Experimental tests of the Standard Model

- Discovery of the $W$ and $Z$ bosons
- Precision tests of the $Z$ sector
- Precision tests of the $W$ sector
- Electro-weak unification at HERA
- Radiative corrections and prediction of the top and Higgs mass
- Top discovery at the Tevatron
- Higgs searches at the LHC

1. Discovery of the $W$ and $Z$ boson

1.1 Boson production in $pp$ interactions

$pp \rightarrow W \rightarrow t\bar{t} + X$

$pp \rightarrow Z \rightarrow t\bar{t} + X$

Similar to Drell-Yan: (photon instead of $W$)

$s = x_q x_{\bar{q}} s$ mit $\langle x_q \rangle = 0.12$

$s = \langle x_q \rangle^2 s = 0.014 \cdot (65 \text{ GeV})^2$

$\rightarrow$ Cross section is small!
1.2 UA-1 Detector

1.3 Event signature: \( p\bar{p} \rightarrow Z \rightarrow f\bar{f} + X \)

High-energy lepton pair:

\[ m_{\ell\ell} = (p_{\ell^+} + p_{\ell^-})^2 = M_Z^2 \]

\( M_Z \approx 91 \text{GeV} \)
1.4 Event signature: \( pp \rightarrow W \rightarrow \ell \bar{\nu}_\ell + X \quad W^\rightarrow \rightarrow e \bar{\nu} \)

**Undetected \( \nu \) – Missing momentum**

**High-energy lepton – Large transverse momentum \( p_t \)**

**How can the \( W \) mass be reconstructed?**

**\( W \) mass measurement**

In the \( W \) rest frame:
- \( |p_\ell| = |p_\bar{\nu}| = \frac{M_W}{2} \)
- \( |p_T| \leq \frac{M_W}{2} \)

Jacobian Peak:
\[
\frac{dN}{dp_t} \sim \frac{2p_t}{M_W} \left( \frac{M_W^2}{4} - p_t^2 \right)^{-1/2}
\]

- Trans. Movement of the \( W \)
- Finite \( W \) decay width
- \( W \) decay is not isotropic

\( M_W \approx 80 \text{ GeV} \)
The Nobel Prize in Physics 1984

Simone van der Meer Carlo Rubbia

"for their decisive contributions to the large project, which led to the discovery of the field particles W and Z, communicators of weak interaction"

S. van der Meer

One of the achievements to allow high-intensity p ¯p collisions, is stochastic cooling of the ¯p beams before inserting them into SPS.

1.5 Production of Z and W bosons in e+e− annihilation

Precision tests of the Z sector

Tests of the W sector

\[ W \rightarrow q\bar{q} \]
2. Precision tests of the Z sector

2.1 Cross section for $e^+ e^- \rightarrow \gamma / Z \rightarrow f \bar{f}$

\[
|M|^2 = \left| \begin{array}{cc}
\gamma & Z \\
\end{array} \right|^2
\]

for $e^+ e^- \rightarrow \mu^+ \mu^-$

\[
M_\gamma = -e^2 (\bar{\gamma} \gamma) \frac{1}{q^\rho} (\bar{\gamma}^\mu e)
\]

\[
M_Z = - \frac{g^2}{\cos^2 \theta_W} \left[ \bar{\gamma} \gamma^\nu \frac{1}{2} \left( g_{\nu\mu} - g_{\nu\rho} g^\nu g^\rho \right) \mu \right] \frac{g_{\nu\rho} - q_\rho q_\nu}{(q^2 - M_Z^2) + i M_Z \Gamma_Z} \left[ \bar{\gamma} \gamma^\nu \frac{1}{2} \left( g_{\nu\mu} - g_{\nu\rho} g^\nu g^\rho \right) e \right]
\]

$Z$ propagator considering a finite $Z$ width

Differential cross section for $e^+ e^- \rightarrow \mu^+ \mu^-$

\[
\frac{d\sigma}{d\cos \theta} = \frac{\pi \alpha^2}{2s} \left[ F_\gamma (\cos \theta) + F_{\gamma Z} (\cos \theta) \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} + F_Z (\cos \theta) \frac{s^2}{(s - M_Z^2)^2 + M_Z^2 \Gamma_Z^2} \right]
\]

\[\gamma\quad \gamma/Z \text{ interference} \quad Z\]

Vanishes at $\sqrt{s}=M_Z$

with

\[
F_\gamma (\cos \theta) = Q_e^2 Q_\mu^2 (1 + \cos^2 \theta) = (1 + \cos^2 \theta)
\]

\[
F_{\gamma Z} (\cos \theta) = \frac{Q_e Q_\mu}{4 s \cos^2 \theta_W} \left[ 2 g_\nu g_{\mu\nu} (1 + \cos^2 \theta) + 4 g_\nu g_{\mu\nu} \cos \theta \right]
\]

\[
F_Z (\cos \theta) = \frac{1}{16 s \cos^2 \theta_W} \left[ (g_{\nu\mu}^2 + g_{\nu\rho}^2)(g_{\nu\rho}^2 + g_{\mu\rho}^2) (1 + \cos^2 \theta) + 8 g_{\nu\rho} g_{\mu\nu} g_{\mu\rho} \cos \theta \right]
\]
At the $Z$-pole $\sqrt{s} \approx M_Z$ → $Z$ contribution is dominant → interference vanishes

$$\sigma_{tot} = \sigma_Z = \frac{4\pi}{3s} \frac{\alpha^2}{16\sin^4\theta_w \cos^4\theta_w} \left[ (g'_\mu)^2 + (g'_\alpha)^2 \right] \left[ (g'_\nu)^2 + (g'_\lambda)^2 \right] \frac{s^2}{(s - M_Z^2)^2 + (M_2\Gamma_Z)^2}$$

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_{\mu}}{M_Z^2} \frac{s}{(s - M_Z^2)^2 + M_2^2\Gamma_Z^2}$$

$$\sigma_Z(\sqrt{s} = M_Z) = \frac{12\pi \Gamma_e \Gamma_{\mu}}{M_Z^2} \frac{s}{\Gamma_Z^2}$$

With partial and total widths:

$$\Gamma_i = \frac{\alpha M_Z}{12 \sin^2\theta_w \cos^2\theta_w} \left[ (g'_\nu)^2 + (g'_\lambda)^2 \right]$$

$$\Gamma_Z = \sum_i \Gamma_i$$

Cross sections and widths can be calculated within the Standard Model if all parameters are known

### 4.2 Measurement of the $Z$ lineshape

**Z resonance curve:**

$$\sigma(s) = 12\pi \frac{\Gamma_e \Gamma_{\mu}}{M_Z^2} \frac{s}{(s - M_Z^2)^2 + M_2^2\Gamma_Z^2}$$

Peak: $\sigma_0 = \frac{12\pi \Gamma_e \Gamma_{\mu}}{M_Z^2} \frac{s}{\Gamma_Z^2}$

- Resonance position $\rightarrow M_Z$
- Height $\rightarrow \Gamma_e \Gamma_{\mu}$
- Width $\rightarrow \Gamma_Z$

Initial state Bremsstrahlung corrections

$$\sigma_{\gamma(Z)} = \frac{1}{4m_e^2/s} \int G(z) \sigma^0_{\gamma(Z)}(zs) \, dz \quad z = 1 - \frac{2E_e}{\sqrt{s}}$$
Resonance looks the same, independent of final state: Propagator is the same.
**Z line shape parameters (LEP average)**

\[
M_Z = 91.1876 \pm 0.0021 \text{ GeV} = \pm 23 \text{ ppm (*)}
\]

\[
\begin{align*}
\Gamma_Z &= 2.4952 \pm 0.0023 \text{ GeV} \\
\Gamma_{\text{had}} &= 1.7458 \pm 0.0027 \text{ GeV} \\
\Gamma_\ell &= 0.08392 \pm 0.00012 \text{ GeV} \\
\Gamma_\mu &= 0.08399 \pm 0.00018 \text{ GeV} \\
\Gamma_\tau &= 0.08408 \pm 0.00022 \text{ GeV}
\end{align*}
\]

3 leptons are treated independently

Assuming lepton universality: \( \Gamma_\ell = \Gamma_\mu = \Gamma_\tau \)

*) error of the LEP energy determination: \( \pm 1.7 \text{ MeV (19 ppm)} \)

http://lepewwg.web.cern.ch/ (Summer 2005)

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**LEP energy calibration: Hunting for ppm effects**

Changes of the circumference of the LEP ring changes the energy of the electrons:

- tide effects
- water level in lake Geneva

Changes of LEP circumference \( \Delta C = 1 \ldots 2 \text{ mm/27km (4} \ldots 8 \times 10^{-8}) \)

![Diagram showing changes in energy and circumference](image-url)
Effect of the French “Train a Grande Vitesse” (TGV)

Vagabonding currents (~1A) from trains

In conclusion: Measurements at the ppm level are difficult to perform. Many effects must be considered!

2.3 Number of light neutrino generations

In the Standard Model:

\[
\Gamma_Z = \Gamma_{\text{had}} + 3\cdot\Gamma_{\nu} + N_{\nu}\cdot\Gamma_{\nu} \quad \text{invisible: } \Gamma_{\text{inv}}
\]

\[
\Gamma_{\text{inv}} = 0.4990 \pm 0.0015 \text{ GeV}
\]

To determine the number of light neutrino generations:

\[
N_{\nu} = \frac{\Gamma_{\text{inv}}}{\Gamma_{\nu}} \cdot \frac{\Gamma_{\nu}^{SM}}{\Gamma_{\nu}}
\]

5.9431 ± 0.0163 = 1.991 ± 0.001 (small theo. uncertainties from \(m_{t,\nu}, M_H\))

\[
N_{\nu} = 2.9840 \pm 0.0082
\]

No room for new physics: \(Z \rightarrow \text{new}\)
4 Forward-backward asymmetry and fermion couplings to Z

Forward-backward asymmetry

\[ \frac{d\sigma}{d\cos\theta} \sim (1 + \cos^2\theta) + \frac{8}{3} A_{FB} \cos\theta \]

with

\[ A_{FB} = \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B} \]

\[ \sigma_F(\theta) = \int_0^{\pi/2} \frac{d\sigma}{d\cos\theta} d\cos\theta \]

At the Z-pole \( \sqrt{s} = M_Z \)

→ Z contribution is dominant
→ interference vanishes

\[ A_{FB} = 3 \cdot \frac{g_V^0g_A^0}{(g_V^0)^2 + (g_A^0)^2} \cdot \frac{g_V^\mu g_A^\mu}{(g_V^\mu)^2 + (g_A^\mu)^2} \]

For the reaction \( e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^- \)

\[ e^+ e^- \rightarrow Z \rightarrow \mu^+ \mu^- \]
Forward-backward asymmetry

- Away from the resonance $A_{FB}$ is large
  $\rightarrow$ interference term dominates
  $$A_{FB} \sim g_A^e g_A^V \cdot \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_{Z^2}^2}$$
- At the $Z$ pole: Interference = 0
  $$A_{FB} \sim g_A^e g_A^V$$
  $\rightarrow$ very small because $g_V$ small in SM

Fermion couplings

- Away from the resonance $A_{FB}$ is large
  $\rightarrow$ interference term dominates
  $$A_{FB} \sim g_A^e g_A^V \cdot \frac{s(s - M_Z^2)}{(s - M_Z^2)^2 + M_{Z^2}^2}$$
- At the $Z$ pole: Interference = 0
  $$A_{FB} \sim g_A^e g_A^V$$
  $\rightarrow$ very small because $g_V$ small in SM

Asymmetries together with cross sections allow the determination of the fermion couplings $g_A$ and $g_V$.

Confirms lepton universality
Higher order corrections seen

Asymmetries together with cross sections allow the determination of the fermion couplings $g_A$ and $g_V$. 

Confirms lepton universality
Higher order corrections seen
I. Precision tests of the W sector (LEP2 and Tevatron)

\[ e^+ e^- \rightarrow WW \rightarrow ffff \]

Threshold behavior of the cross section (phase space) for \(ee\rightarrow WW\) production:

Phase space factor = \(f(M_W, \sqrt{s})\):
- Allows determination of \(M_W\)

\[ \sigma_{WW}(\text{pb}) \]

\(\sqrt{s}\) (GeV)

\[ \sigma_{WW}(\text{pb}) \]

\(\sqrt{s}\) (GeV)

W decays

\(W\) \[ q^-, q_d, \bar{\nu}_d, \bar{\nu}_d \]

WW \[ qq\nu \ 44\% \]
\[ qqqq \ 45\% \]
\[ l\nu\nu \ 11\% \]
W branching ratios

ALEPH  
DELPHI  
L3      
OPAL    

LEP \( W \rightarrow e\nu \) \( 10.59 \pm 0.17 \)
ALEPH  
DELPHI  
L3      
OPAL    

LEP \( W \rightarrow \mu\nu \) \( 10.55 \pm 0.16 \)
ALEPH  
DELPHI  
L3      
OPAL    

LEP \( W \rightarrow \tau\nu \) \( 11.20 \pm 0.22 \)
ALEPH  
DELPHI  
L3      
OPAL    

LEP \( W \rightarrow l\nu \) \( 10.74 \pm 0.09 \)

\( Br(W \rightarrow q\bar{q}) = (67.77 \pm 0.28)\% \)

Lepton universality tested to 2%

Invariant W mass reconstruction

ALEPH Preliminary \( q\bar{q}q\bar{q} \)

ALEPH Preliminary \( e\nu\nu \)

More difficult: pairing ambiguities

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Mass of the W Boson ( M_W ) [GeV]</th>
</tr>
</thead>
<tbody>
<tr>
<td>ALEPH</td>
<td>80.412 \pm 0.042</td>
</tr>
<tr>
<td>DELPHI</td>
<td>80.404 \pm 0.074</td>
</tr>
<tr>
<td>L3</td>
<td>80.376 \pm 0.077</td>
</tr>
<tr>
<td>OPAL</td>
<td>80.459 \pm 0.065</td>
</tr>
<tr>
<td>LEP</td>
<td>80.379 \pm 0.058</td>
</tr>
</tbody>
</table>

p\bar{p}-colliders

\( 80.454 \pm 0.059 \)

LEP2

\( 80.412 \pm 0.042 \)

Average

\( 80.426 \pm 0.034 \)

NuTeV

\( 80.136 \pm 0.084 \)

LEP1/SLD

\( 80.373 \pm 0.033 \)

LEP1/SLD/m_{t}

\( 80.378 \pm 0.023 \)
Effect of triple gauge coupling

Data confirms the existence of the γ/ZWW triple gauge boson vertex

4. Electro-weak unification, as visible at HERA

\[
\frac{d\sigma_{NC}}{dQ^2} \sim \frac{1}{(Q^2)^2} + \frac{1}{Q^2(Q^2 + M_Z^2)} + \frac{1}{(Q^2 + M_Z^2)^2}
\]

\[
\gamma \quad \gamma/Z \quad Z
\]

\[
\frac{d\sigma_{CC}}{dQ^2} \sim \frac{1}{(Q^2 + M_W^2)^2}
\]

\[
\gamma
\]
4. Electro-weak unification, as visible at HERA

\[
\frac{d\sigma_{\text{NC}}}{dQ^2} \sim \frac{1}{(Q^2)^2} + \frac{1}{Q^2(Q^2 + M_Z^2)} + \frac{1}{(Q^2 + M_Z^2)^2}
\]

HERA

\[
\frac{d\sigma_{\text{CC}}}{dQ^2} \sim \frac{1}{(Q^2 + M_W^2)^2}
\]

5. Higher order corrections and the Higgs mass

Lowest order SM predictions

\[
\rho = \frac{m_W^2}{m_Z^2 \cos^2 \theta_W} = 1
\]

\[
\sin^2 \theta_W = 1 - \frac{m_W^2}{m_Z^2} = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W G_F}
\]

\[
\alpha(m_Z^2) = \frac{\alpha(0)}{1 - \Delta \alpha}
\]

Including radiative corrections

\[
\bar{\rho} = 1 + \Delta \rho
\]

\[
\sin^2 \theta_{\text{eff}} = (1 + \Delta \kappa) \sin^2 \theta_W
\]

\[
m_W^2 = \frac{\pi \alpha}{\sqrt{2} \sin^2 \theta_W G_F} (1 + \Delta r)
\]

\[
\Delta \rho, \Delta \kappa, \Delta r = f(m_H^2, \log(m_H), ...)
\]
Top mass prediction from radiative corrections

The measurement of the radiative corrections:

\[ \sin^2 \theta_{\text{eff}} = \frac{1}{4} \left( 1 - \frac{\mathcal{G}_f}{\mathcal{G}_A} \right) \]

\[ \sin^2 \theta_{\text{eff}} = (1 + \Delta x) \sin^2 \theta_w \]

allows an indirect determination of the unknown parameters \( m_t \) and \( M_H \).

Direct measurement of \( m_t \):

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Mass (GeV)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CDF</td>
<td>176.1 ± 6.8</td>
</tr>
<tr>
<td>DØ</td>
<td>179.0 ± 5.1</td>
</tr>
<tr>
<td>Average</td>
<td>178.0 ± 4.3</td>
</tr>
</tbody>
</table>

Prediction of \( m_t \) by LEP before the discovery of the top at TEVATRON.

Good agreement between the indirect prediction of \( m_t \) and the value obtained in direct measurements confirm the radiative corrections of the SM.

Observation of the top quark at TEVATRON (1995)

\( p\bar{p} \rightarrow 2 \text{ TeV} \)

- \( q\bar{q} \) annihilation (85%)
- Gluon fusion (15%)

Top decay

Channel used for mass reconstruction:

\[ m_t = m_H(b - \text{jet}, W \rightarrow \text{jet} + \text{jet}) \]
Higgs mass prediction from radiative corrections

Awaiting the discovery of the Higgs at the LHC

- $M_H > 114$ GeV (from direct searches)
- $M_H < 144$ GeV (from EW fits)