states)}$$

Differential cross section:

$$d\sigma = \frac{W_{fi}}{F_i} dn_f$$

dn_f number of final states
F_i incident particle flux of A and B

4.1 Number of final states

Quantum mechanics restricts the number of final states dn_f of a single particle in a volume V with momentum $\in [\vec{p}, \vec{p} + d\vec{p}]$

$$dn_f = \frac{Vd^3p}{2E\,h^3} = \frac{Vd^3p}{2E\,\hbar^3 (2\pi)^3} \overset{\hbar = 1}{=} \frac{Vd^3p}{2E\,(2\pi)^3}$$

Factor 2E is result of normalization of the wave function: 2E particles / V

For particle C and D scattered into momentum elements d^3p_C and d^3p_D

$$dn_f(A + B \rightarrow C + D) = \frac{Vd^3p_C}{2E_C(2\pi)^3} \frac{Vd^3p_D}{2E_D(2\pi)^3}$$

4.2 Incident particle flux F_i

Choose system where particle B is at rest

F_i = (flux density A) × (density B)

$A \xrightarrow{\vec{v}_A} \bullet\ B$

$$F_i = |\vec{v}_A| \frac{2E_A}{V} \cdot \frac{2E_B}{V}$$

with $\vec{v}_A = \frac{\vec{p}_A}{E_A}$

<u>Other cases:</u>

$A \xrightarrow{\vec{v}_A} B \xrightarrow{\vec{v}_B}$

$$F = \frac{1}{V^2}|\vec{v}_A - \vec{v}_B| \cdot 2E_A \cdot 2E_B = \frac{4}{V^2}\left((p_A p_B)^2 - m_A^2 m_B^2\right)^{1/2}$$

$A \xrightarrow{\vec{p}_A} \quad \xleftarrow{\vec{p}_B} B$

$\vec{p}_A = -\vec{p}_B = \vec{p}_i$

$$F = \frac{4}{V^2}|\vec{p}_i| \cdot (E_A + E_B) = \frac{4}{V^2}|\vec{p}_i|\sqrt{s}$$

4.3 Lorentz invariant phase space factor

Putting everything together $\quad d\sigma = \frac{W_{fi}}{F_i} dn_f$

$$d\sigma = \frac{(2\pi)^4}{V^4}\delta^4(p_A + p_B - p_C - p_D) \cdot |M_{fi}|^2 \frac{V^2}{|\vec{v}_A| 2E_A 2E_B} \cdot \frac{V d^3 p_c}{2E_C (2\pi)^3} \cdot \frac{V d^3 p_D}{2E_D (2\pi)^3}$$

$$= \frac{|M_{fi}|^2}{|\vec{v}_A| 2E_A 2E_B} \cdot (2\pi)^4 \delta^4(p_A + p_B - p_C - p_D) \cdot \frac{d^3 p_c}{2E_C (2\pi)^3} \cdot \frac{d^3 p_D}{2E_D (2\pi)^3}$$

Particle flux $\boldsymbol{F_i}$ — Lorentz invariant phase space factor $\boldsymbol{d\Phi_n}$

Remark: volume V drops out !

Phase space factor **dΦ** for two-particles final-state

CM System: $\vec{p}_A = -\vec{p}_B \quad \vec{p}_C = -\vec{p}_D$

$$d\Phi \xrightarrow{\int} \int d\Phi = \frac{1}{4\pi^2}\int \delta^3(\vec{p}_C + \vec{p}_D)\delta(E_A + E_B - E_C - E_D)\frac{d^3p_C}{2E_C}\frac{d^3p_D}{2E_D}$$

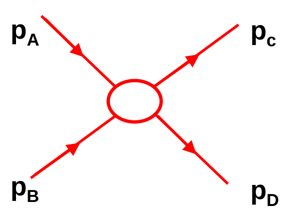
$$\int d\Phi = \frac{1}{4\pi^2}\int \frac{|\vec{p}_f|}{4\sqrt{s}}\, d\Omega$$

$\vec{p}_f = \vec{p}_C = -\vec{p}_D$

$s = (E_A + E_B)^2$

4.4 Differential cross section …putting everything together

$$A + B \rightarrow C + D$$

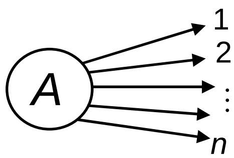


$$d\sigma = \frac{|M_{fi}|^2}{F}d\Phi = \frac{1}{64\pi^2}\cdot\frac{1}{s}\cdot\frac{|\vec{p}_f|}{|\vec{p}_i|}\cdot|M_{fi}|^2 d\Omega$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\cdot\frac{1}{s}\cdot\frac{|\vec{p}_f|}{|\vec{p}_i|}\cdot|M_{fi}|^2$$

5. Decay width, lifetime and Dalitz plots

5.1 Decay width



$$\tau = \frac{1}{\Gamma} \qquad \Gamma = \sum \Gamma_i$$

Diff. decay width:

$$d\Gamma_i(A \to 1 + 2 + ... + n) = \frac{W_{fi}}{n_A} dn_f$$

$$d\Gamma_i = \frac{|M_{fi}|^2}{2E_A} \cdot (2\pi)^4 \delta^4 (p_A - p_1 - p_2 - ... - p_n) \cdot \frac{d^3p_1}{2E_1(2\pi)^3} \cdot \frac{d^3p_2}{2E_2(2\pi)^3} \cdot ... \cdot \frac{d^3p_n}{2E_n(2\pi)^3}$$

Two-body decay:

$$A \to 1 + 2$$

$$d\Gamma_i(A \to 1 + 2) = \frac{|M_{fi}|^2}{2E_A} \cdot d\Phi = \frac{|M_{fi}|^2}{2E_A} \frac{1}{4\pi^2} \frac{|\vec{p}_f|}{4\sqrt{s}} d\Omega$$

wie oben

$$\sqrt{s} = E_A = m_A$$

$$d\Gamma_i(A \to 1 + 2) = \frac{|\vec{p}_f|}{32\pi^2 m_A^2} \int |M_{fi}|^2 d\Omega$$

Three-body decay:

$$A \to 1 + 2 + 3$$

↑ scalar

$$\int d\Phi = \frac{1}{32\pi^3} \int dE_1 dE_2$$

← flat in E_1 and E_2

$$d\Gamma_i(E_1, E_2) = \frac{1}{64\pi^3} \frac{1}{2m_A} |M_{fi}|^2 dE_1 dE_2$$

<u>Remark:</u> Instead of variables E_1 and E_2 one can use variables m_{12}^2 and m_{23}^2 = invariant mass of pairs (i,j)

$$m_{ij}^2 = (p_i + p_j)^2$$

$$dE_1 dE_1 = C \cdot dm_{12}^2 dm_{23}^2$$

If phase space is flat in E_i then it is also flat in m_{ij}

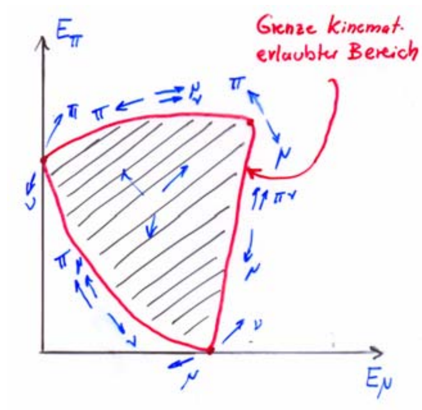
$$d\Gamma_i(m_{12}, m_{23}) = \frac{1}{256\pi^3} \frac{1}{m_A^3} |M_{fi}|^2 dm_{12} dm_{23}$$

Experimental method to explore behavior of $\boldsymbol{M_{fi}}$: Dalitz Analysis

5.2 Dalitz Plots

Method:

Put every measured decay into a 2-dim,. (E_1,E_2) distribution. A flat distribution over the allowed region corresponds to a “flat matrix element”. Structures in the distribution point to a varying matrix elem.



Ex.: $K^0 \to \pi^\pm \mu^\mp \nu$

Dalitz Plot: $\pi^+ \overline{K}^0 p$ final state at 3 GeV

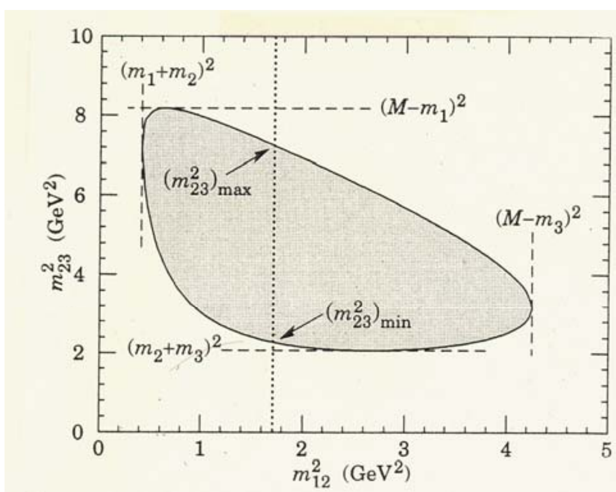


Figure 34.3: Dalitz plot for a three-body final state. In this example, the state is $\pi^+\overline{K}^0p$ at 3 GeV. Four-momentum conservation restricts events to the shaded region.

Dalitz Plot: Example

$K^- + p \to \pi^+ \pi^- \Lambda$

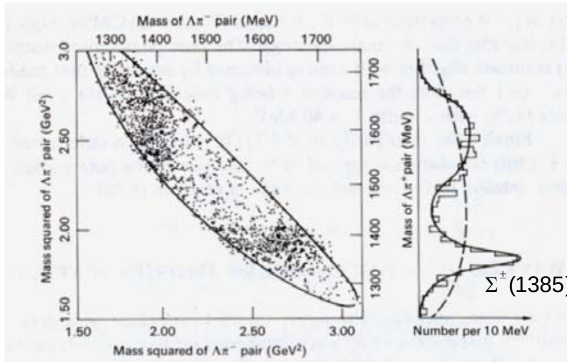
$\quad\quad\quad\quad\quad\quad\quad\quad \hookrightarrow p\pi$

$\Rightarrow$ 2 bands at $\Lambda\pi^\pm$ inv. masses of ~1385 MeV

<u>Explanation:</u>

$\Lambda\pi^\pm$ form an intermediate resonance state:

$$\Sigma^\pm(1385) \to \Lambda\pi^\pm$$



4.8 Dalitz plot of the $\Lambda\pi^+\pi^-$ events from reaction (4.36), as measured by Shafer , for 1.22-GeV/c incident momentum. The effective $\Lambda\pi^+$ mass spectrum is shown at ished curve is that expected for a phase-space distribution (ordinate equal to the inter within the Dalitz-plot boundary), while the full curve corresponds to a Breit-W nce expression fitted to the $\Lambda\pi^+$ and $\Lambda\pi^-$ systems.

Experiment: Kaon beam on a liquid H_2 bubble chamber