IX. Flavor oscillation and CP violation

1. Quark mixing and the CKM matrix
2. Flavor oscillations: Mixing of neutral mesons
3. CP violation
4. Neutrino oscillations

1. CKM Matrix

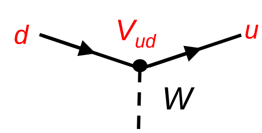
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak eigenstates — **CKM matrix** — **mass eigenstates**

Unitarity

$$V_{CKM} V_{CKM}^{+} = 1$$

Charged currents:



$$J_\mu^+ \propto \overline{\psi_u}\, \gamma_\mu (1-\gamma_5) \psi_{d'}$$

$$J_\mu^+ \propto (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu (1-\gamma_5) \begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = (\bar{u}, \bar{c}, \bar{t})\, \gamma_\mu (1-\gamma_5) \mathbf{V_{CKM}} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

weak — **mass/ flavor**

1.1 Parameters of CKM matrix

Number of independent parameters: →

18 parameter (9 complex elements)
-5 relative quark phases (unobservable)
-9 unitarity conditions

=4 independent parameters: 3 angles + 1 phase

PDG parametrization

3 Euler angles
$\theta_{23}, \theta_{13}, \theta_{12}$

1 Phase
δ

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

$$\begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

Unobservable Phases

Phases of left-handed fields in J^{cc} are unobservable: possible redefinition

$$u_L \to e^{i\phi(u)}u_L \qquad c_L \to e^{i\phi(c)}c_L \qquad t_L \to e^{i\phi(t)}t_L$$

$$d_L \to e^{i\phi(d)}d_L \qquad s_L \to e^{i\phi(s)}s_L \qquad b_L \to e^{i\phi(b)}b_L$$

↑ Real numbers

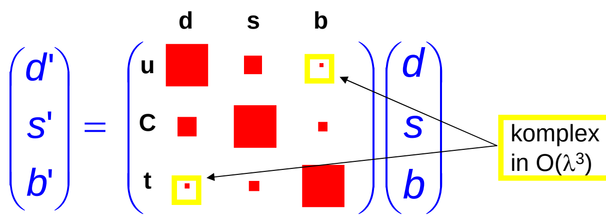
Under phase transformation:

$$V \to \begin{pmatrix} e^{-i\phi(u)} & 0 & 0 \\ 0 & e^{-i\phi(c)} & 0 \\ 0 & 0 & e^{-i\phi(t)} \end{pmatrix} \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} e^{i\phi(d)} & 0 & 0 \\ 0 & e^{i\phi(s)} & 0 \\ 0 & 0 & e^{i\phi(b)} \end{pmatrix}$$

$$V\alpha j \to \exp[i(\phi(j) - \phi(\alpha))]V\alpha j$$

5 unobservable phase differences !

Magnitude of elements



Wolfenstein Parametrization $\lambda, A, \rho, \eta, \lambda = 0.22$

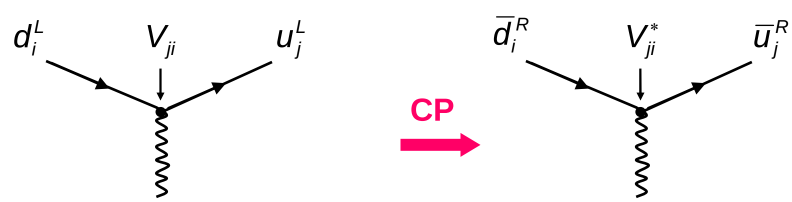
$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

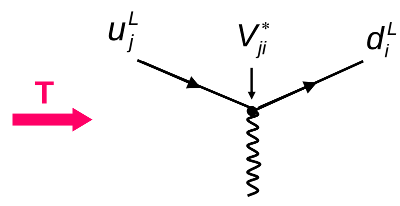
$V_{ub} = |V_{ub}| e^{-i\gamma}$

$V_{td} = |V_{td}| e^{-i\beta}$

Reflects hierarchy of elements in O(λ)

Complex CKM elements and CP violation





CP (T) violation $\Leftrightarrow$ $V_{ji} \neq V_{ji}^*$

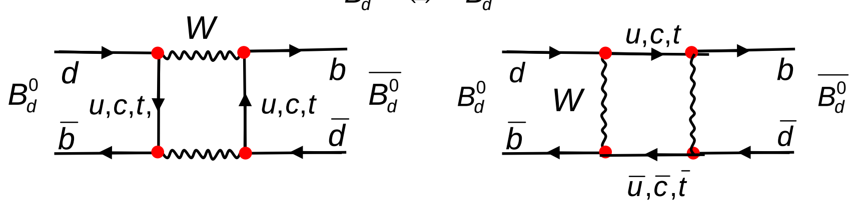
i.e. Complex elements

Remark: For 2 quark generations the mixing is described by the **real 2x2** Cabbibo matrix → **no CP violation** !!. To explain **CPV** in the SM Kobayashi and Maskawa have predicted a **third quark generation**.

2. Mixing of neutral mesons

As result of the quark mixing the Standard Model predicts oscillations of neutral mesons:

$$B_d^0 \Leftrightarrow \overline{B_d^0}$$



Similar graphs for other neutral mesons:

Neutral mesons: $\lvert P^0\rangle$:	$K^0 = \lvert d\bar{s}\rangle$	$D^0 = \lvert \bar{u}c\rangle$	$B_d^0 = \lvert d\bar{b}\rangle$	$B_s^0 = \lvert s\bar{b}\rangle$
$\lvert \overline{P^0}\rangle$:	$\overline{K^0} = \lvert \bar{d}s\rangle$	$\overline{D^0} = \lvert u\bar{c}\rangle$	$\overline{B_d^0} = \lvert \bar{d}b\rangle$	$\overline{B_s^0} = \lvert \bar{s}b\rangle$
discovery of mixing	1960	2007	1987	2006

2.1 Phenomenological description of mixing

Schrödinger equation for unstable meson:

$$i\frac{d}{dt}\lvert\psi\rangle = H\lvert\psi\rangle = \left(m - \frac{i}{2}\Gamma\right)\lvert\psi\rangle \quad \Rightarrow \quad \begin{cases} \lvert\psi(t)\rangle = \lvert\psi_0\rangle \cdot e^{-imt} \cdot e^{-\frac{1}{2}\Gamma t} \\ \left\lVert\psi(t)\rangle\right\rVert^2 = \left\lVert\psi_0\rangle\right\rVert^2 \cdot e^{-\Gamma t} \end{cases}$$

For neutral mesons $(K^0,\overline{K}^0),(D^0,\overline{D}^0),(B^0,\overline{B}^0),(B_s^0,\overline{B}_s^0)$ consider 2 components

$$i\frac{d}{dt}\begin{pmatrix} P^0 \\ \overline{P^0} \end{pmatrix} = \mathbf{H}\begin{pmatrix} P^0 \\ \overline{P^0} \end{pmatrix} = \begin{pmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{pmatrix}\begin{pmatrix} P^0 \\ \overline{P^0} \end{pmatrix} = \left(\mathbf{M} - \frac{i}{2}\Gamma\right)\begin{pmatrix} P^0 \\ \overline{P^0} \end{pmatrix} = \begin{pmatrix} m_{11} - \frac{i}{2}\Gamma_{11} & m_{12}^* - \frac{i}{2}\Gamma_{12}^* \\ m_{12} - \frac{i}{2}\Gamma_{12} & m_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}\begin{pmatrix} P^0 \\ \overline{P^0} \end{pmatrix}$$

$\langle P^0 \lvert \mathbf{H}_W \rvert \overline{P^0}\rangle$

M and **Γ** hermitian:

$$m_{21} = m_{12}^*$$
$$\Gamma_{21} = \Gamma_{12}^*$$

CP $\Rightarrow H_{12} = H_{21}$ CPT $\Rightarrow$ $H_{11} = H_{22} = H$, $m_{11} = m_{22} = m$, $\Gamma_{11} = \Gamma_{22} = \Gamma$

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Mass eigenstates (by diagonalizing matrix)

Heavy and light mass eigenstate:

$$|P_L\rangle = p|P^0\rangle + q|\overline{P^0}\rangle \quad \text{with } m_L, \Gamma_L$$

$$|P_H\rangle = p|P^0\rangle - q|\overline{P^0}\rangle \quad \text{with } m_H, \Gamma_H$$

$|p|^2 + |q|^2 = 1$ complex coefficients

$$|P^0\rangle = \frac{1}{2p}(|P_L\rangle + |P_H\rangle)$$

$$|\overline{P^0}\rangle = \frac{1}{2q}(|P_L\rangle - |P_H\rangle)$$

Flavor eigenstates

$|P_L\rangle = |P_1\rangle$ and $|P_H\rangle = |P_2\rangle$

Parameters of the mass states

$$m_{H,L} = m \pm \mathrm{Re}\sqrt{H_{12}H_{21}}$$

$$\Gamma_{H,L} = \Gamma \mp 2\mathrm{Im}\sqrt{H_{12}H_{21}}$$

$$\Delta m = m_H - m_L = 2\mathrm{Re}\sqrt{H_{12}H_{21}}$$

$$\Delta\Gamma = \Gamma_H - \Gamma_L = -4\mathrm{Im}\sqrt{H_{12}H_{21}}$$

$$x \equiv \frac{\Delta m}{\Gamma} \quad \text{und} \quad y \equiv \frac{\Delta\Gamma}{2\Gamma}$$

Neutral B mesons

$\Delta m_d = 0.502 \pm 0.007\ \mathrm{ps}^{-1}$ (PDG)
$= (3.337 \pm 0.033) \times 10^{-10}\ \mathrm{MeV}$

$\Delta m_s = 17.8 \pm 0.1\ \mathrm{ps}^{-1}$ (CDF 2006)
$= 118 \times 10^{-10}\ \mathrm{MeV}$

$\Delta\Gamma / \Gamma < 0.07$ (90% *CL*)

$\Delta\Gamma_s / \Gamma_s = 0.09 \pm 0.64$ (Moriond07)

Time evolution

"Generic particle" ($P_{H,L}$)

$$|B_{H,L}, t\rangle = b_{H,L}(t)|B_{H,L}\rangle \quad \text{mit} \quad b_{H,L}(t) = e^{-\Gamma_{H,L}t/2} e^{-i m_{H,L} t}$$

$$|B^0\rangle = \frac{1}{2p}(|B_L\rangle + |B_H\rangle)$$

$$|\psi_{B^0}(t)\rangle = \frac{|B_L,t\rangle + |B_H,t\rangle}{2p} = \frac{1}{2p}\left(b_L(t)\cdot\left(p|B^0\rangle + q|\overline{B^0}\rangle\right) + b_H(t)\cdot\left(p|B^0\rangle - q|\overline{B^0}\rangle\right)\right)$$

$$= f_+(t)\cdot|B^0\rangle + \frac{q}{p} f_-(t)\cdot|\overline{B^0}\rangle$$

$$|\psi_{\overline{B}^0}(t)\rangle = f_+(t)\cdot|\overline{B}^0\rangle + \frac{p}{q} f_-(t)\cdot|B^0\rangle$$

$$f_\pm(t) = \frac{1}{2}\cdot\left[e^{-i m_H t} e^{-\Gamma_H t/2} \pm e^{-i m_L t} e^{-\Gamma_L t/2}\right]$$

B^0:

$$P(B^0 \to B^0, t) = |f_+(t)|^2$$

$$P(B^0 \to \overline{B}^0, t) = \left|\frac{q}{p}\right|^2 |f_-(t)|^2$$

$\overline{B}^0$:

$$P(\overline{B}^0 \to \overline{B}^0, t) = |f_+(t)|^2$$

$$P(\overline{B}^0 \to B^0, t) = \left|\frac{p}{q}\right|^2 |f_-(t)|^2$$

Time evolution of neutral meson states

No mixing part:

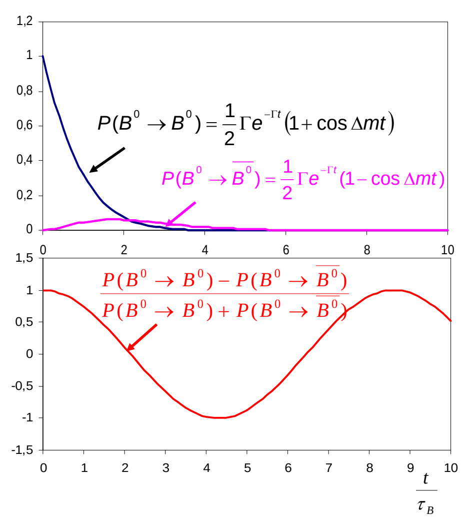
$$\underbrace{P(B^0 \to B^0) = P(\overline{B^0} \to \overline{B^0})}_{\text{CPT}} = \frac{1}{4}\left[e^{-\Gamma_L t} + e^{-\Gamma_H t} + 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos \Delta m t\right]$$

With mixing:

$$P(B^0 \to \overline{B^0}) = \frac{1}{4}\left|\frac{q}{p}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos \Delta m t\right]$$

$$P(\overline{B^0} \to B^0) = \frac{1}{4}\left|\frac{p}{q}\right|^2 \left[e^{-\Gamma_L t} + e^{-\Gamma_H t} - 2e^{-(\Gamma_L+\Gamma_H)t/2}\cos \Delta m t\right]$$

For $\Delta\Gamma$ very small: $\Gamma_H \approx \Gamma_L \approx \Gamma$ (e.g. B^0)



In general:

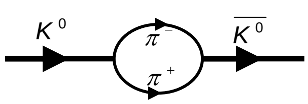
Two mixing mechanisms:

- Mixing through decays → $y = \frac{\Delta\Gamma}{2\Gamma} \approx O(1)$
- Mixing through oscillation → $x = \frac{\Delta m}{\Gamma} \approx O(1)$

$(K^0,\overline{K}^0),(D^0,\overline{D}^0),(B^0,\overline{B}^0),(B_s^0,\overline{B}_s^0)$ show different oscillation behavior

CP, T- violation in mixing: $P(B^0 \to \overline{B}^0) \neq P(\overline{B}^0 \to B^0) \Rightarrow \left|\frac{q}{p}\right| \neq 1$

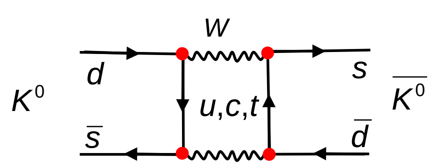
Mixing mechanisms: e.g. K^0



„long distant, on-shell states“

For K^0 important, for B^0 negligible

⇒ $\Delta\Gamma$



„short distant, virtual states“

⇒ Δm

	$K^0/\overline{K}^0$	$D^0/\overline{D}^0$	$B^0/\overline{B}^0$
τ [ps]	89.4 ± 0.1; 51700 ± 400	$0.413 \pm .003$	1.548 ± 0.021
Γ [s^{-1}]	$5.61 \cdot 10^9$	$2.4 \cdot 10^{12}$	$(6.41 \pm 0.16) \cdot 10^{11}$
$y = \frac{\Delta\Gamma}{2\Gamma}$	-0.9966	$\lvert y\rvert < 0.06$	$\lvert y\rvert \lesssim 0.01^*$
Δm [s^{-1}]	$(5.300 \pm 0.012) \cdot 10^9$	$< 7 \cdot 10^{10}$	$(4.89 \pm 0.09) \cdot 10^{11}$
Δm [eV]	$(3.49 \pm 0.01) \cdot 10^{-6}$	$< 5 \cdot 10^{-6}$	$(3.2 \pm 0.1) \cdot 10^{-4}$
$x = \frac{\Delta m}{\Gamma}$	0.945 ± 0.002	< 0.03	0.76 ± 0.02

2.2 Neutral kaons

Observation of two neutral kaons $\mathbf{K_L}$ (long) and $\mathbf{K_S}$ (short) with different lifetimes:

$$\tau(K_L^0) = (51.7 \pm 0.4)\,\text{ns} \gg \tau(K_S^0) = (0.089 \pm 0.001)\,\text{ns}$$

$$K_L^0 \to 3\pi \qquad\qquad K_S^0 \to 2\pi$$

$$\text{CP} = -1 \qquad\qquad \text{CP} = +1$$

$\mathbf{K_L}$ and $\mathbf{K_S}$ can be identified with the mass eigenstates (ignoring CP violation)

$$|K_L\rangle = \text{"}|K_2\rangle\text{"} \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right)$$

$$|K_S\rangle = \text{"}|K_1\rangle\text{"} \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right)$$

$$\text{CP}|K_2\rangle = -|K_2\rangle \qquad \text{CP}|K_1\rangle = +|K_1\rangle$$

Phase convention:

$$CP|K^0\rangle = |\overline{K^0}\rangle \qquad CP|\overline{K^0}\rangle = |K^0\rangle$$

Large differences between lifetimes

$$\Delta m = (0.5303 \pm 0.0009) \cdot 10^{10}\,\hbar s^{-1} = (3.49 \pm 0.006) \cdot 10^{-12}\,\text{MeV}$$

$$\Delta\Gamma = -11.182 \cdot 10^{9}\,\hbar s^{-1}$$

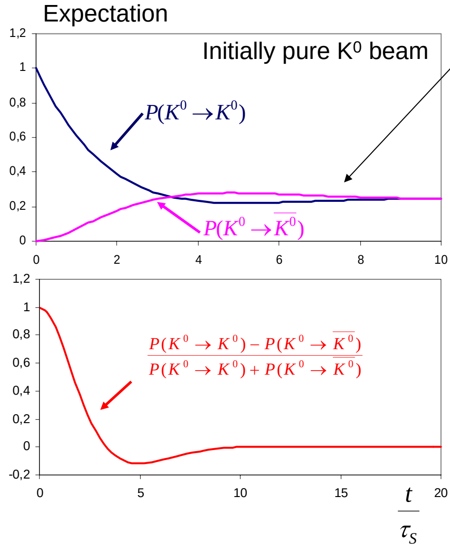
$$x = \frac{\Delta m}{\Gamma} = 0.942 \pm 0.007$$

$$y = \frac{\Delta\Gamma}{2\Gamma} = -0.9966$$

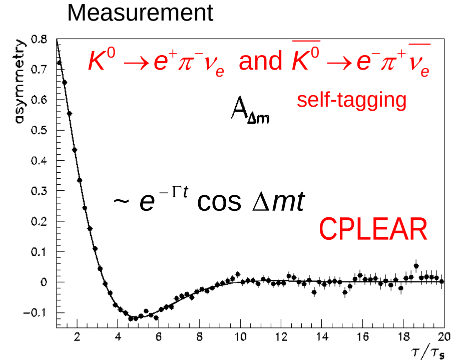
Neutral kaon system

After the lifetime of the K_s the K^0 consists entirely out of K_l's, which are essentially an equal mixture of K^0 and $\overline{K^0}$.

Expectation



Measurement



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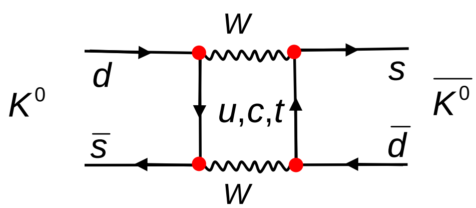
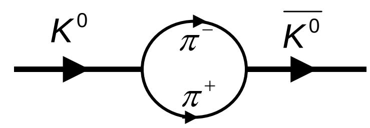
K^0 - $\bar{K}^0$ (strangeness) oscillation in the SM





Short range effects

Long range effects: difficult to calculate

Oscillation frequency Δm:

$$\Delta m \sim \frac{G_F^2}{4\pi} m_K f_K^2 \sum_{q=u,c,t} m_q^2 \left|V_{qs} V_{qd}\right|^2 \approx \frac{G_F^2}{4\pi} m_K f_K^2 m_c^2 \left|V_{cs} V_{cd}\right|^2$$

c quark contribution dominant: although m_t^2 is very large, the factor $|V_{ts}V_{td}|^2 \sim \lambda^5$ is very small !

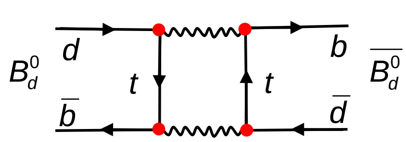
2.3 Neutral B Meson

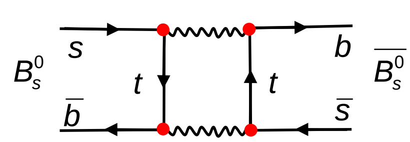
Mixing mechanisms:

- **Mixing through decay:** many possible hadronic decays $\rightarrow \Gamma$ is large

$$\Rightarrow y = \frac{\Delta\Gamma}{2\Gamma} \text{ is small} = \begin{cases} \approx 0 \text{ for } B_d^0 \\ \approx O(0.1) \text{ for } B_s^0 \end{cases} \Rightarrow \text{don't expect mixing via decay}$$

- **Mixing through oscillation** $\rightarrow$ Significant contribution only from top loop





$$\Delta m \sim m_t^2 \left|V_{tb} V_{td}\right|^2 \sim m_t^2 \cdot O(\lambda^6) \qquad \Delta m \sim m_t^2 \left|V_{tb} V_{ts}\right|^2 \sim m_t^2 \cdot O(\lambda^4)$$

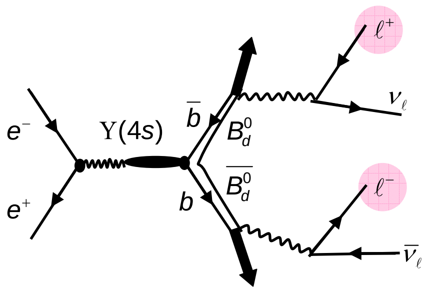
Large $\Delta m_{s,d}$: $\Delta m_s \sim 1/\lambda^2 \, \Delta m_d \rightarrow B_s$ osc. is about 35 times faster than B_d osc.

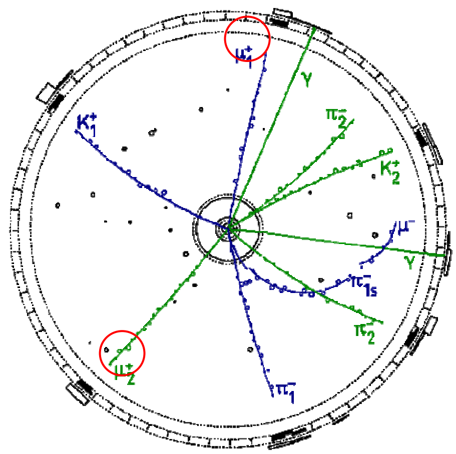
Discovery of B^0 mixing

ARGUS 1987

First e^+e^- B factory at DESY:

at $\sqrt{s} = 10.58$ GeV : $e^+e^- \to \Upsilon(4S) \to B^0\overline{B}^0$ $\}$ $\sigma(B\overline{B}) \approx 1$nb





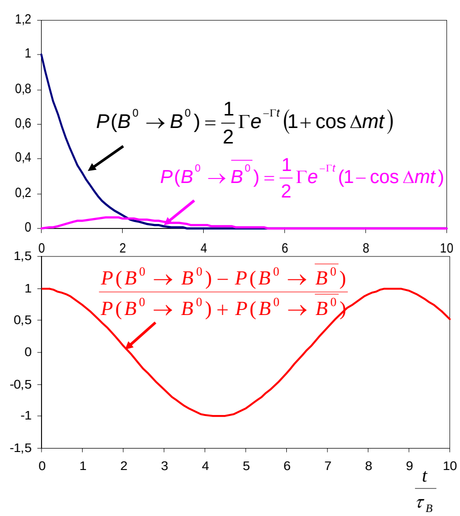
$B^0 \to D^{*-}\mu^+\nu_\mu$, $\hookrightarrow \overline{D}^0\pi_S^-$, $\hookrightarrow K^+\pi^-$

$B^0 \to D^{*-}\mu^+\nu_\mu$, $\hookrightarrow D^-\pi^0$, $\hookrightarrow \gamma\gamma$, $\hookrightarrow K^+\pi^-\pi^-$

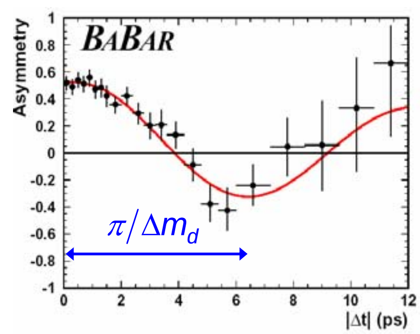
Unmixed: $B^0\overline{B}^0 \to \ell^+\ell^-$

Mixed: $B^0B^0 \to \ell^+\ell^+$, $\overline{B}^0\overline{B}^0 \to \ell^-\ell^-$ } Same charge

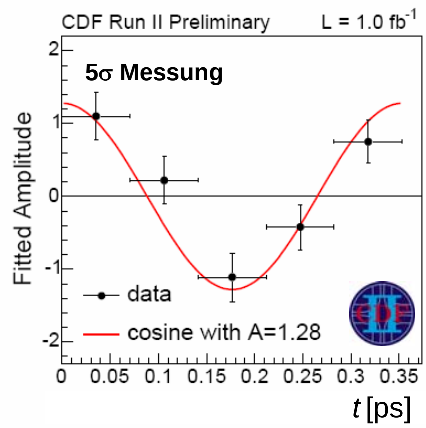
Mixing of neutral B mesons



$$A = \frac{unmixed - mixed}{unmixed + mixed}$$



$$\Delta m_d = 0.506 \pm 0.006 \pm 0.004 \text{ ps}^{-1} \approx \frac{0.774}{\tau_B}$$



Observation:

Spring 2006

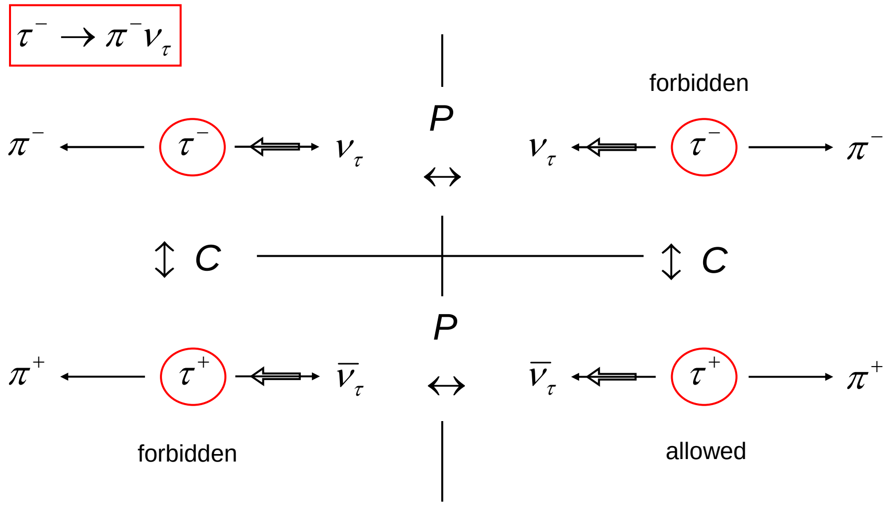
$$\Delta m_s = 17.77 \pm 0.10\,(\text{stat.}) \pm 0.07\,(\text{syst.})\ \text{ps}^{-1} = \frac{26}{\tau}$$

(CDF Collaboration, September 2006)

35 times faster than B^0

3. CP violation in the K^0 and B^0 system

$\tau^- \to \pi^- \nu_\tau$



- C and P violated in weak decays
- CP conserved in weak interaction ? $\to$ No !

3.1 Observation of CP violation (CPV) in K_L decays

Reminder:

If no CPV:

$$|K_L\rangle = "|K_2\rangle" \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right)$$

$$|K_S\rangle = "|K_1\rangle" \equiv \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right)$$

$\text{CP}|K_2\rangle = -|K_2\rangle$

$\text{CP}|K_1\rangle = +|K_1\rangle$

Phase convention:

$CP|K^0\rangle = |\overline{K^0}\rangle$

$CP|\overline{K^0}\rangle = |K^0\rangle$

If no CPV:

$$|K_L\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right) \quad \text{CP} = -1$$

should always decay into 3π:

CP(|3π>)= -1

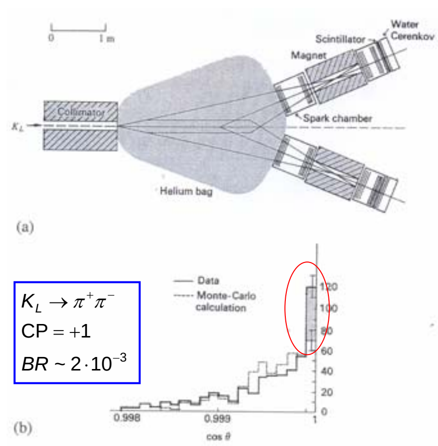
and never into 2π CP(|2π>)= +1

Explanation:

$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(\underbrace{|K_2\rangle}_{CP=-1} - \varepsilon \underbrace{|K_1\rangle}_{CP=+1}\right)$$

Not a CP eigenstate: CP violation !

Christenson, Cronin, Fitch, Turlay, 1964



$K_L \to \pi^+\pi^-$

$CP = +1$

$BR \sim 2\cdot 10^{-3}$

After 35 years of kaon physics:

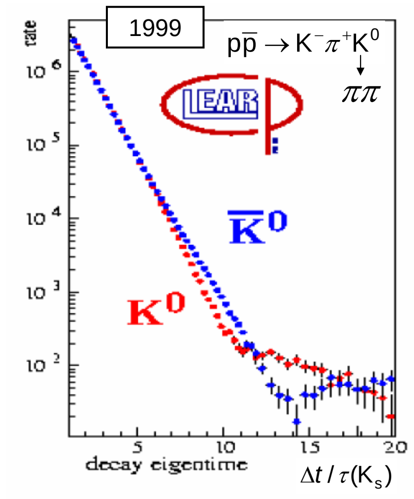
$$|K_L\rangle = \frac{1}{\sqrt{1+|\varepsilon|^2}}\left(|K_2\rangle + \varepsilon|K_1\rangle\right)$$

$|K_2\rangle$ → ε' → $\pi\pi$ (Direct CPV)

$|K_1\rangle$ → $\pi\pi$ (mixing)

$$|\varepsilon| = (2.284 \pm 0.014)\cdot 10^{-3}$$

$$\mathrm{Re}(\varepsilon'/\varepsilon) = (1.67 \pm 0.26)\cdot 10^{-3}$$



Interpretation of CPV measured in the kaon system is difficult. Much easier to understand and to predict in the SM is the B meson system:

CPV in the B^0 system was observed in 2000.

Comments:

$$i\frac{d}{dt}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \mathbf{H}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix} = \begin{pmatrix} H_{11} & H_{21} \\ H_{12} & H_{22} \end{pmatrix}\begin{pmatrix} K^0 \\ \overline{K}^0 \end{pmatrix}$$

CPT conservation: $H_{11} = H_{22}$

CP conservation: $H_{12} = H_{21}$

CP Eigenstates

$$|K_1\rangle = p|K^0\rangle + q|\overline{K^0}\rangle = |K^0\rangle + |\overline{K^0}\rangle$$

$$|K_2\rangle = p|K^0\rangle - q|\overline{K^0}\rangle = |K^0\rangle - |\overline{K^0}\rangle$$

CP violation: $H_{12} \neq H_{21}$ $\quad \langle K^0|\mathbf{H}_w|\overline{K^0}\rangle \neq \langle \overline{K^0}|\mathbf{H}_w|K^0\rangle$

$$|K_S\rangle = p|K^0\rangle + q|\overline{K^0}\rangle = \left((1+\varepsilon)|K^0\rangle + (1-\varepsilon)|\overline{K^0}\rangle\right)/\sqrt{2(1+\varepsilon^2)} \neq K_1$$

$$|K_L\rangle = p|K^0\rangle - q|\overline{K^0}\rangle = \left((1+\varepsilon)|K^0\rangle - (1-\varepsilon)|\overline{K^0}\rangle\right)/\sqrt{2(1+\varepsilon^2)} \neq K_2$$