Neutrino oscillations and masses

1. Neutrino oscillations
2. Atmospheric neutrinos
3. Solar neutrinos, MSW effect
4. Reactor neutrinos
5. Accelerator neutrinos
6. Neutrino masses, double beta decay

1. Neutrino Oscillations

For massive neutrinos, one can introduce in analogy to the quark mixing a mixing matrix describing the relation between mass and flavor states:

$$\begin{pmatrix} \nu_e \\ \nu_\mu \\ \nu_\tau \end{pmatrix} = \begin{pmatrix} U_{e1} & U_{e2} & U_{e3} \\ U_{\mu 1} & U_{\mu 3} & U_{\mu 3} \\ U_{\tau 1} & U_{\tau 2} & U_{\tau 3} \end{pmatrix} \cdot \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix}$$

***Pontecorvo-Maki-Nakagawa-Sakata* matrix**

$$\underbrace{\nu_e = U_{e1}\nu_1 + U_{e2}\nu_2 + U_{e3}\nu_3}$$

Constant for massless ν:
mixing is question of convention

Massive neutrinos develop differently in time.

$$|\nu_i(t)\rangle = |\nu_i(0)\rangle e^{-iE_i t} = |\nu_i(0)\rangle e^{-i\left(p_i + \frac{m_i^2}{2p_i}\right)}$$

for masses $m_i \ll E_i$:

$$E_i = \sqrt{p^2 + m_i^2} = p_i + \frac{m_i^2}{2p_i}$$

→ there will be a mixing of the flavor states with time.

$$|\nu(t)\rangle_\alpha = \sum_i U_{\alpha i} e^{-iE_i t} |\nu_i(0)\rangle = \sum_{i,\beta} U_{\alpha i} U^*_{\beta i} e^{-iE_i t} |\nu_\beta\rangle$$

Two-Flavor mixing (for simplicity)

$$\begin{pmatrix}\nu_\alpha\\ \nu_\beta\end{pmatrix}=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}\cdot\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

Time development for an initially pure $|\nu_\alpha\rangle$ beam:

$$|\nu_\alpha(t)\rangle=\cos\theta\, e^{-iE_1 t}|\nu_1\rangle+\sin\theta\, e^{-iE_2 t}|\nu_2\rangle$$

$$=\left[\cos^2\theta\, e^{-iE_1 t}+\sin^2\theta\, e^{-iE_2 t}\right]\cdot|\nu_\alpha\rangle$$

$$+\left[\cos\theta\sin\theta(\,e^{-iE_1 t}-e^{-iE_2 t})\right]\cdot|\nu_\beta\rangle$$

Definite momentum p; same for all mass eigenstate components

$$E_i=\sqrt{p^2+m_i^2}=p+\frac{m_i^2}{2\mathrm{p}}$$

$$E_2-E_1=\frac{m_1^2-m_2^2}{2\mathrm{p}}\approx\frac{\Delta m^2}{2E}$$

$$(\text{assuming } \mathrm{p}_i \text{ is the same})$$

$$t=\mathrm{L}/\beta \quad \text{w/ } \beta\approx 1 :$$

$$(E_2-E_1)\,t=\frac{\Delta m^2}{2E}L$$

Mixing probability:

$$P(\nu_\alpha\to\nu_\beta,\,t)=|\langle\nu_\beta|\nu_\alpha(t)\rangle|^2=2(\cos\theta\sin\theta)^2\left[1-\cos^2\frac{E_2-E_1}{2}t\right]$$

$$\boxed{P(\nu_\alpha\to\nu_\beta,\,t)=\sin^2 2\theta\sin^2\left(\frac{\Delta m^2}{4E}L\right)}=\sin^2 2\theta\sin^2\left(\frac{1.27\cdot\Delta m^2[eV]}{4E[GeV]}L[km]\right)$$

Search for Neutrino Oscillations (PDG 1996)

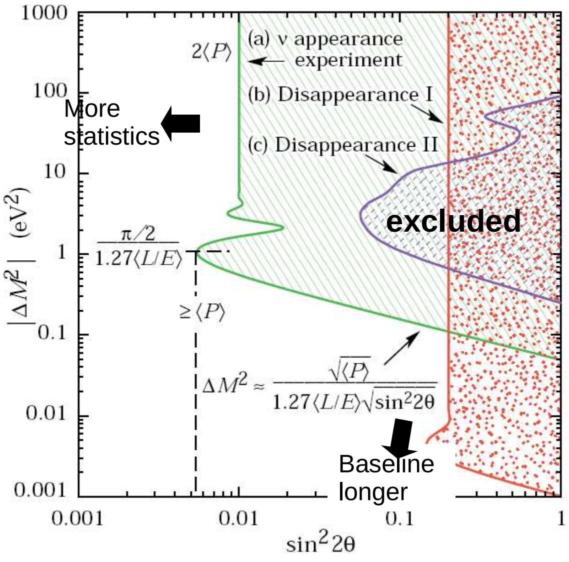
$$P(\nu_\alpha\to\nu_\beta,\,t)=\sin^2 2\theta\sin^2\left(\frac{\Delta m^2}{4E}L\right)$$

- Disappearance: reactor experiments. Nuclear reactors are most intensive sources of $\bar{\nu}_e$ on Eearth.
  - (I) With known neutrino flux: measure flux at distance *L*. Only $\bar{\nu}_e$ flux is measured via $\bar{\nu}_e + p \to e^+ + n$
  - (II) Measure neutrino flux at position 1 and verify flux after distance *L*.

  More sensitive to small Δm^2 , as longer *L* can be used due to high flux

- Appearance: Use neutrino beam of type A $(\overset{(-)}{\nu}_\mu)$ and search at distance *L* for neutrinos of type B $(\overset{(-)}{\nu}_e)$. More sensitive to small sinθ due to appearance

Exclusion plots

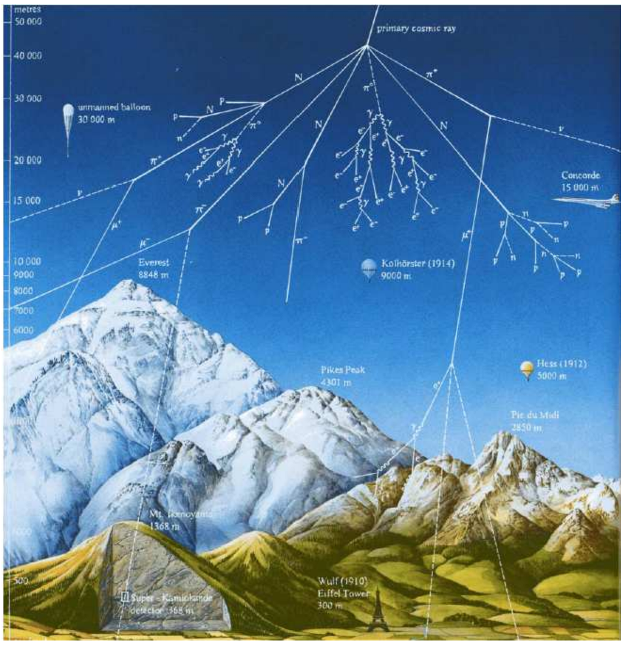


Observation of Neutrino Oscillations

Neutrino source	Experiment	Comments
Solar neutrinos	Radio-chemical exp.: Homestake Cl exp., GALLEX, SAGE	First observation of “neutrino disappearance” dates more than 20 years ago: “Solar neutrino problem”
	Water experiments: (Super)Kamiokande, IMB	Confirm disappearance of solar neutrinos
	Heavy water: SNO	Ultimate “solar neutrino experiment”: proves the oscillation of solar ν
Atmospheric neutrinos	(Super)Kamiokande	Oscillation signal
Accelerator	LSND much disputed KARMEN and MiniBooNE	Oscillation signal refuted
	K2K	Clear disappearance signal
	MINOS	Clear disappearance signal
Reactor	KamLAND, CHOOZ	Clear disappearance signal

---

2 Atmospheric neutrino problem



Cosmic radiation: Air shower

$$p+N \to \pi^{\pm},\ K^{\pm}$$

$$\pi^{\pm},\ K^{\pm} \to \mu^{\pm}+\nu_{\mu}(\bar{\nu}_{\mu})$$

$$\mu^{\pm} \to e^{\pm}+\nu_{e}(\bar{\nu}_{e})+\bar{\nu}_{\mu}(\nu_{\mu})$$

$$\Downarrow$$

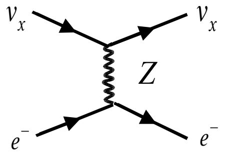
$$R=\frac{\nu_{\mu}+\bar{\nu}_{\mu}}{\nu_{e}+\bar{\nu}_{e}}=2$$

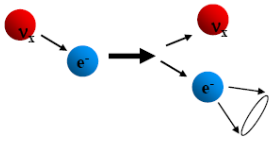
Exact calculation: R=2.1
(E_ν<1GeV)
(For larger energies R>2.1)

Neutrino detection with water detectors [E_ν~O(GeV)]

Water = “active target”

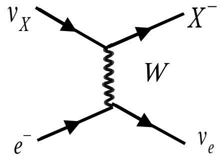
Elastic scattering





Cherenkov Light

Charged current



Kinematical limit for ν_μ: $E_\nu > m_\mu$

Detection of Cherenkov photons: photomultiplier

Experiments: (Super)-Kamiokande in Kamioka Mining and Smelting Co.'s Mine in Japanese Alpes,

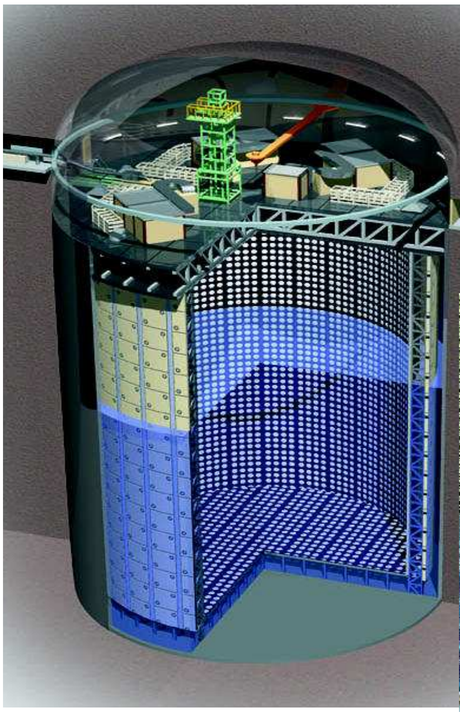
IMB (Irvine-Michigan-Brookhaven) in salt mine on the shore of lake Erie (USA)

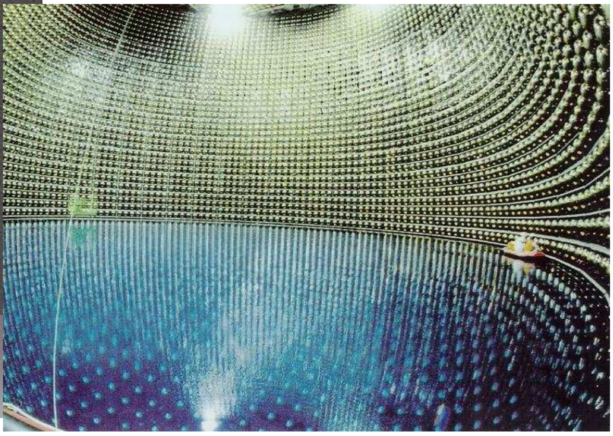
Primary goal of both experiments: discover proton decay.
KamiokaNDE = Kamioka Nucleon Decay Experiment.

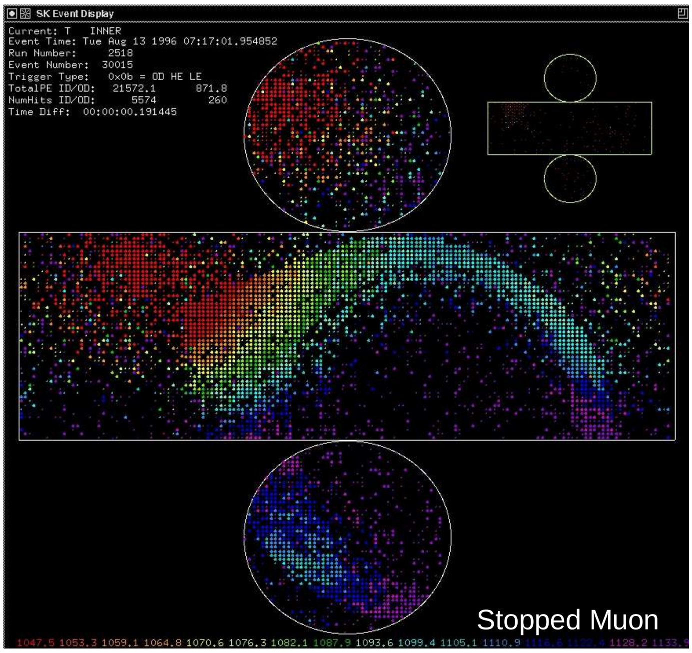
Super-Kamiokande

located 1 km underground

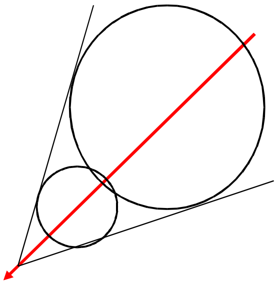
- Largest artificial water detector (50 kt), 41 m height and 39 m diameter
- Until the 2001 accident: 11000 PMTs (50 cm tubes!): 40% of surface covered with photo-cathode
- Back in operation since 2003, fully restored 2006







```
Current: T   INNER
Event Time: Tue Aug 13 1996 07:17:01.954852
Run Number:     2518
Event Number:  30015
Trigger Type:   0x0b = OD HE LE
TotalPE ID/OD:   21572.1       871.8
NumHits ID/OD:      5574         260
Time Diff:  00:00:00.191445
```



Cherenkov cone:

$$\cos\theta = \frac{1}{\beta n}$$

$$\Leftrightarrow \theta = 42^o \; (\beta = 1)$$

Experiment can distinguish electron and muon events, and can measure energy

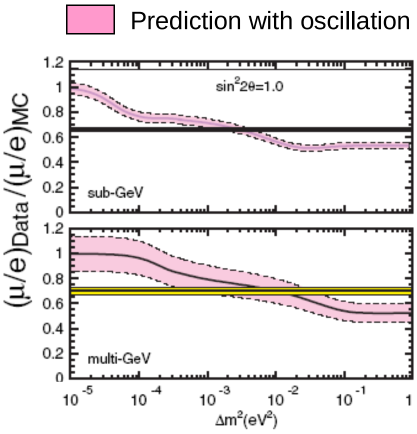
Electrons suffer multiple interactions – fuzzy ring.
Muons fly straight through – sharp edge ring.

Ratio of muon to electron neutrinos

$$R \equiv \frac{(\mu/e)_{DATA}}{(\mu/e)_{M.C.}}$$

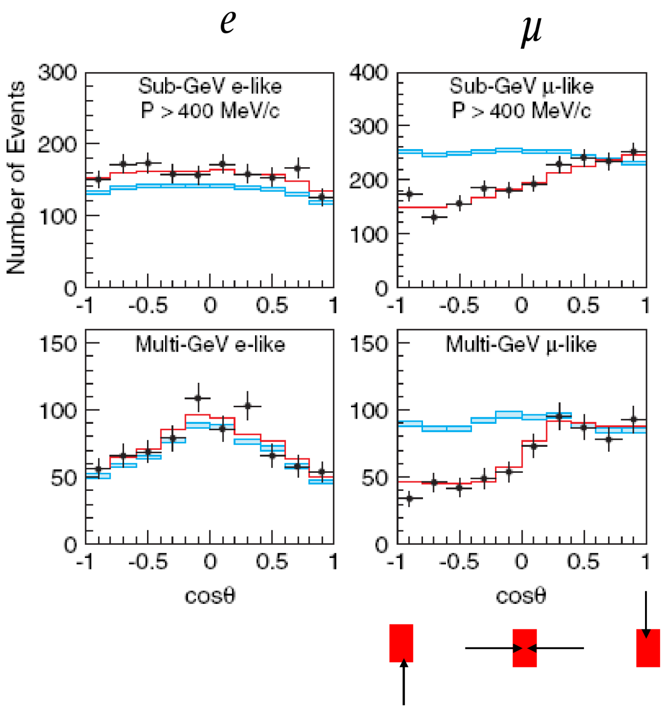
$$R_{sub-GeV} = 0.658 \pm 0.016(stat) \pm 0.032(sys)$$

$$R_{multi-GeV} = 0.702^{+0.032}_{-0.030}(stat) \pm 0.099(sys)$$



- Too few muon neutrinos observed
- Can be explained by oscillations

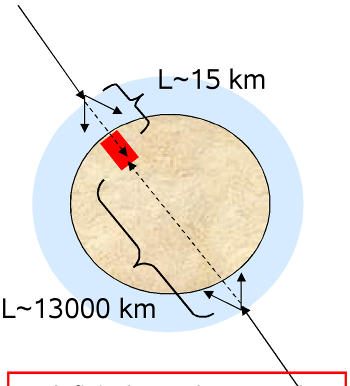
Zenith angle dependence of the neutrino flux



Theoretical predicton

🟦 w/o oscillation

🟥 w/ oscillation



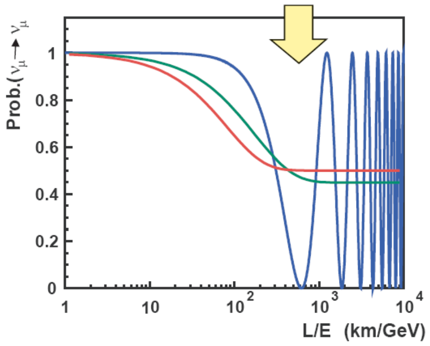
ν_μ deficit depends on angle

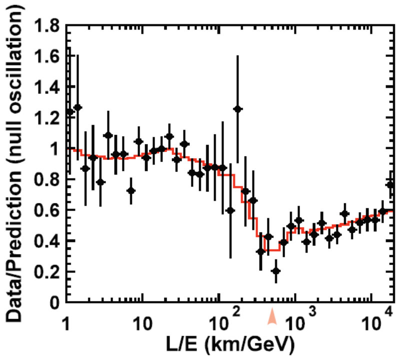
ν_e flux okay

Oscillation: $\nu_\mu \leftrightarrow \nu_\tau$

Oscillation pattern of atmospheric neutrinos

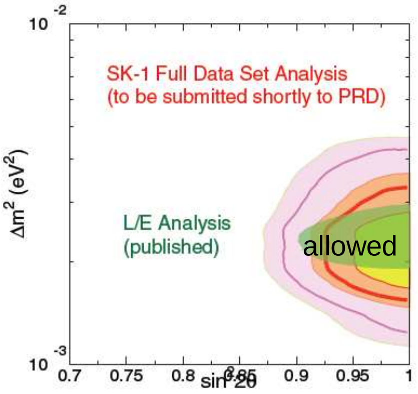
Survival probability: $$P(\nu_\mu \to \nu_\mu) = 1 - \sin^2 2\theta \sin^2\left(\frac{1.27\,\Delta m^2\,(\mathrm{eV})^2}{E\,(\mathrm{GeV})} L\,(\mathrm{km})\right)$$





oscillation dip seen at ~500 km/GeV

Oscillation pattern of atmospheric neutrinos



$\nu_\mu \leftrightarrow \nu_\tau$ mixing of atmos. neutrinos

$$\Delta m^2 = (2.4 \pm 0.4) \times 10^{-3}\,\mathrm{eV}$$

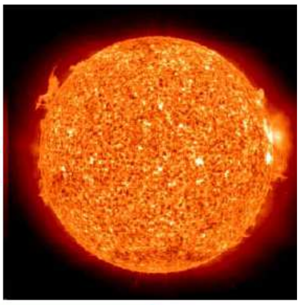
$$\sin^2 2\theta > 0.92 \text{ @ } 90\%\ \mathrm{CL}$$

---

3 Solar neutrino problem

Neutrino production

ν_e are abundant by-products of nuclear fusion in the sun



$p + p \rightarrow {}^2H + e^+ + \nu_e + 0.42 MeV$

$p + e^- + p \rightarrow {}^2H + \nu_e + 1.44 MeV$

"pp" 99.75%

"pep" 0.25%

${}^2H + p \rightarrow {}^3He + \gamma + 5.49\ MeV$

86%

14%

"hep" 2.4*10^{-5}

${}^3He + {}^3He \rightarrow \alpha + 2p + 12.86 MeV$

${}^3He + \alpha \rightarrow {}^7Be + \gamma + 1.59 MeV$

${}^3He + p \rightarrow \alpha + e^+ + \nu_e$

"^{7}Be" 99.89%

0.11%

${}^7Be + e^- \rightarrow {}^7Li + \gamma + \nu_e + 0.8617 MeV$

${}^7Be + p \rightarrow {}^8B + \gamma + 0.14 MeV$

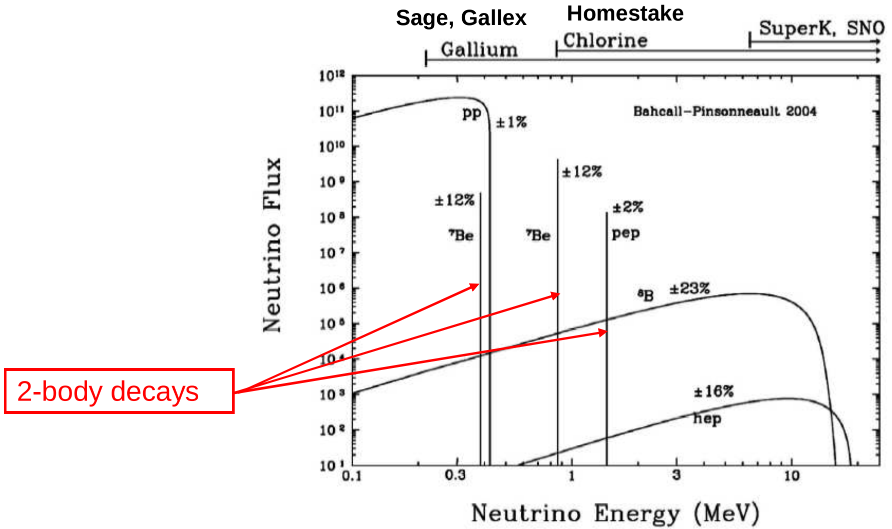
"^{8}B" 0.11%

${}^7Li + p \rightarrow \alpha + \alpha + 17.35 MeV$

${}^8B \rightarrow {}^8Be + e^+ + \nu_e + 14.6 MeV$

${}^8Be \rightarrow \alpha + \alpha + 3 MeV$

Neutrino energy spectrum



Neutrino experiments:

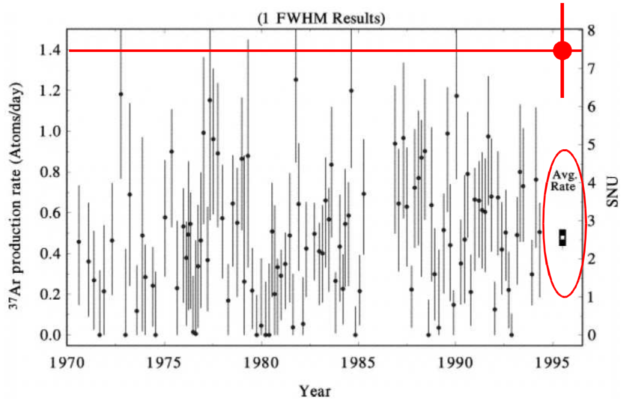
Cl_2 detectors	$\nu_e + {}^{37}Cl \rightarrow {}^{37}Ar + e$, ${}^{37}Ar \rightarrow {}^{37}Cl$ (EC)	$E_\nu > 0.8$ MeV
Ga detectors	$\nu_e + {}^{71}Ga \rightarrow {}^{71}Ge + e$	$E_\nu > 0.2$ MeV
H_2O detectors	Elastic scattering: $\nu_e + e \rightarrow \nu_e + e$	$E_\nu > 5$ MeV (detection)
Scintillator	(Borexino started 2007, SNO+ under construction)	$E_\nu > 0.25$ MeV

Radio-chemical experiments:

Homestake, SAGE, GALLEX

- Homestake mine, 1400 m underground
- 615 t of C_2Cl_4 (perchloroethilene) = 2.2×10^{30} atoms of ^{37}Cl
- Use He and ^{36}Ar and ^{38}Ar to carry-out the few atoms of ^{37}Ar (~ 1 atom/day)
- Count radioactive ^{37}Ar decays

SSM prediction



Homestake Cl_2 experiment
(Homestake gold mine, South Dakota)

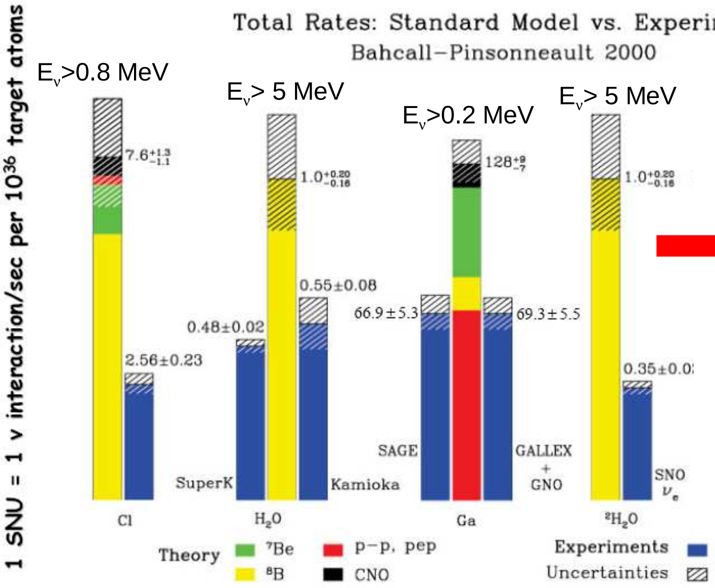
The Nobel Prize in Physics 2002

Raymond Davis Jr.	Masatoshi Koshiba	Riccardo Giaconi

"for pioneering contributions to astrophysics, in particular for the detection of cosmic neutrinos"

"for pioneering contributions to astrophysics, which have led to the discovery of cosmic X-ray sources"

Solar Neutrino Problem: Experimental summary



CC ES CC ES

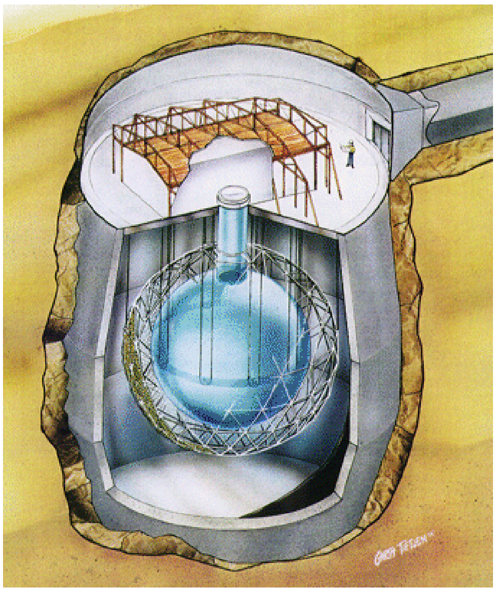
Neutrino disappearance

Can one measure the oscillated neutrinos?

Sudbury Neutrino Observatory

in Vale Inco's Creighton Mine in Sudbury, Ontario, Canada, located 2 km underground

- 6 m radius transparent acrylic vessel
- 1000 t of heavy water (D_2O)
- 9456 inward looking photo multipliers
- Add 2 t of NaCl to improve detection of neutrons (salt phase)
- Surrounded by normal water (H_20) shielding external radiation



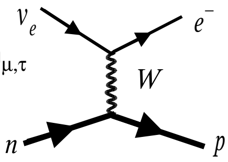
Neutrino detection with SNO

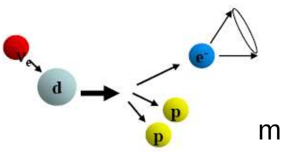
Charged current

$E_\nu < m_{\mu,\tau}$

$$\sigma(\nu_\mu)=\sigma(\nu_\tau)=0$$

$$\varphi_\nu=\varphi_{\nu_e}$$





Cherenkov light

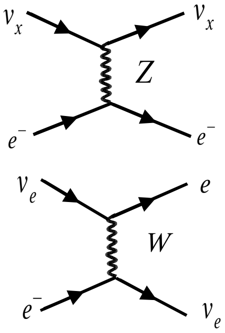
measure E_ν

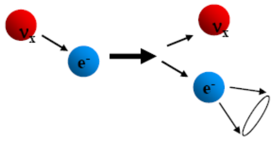
$E_\nu = E_e + 1.442$ MeV

Elastic scattering

$$0.154\cdot\sigma(\nu_e)=\sigma(\nu_\mu)=\sigma(\nu_\tau)$$

$$\varphi_\nu=\varphi_{\nu_e}+(\varphi_{\nu_\mu}+\varphi_{\nu_\tau})/6$$





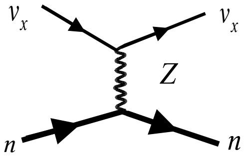
Cherenkov light

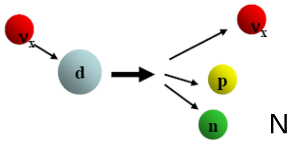
lower rate, strong forward peak

Neutral current

$$\sigma(\nu_e)=\sigma(\nu_\mu)=\sigma(\nu_\tau)$$

$$\varphi_\nu=\varphi_{\nu_e}+\varphi_{\nu_\mu}+\varphi_{\nu_\tau}$$



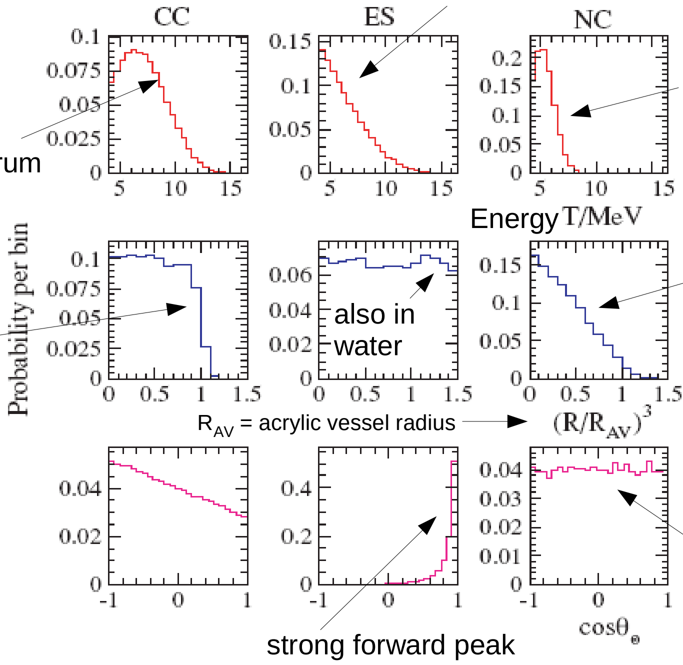


Neutron

${}^2\mathrm{H}(n,\gamma)^3\mathrm{H}$ or ${}^{35}\mathrm{Cl}(n,\gamma)^{36}\mathrm{Cl}$

γ–ray 6.25 MeV or 8.5 MeV

Signals are determined not event-by-event but on statistical basis



softer than CC

γ-ray in NC makes Compton scattering or e^+e^- pair and Cerenkov light is observed. 6.25 MeV line within resolution

resembles ^{8}B ν spectrum

Energy

NC only on deuterons in heavy water decreases due to long thermal diffusion of *n.* In normal water E_γ = 2.2 MeV – below analysis threshold

CC only on deuterons within heavy water vessel

also in water

R_{AV} = acrylic vessel radius →

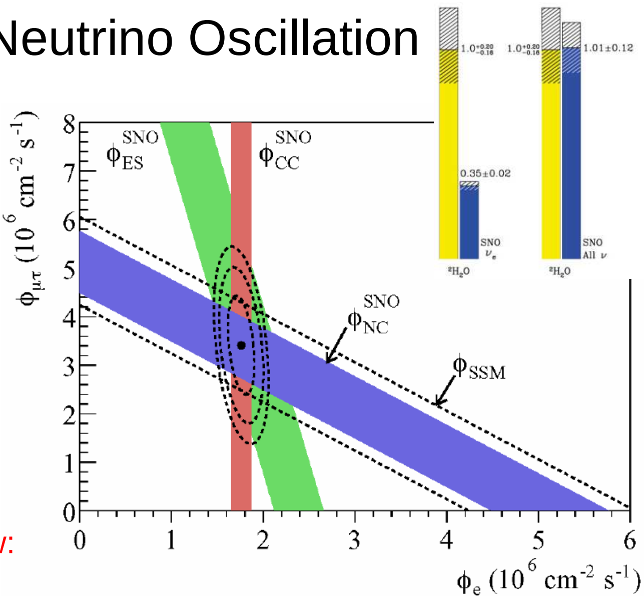
strong forward peak direction from the Sun

no correlation

SNO Evidence for Neutrino Oscillation

$$\phi^{\rm SNO}_{\rm CC} = 1.76^{+0.06}_{-0.05}({\rm stat})^{+0.09}_{-0.09}({\rm syst})\,,$$
$$\phi^{\rm SNO}_{\rm ES} = 2.39^{+0.24}_{-0.23}({\rm stat})^{+0.12}_{-0.12}({\rm syst})\,,$$
$$\phi^{\rm SNO}_{\rm NC} = 5.09^{+0.44}_{-0.43}({\rm stat})^{+0.46}_{-0.43}({\rm syst})\,.$$

Phys. Rev.Lett. 89 (2002) 011301



Electron neutrino flux is too low:

$$P_{veve} = (35 \pm 2)\,\%$$

Total flux of neutrinos is correct.

Interpreted as $\nu_e \leftrightarrow \nu_\mu$ or ν_τ oscillation

But in case of simple “vacuum oscillation”: $P_{veve} \geq 1 - \frac{1}{2}\sin^2 2\theta = 50\%$?

Neutrino oscillations in matter: MSW-effect

Mikheyev, Smirnov (1986), Wolfenstein (1976)

Neutrino oscillation in vacuum:

time development of
mass eigenstates

$$i\frac{d}{dt}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}=\frac{1}{2\mathrm{p}}\underbrace{\begin{pmatrix}m_1^2 & 0\\ 0 & m_2^2\end{pmatrix}}_{M}\begin{pmatrix}\nu_1\\ \nu_2\end{pmatrix}$$

With unitary transformation U
one obtains for the flavor oscillation
in vacuum:

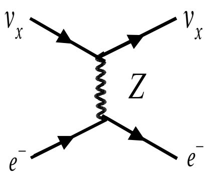
$$U_\theta=\begin{pmatrix}\cos\theta & \sin\theta\\ -\sin\theta & \cos\theta\end{pmatrix}$$

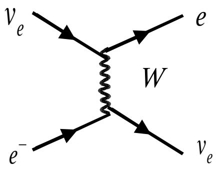
$$i\frac{d}{dt}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}=\frac{1}{2\mathrm{p}}UMU^T\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$=\frac{M^2}{2\mathrm{p}}\begin{pmatrix}\nu_e\\ \nu_\mu\end{pmatrix}$$

$$UMU^T=\frac{\Delta m^2}{2}\begin{pmatrix}-\cos 2\theta & \sin 2\theta\\ \sin 2\theta & \cos 2\theta\end{pmatrix}$$

Neutrinos in matter:





Electron neutrinos suffer an additional potential V_e affecting the forward scattering amplitude which leads to change in the effective mass for ν_e:

$$V_e=G_F\sqrt{2}\,N_e$$

N_e = electron density

$$m^2=E^2-p^2\rightarrow(E+V_e)^2-p^2\approx m^2+2EV_e$$

$$\Delta m_M^2=2\sqrt{2}\,G_F N_e E$$

Neutrino oscillation in matter:

$$\Delta m_M^2 = 2\sqrt{2}\, G_F N_e E$$

$$M^2 \to M_M^2 = \frac{\Delta m^2}{2}\begin{pmatrix} -\cos 2\theta & \sin 2\theta \\ \sin 2\theta & \cos 2\theta \end{pmatrix} + \begin{pmatrix} \Delta m_M^2 & 0 \\ 0 & -\Delta m_M^2 \end{pmatrix}$$

Go the opposite direction…

Define the matter mass eigenstates which one obtains by diagonalizing M_M

$$\begin{pmatrix} \nu_{1\mathrm{m}} \\ \nu_{2\mathrm{m}} \end{pmatrix} = U^T_{\theta_m} \begin{pmatrix} \nu_e \\ \nu_x \end{pmatrix}$$

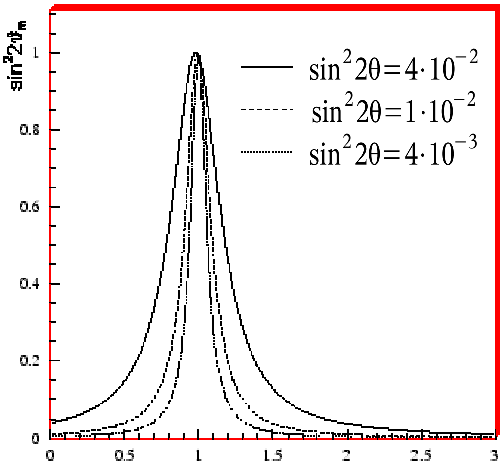
$$U^T_{\theta_m} M^2\, U_{\theta_m} = \frac{1}{2}(m_1^2 + m_1^2)\begin{pmatrix} -\Delta_m & 0 \\ 0 & \Delta_m \end{pmatrix}$$

$$\Delta_m = \Delta m^2 \sqrt{(a - \cos 2\theta)^2 + \sin^2 2\theta}$$

$$a = 2\sqrt{2}\, E G_F N_e / \Delta m^2$$

$$U_{\theta_m} = \begin{pmatrix} \cos\theta_m & \sin\theta_m \\ -\sin\theta_m & \cos\theta_m \end{pmatrix}$$ with

$$\sin^2 2\theta_m = \frac{\sin^2 2\theta}{(\cos 2\theta - a)^2 + \sin^2 2\theta}$$



$$a = 2\sqrt{2}\, E G_F N_e / \Delta m^2$$

Matter mixing angle can go through a resonance:

$$\cos 2\theta - a = 0 \quad i.e. \quad N_e = \Delta m^2 \frac{\cos 2\theta}{2\sqrt{2}\, E G_F}$$

As in the core of the sun, N_e is larger than the critical density, the resonance condition will always be fulfilled: oscillation is largely modified by matter.

Explains $P_{veve} = (35 \pm 2)\,\%$ observed with SNO