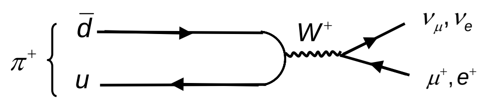
b) Pion decay

$$\pi^+ \to \mu^+ \nu_\mu$$
$$\pi^+ \to e^+ \nu_e$$



<u>Naïve expectation:</u>

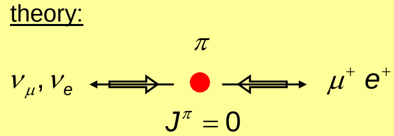
Assuming the same decay dynamics the decay rate to e^+ should be much larger than to μ^+ as the phase space is much bigger.

<u>Measurement:</u> (PDG)

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \cdot 10^{-4}$$

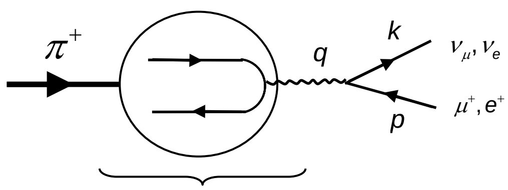
Large suppression due to a dynamic effect.

<u>Qualitative explanation within V-A theory:</u>



Angular momentum conservation forces the lepton into the "wrong" helicity state: suppressed ~ (1-v/c) i.e. for vanishing lepton masses the pion could not decay into leptons.

<u>Determination of decay rates:</u>



Quarks in pion are bound

$$M = \frac{G_F}{\sqrt{2}} \cdot \left(\pi \right)_\mu \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu \right]$$

Reminder: scalar particles

$$M = \frac{G_F}{\sqrt{2}} \cdot \left(p_\mu + k_\mu \right) \cdot f_\pi \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu \right]$$

As the pion spin s_π=0, q is the only relevant 4-vector:

$$q^\mu = p^\mu + k^\mu$$
$$\left(\pi \right)_\mu = q_\mu f_\pi(q^2)$$

Pion form factor:

$$q^2 = m_\pi^2 : \quad f_\pi(q^2) = f_\pi(m_\pi^2) = f_\pi$$

Must be measured !

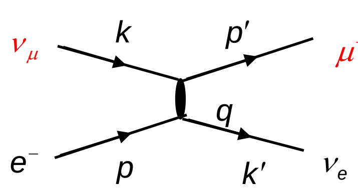
$$\Gamma(\pi^+ \to \mu^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_\mu^2 (1 - \frac{m_\mu^2}{m_\pi^2})$$

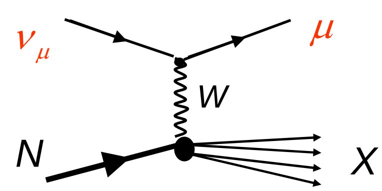
$$\Gamma(\pi^+ \to e^+ \nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_e^2 (1 - \frac{m_e^2}{m_\pi^2})$$

$$\frac{\Gamma(\pi^+ \to e^+ \nu_\mu)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) = 1.275 \cdot 10^{-4} \qquad (1.230 \pm 0.004) \cdot 10^{-4}{}_{PDG}$$

The prediction of the V-A theory is confirmed by the experimental observation. The pion decay rates - although in agreement with the V-A theory - are not a proof of the V-A coupling. Pure V or pure A coupling together with LH neutrinos would result to the same rates.

3.7 Neutrino scattering in V-A theory





Very small cross section for νN scattering: $\sigma(\nu N) \approx E_\nu[\text{GeV}] \text{x} 10^{-38}\ \text{cm}^2$
$= E_\nu[\text{GeV}] \text{x} 10\ \text{fb}$

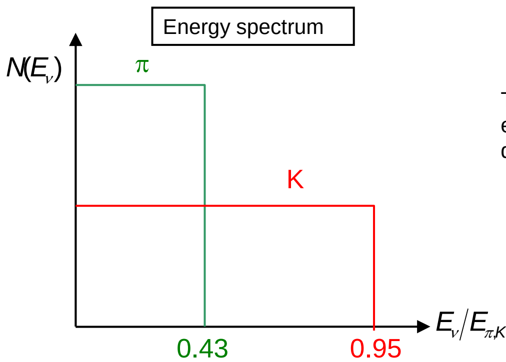
- intense neutrino beams
- large instrumented targets

Neutrino beams

Sources of intense neutrino beams are 2-body decays of intense hadron beams

$$\pi^{\pm} \to \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu}) \qquad K^{\pm} \to \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu})$$

where the pions/kaons are generated in proton-nucleon interactions: p+N→π, K

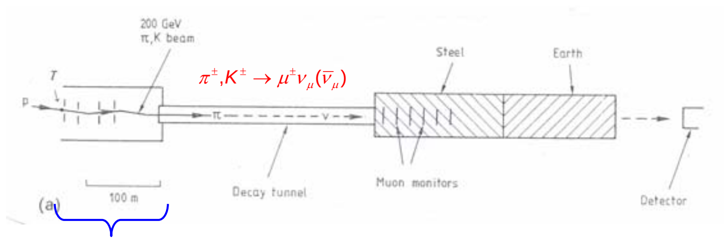


The Lorentz boost transforms the mono-energetic neutrino of the two-body decay into a flat energy spectrum:

$$0 < \frac{E_{\nu}}{E_{\pi,K}} < 1 - \frac{m_{\mu}^2}{m_{\pi,K}^2}$$

Beams contain small admixture (0.5%) of electron neutrinos !

Generation of neutrino beams

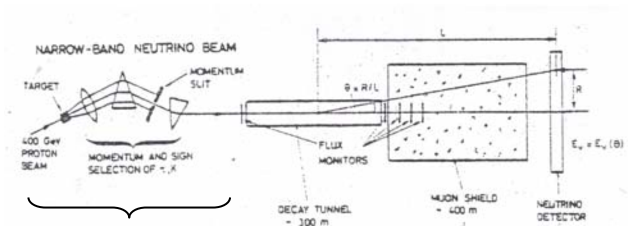


Focussing, momentum & charge selection

1. **~400 GeV proton** beam on a (Be) target: secondary hadrons π, K
2. Momentum and charge selection of π's and K's using a focusing system
3. Selected π's and K's enter a decay tunnel: $\pi^{\pm},K^{\pm} \to \mu^{\pm}\nu_{\mu}(\bar{\nu}_{\mu})$
4. Remaining hadrons and decay muons are filtered by a massive absorber (~400 m iron, concrete, earth): only neutrinos after absorber

There exist 2 different focusing systems for the selection of π's and K's: the two systems lead to neutrino beams with much different energy spectra and fluxes.

Narrow-band neutrino beam:



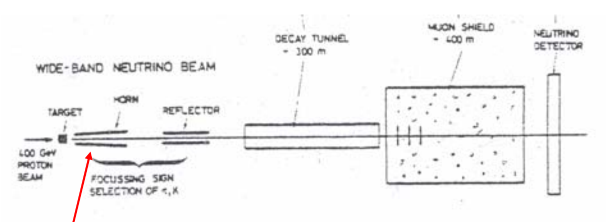
Deflection and focusing magnets to select and focus hadrons (one charge) of a narrow momentum range

➡ $\frac{\Delta p_{K,\pi}}{p_{K,\pi}} \approx 7\%$ (at SPS $p_{K,\pi}$ ~ 200 GeV)

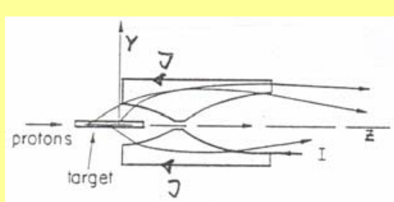
➡ One gets a neutrino beam with a 2-component spectrum

Narrow-band neutrino beam used if one needs to know the exact neutrino flux and wants to achieve max. neutrino energies

Wide-band neutrino beam: Magnetic Horn



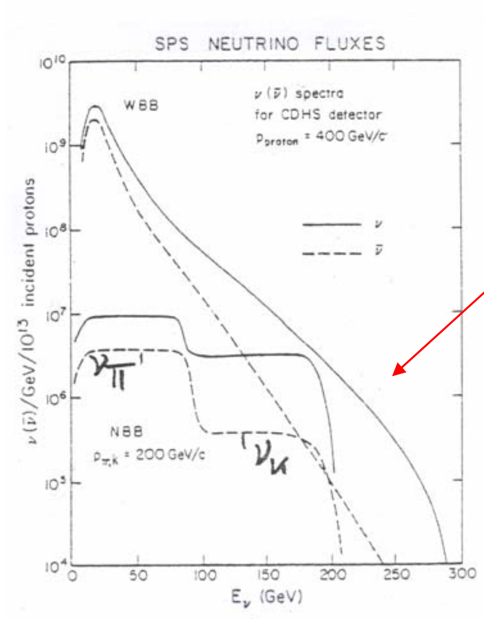
Magnetic Horn (S. van der Meer)



Horn formed from thin aluminum skin

- Short current pulses of 100 to 180 kA
  → large short-time magnetic field perpendicular to particle direction

  → magnetic deflection similar to a paraboloid for hadrons of one charge
- Advantages: use all π^+/K^+ → **large ν flux**
- Disadvantage: large background of wrong “sign” ν’s



At high energies, there are much more ν than $\bar{\nu}$?

Reason:

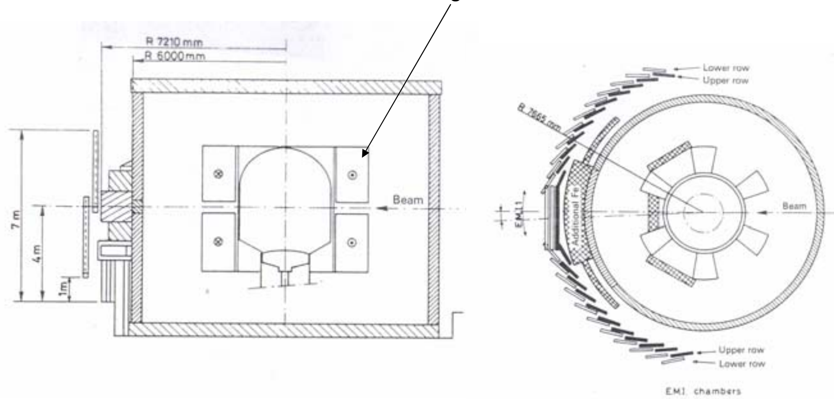
•High energy neutrinos are produced from K decays:

$$pN \to \Lambda \quad K^+ \quad \text{with} \quad K^+ \to \mu^+\nu_\mu$$
$$|uds\rangle |u\bar{s}\rangle$$

(associated strange baryon production: no anti-baryon production !!)

Neutrino detector: Bubble chamber BEBC (CERN)

Magnet coil



3.7 m diameter bubble chamber, filled with lq. hydrogen, deuterium
+ additional muon identifier consisting out of 150 m² of MWPC

Neutrino event in bubble chamber: BEBC (CERN)

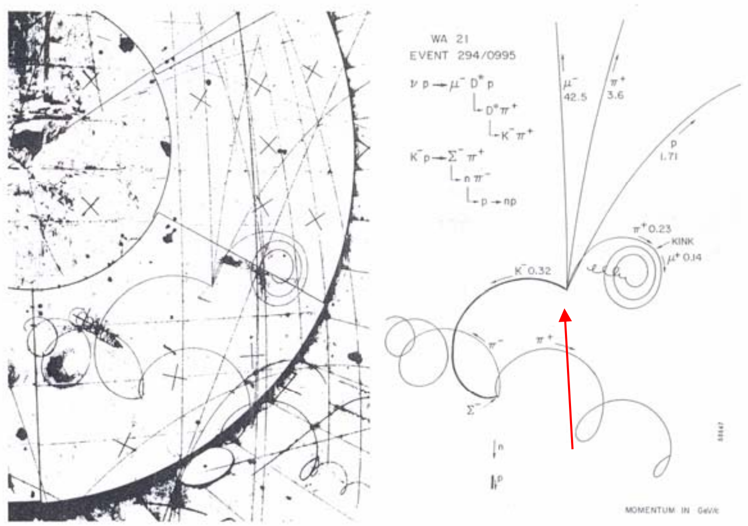
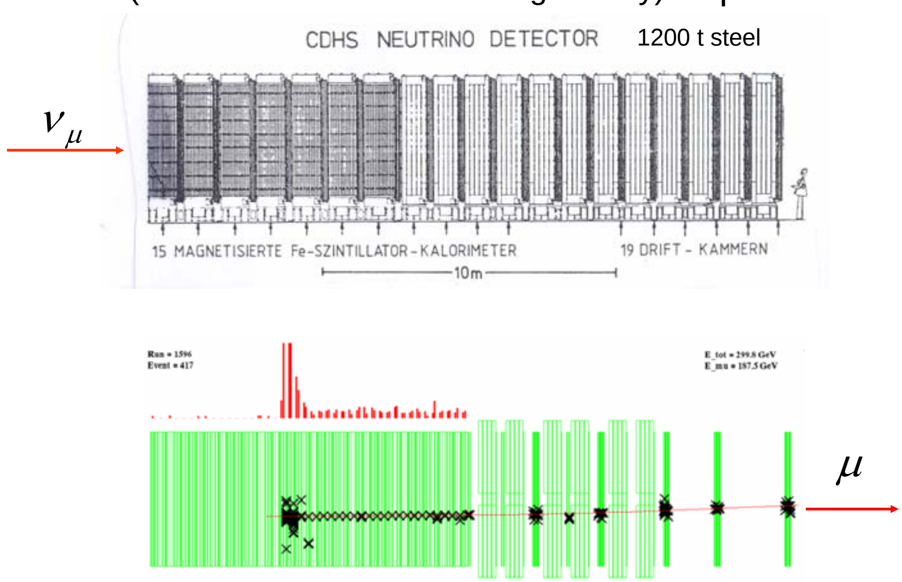
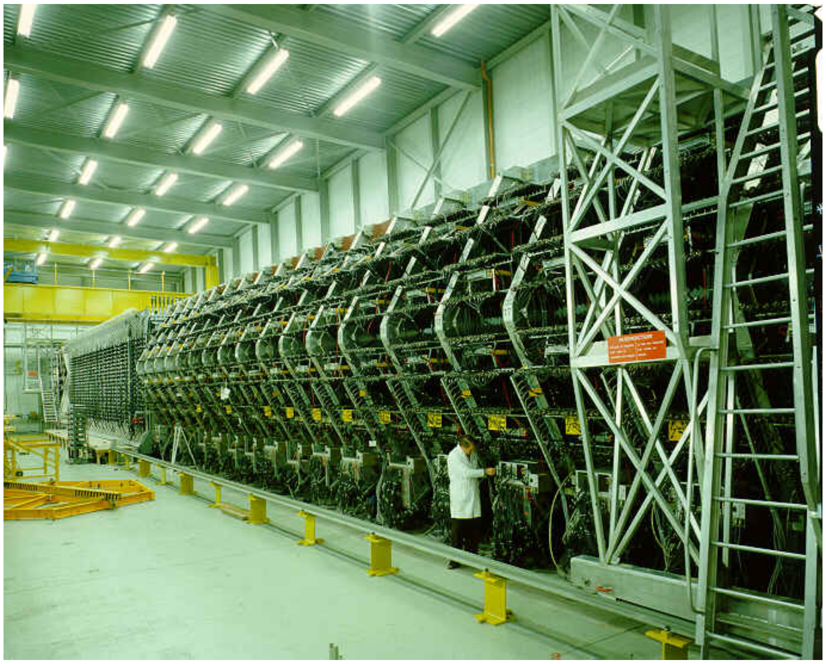


Figure 2.15 Example of charmed-particle production and decay in the hydrogen bubble chamber BEBC exposed to a neutrino beam at the CERN SPS. (Courtesy CERN.)

CHDS (CERN-Dortmund-Heidelberg-Saclay) Experiment

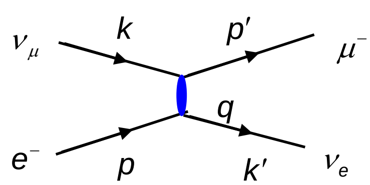




a) <u>Neutrino-electron scattering</u>

$$\nu_\mu e^- \rightarrow \mu^- \nu_e$$

ν_e e^-



$$M = \frac{G_F}{\sqrt{2}} \left[\bar{u}_\nu(k')\gamma_\alpha(1-\gamma^5)u_e(p)\right]\left[\bar{u}_\mu(p')\gamma^\alpha(1-\gamma^5)u_\nu(k)\right]$$

$$\overline{|M|^2} = \frac{1}{2}\sum_{Spins}|M|^2 = \ldots = 64G_F^2(k\cdot p)(k'\cdot p') = 16G_F^2\cdot s^2$$

Limit $m_e \approx m_\mu \approx 0$ $\quad s=(k+p)^2=2kp=2k'p'$

Using the phase space factor of chapter II:

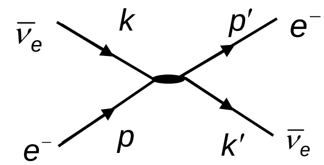
$$\frac{d\sigma}{d\Omega}(\nu_\mu e^-) = \frac{1}{64\pi^2 s}\overline{|M|^2} = \frac{G_F^2 s}{4\pi^2}$$

$$\sigma(\nu_\mu e^-) = \frac{G_F^2 s}{\pi} \longleftarrow = 2m_e E_\nu$$

Although effective 4-fermion theory works well for low q^2 it violates unitarity bound for high q^2!

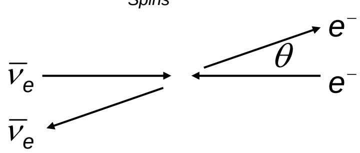
b) Anti-Neutrino-electron scattering (V-A)

$\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$



Crossing: s ⇔ t (u)

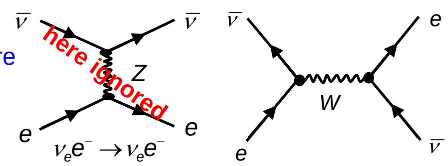
$$\overline{|M|^2} = \frac{1}{2}\sum_{Spins}|M|^2 = 16G_F^2 \cdot t^2 = 4G_F^2 \cdot s^2(1-\cos\theta)^2$$



$$\frac{d\sigma}{d\Omega}(\bar{\nu}e^-) = \frac{G_F^2 s}{16\pi^2}(1-\cos\theta)^2$$

$$\sigma(\bar{\nu}e^-) = \frac{G_F^2 s}{3\pi}$$

Beside the charged current contribution there is of course a neutral current contribution which is ignored in this reasoning.



Result of V-A structure

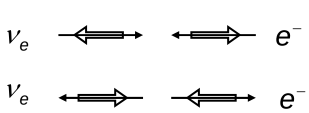
For the charged current (CC) contribution to the (anti) neutrino electron scattering one finds $\frac{\sigma_{\nu e}^{cc}}{\sigma_{\bar{\nu} e}^{cc}} = 3$

Different angular distribution of (anti) neutrino scattering can be understood from a helicity discussion

$$\frac{d\sigma}{d\Omega}(\nu_e e^-) = \frac{G_F^2 s}{4\pi^2}$$

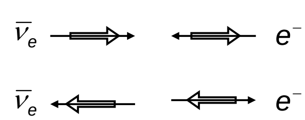
$$\frac{d\sigma}{d\Omega}(\bar{\nu}e^-) = \frac{G_F^2 s}{16\pi^2}(1-\cos\theta)^2$$

$\nu_e e^- \rightarrow \nu_e e^-$ | $\bar{\nu}_e e^- \rightarrow \bar{\nu}_e e^-$



initial

final state

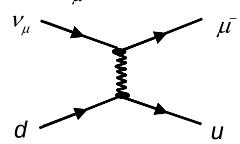


Allowed → isotropic

Forbidden (angular mom.) → (1-cosθ) suppression

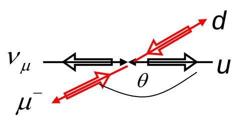
c) (Anti) neutrino-quark scattering

$\nu_\mu d \to \mu^- u$

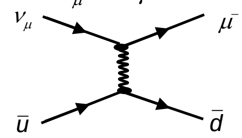


$$\frac{d\sigma}{d\Omega}(\nu_\mu d) = \frac{G_F^2 s}{4\pi^2}$$

$$\sigma(\nu_\mu d) = \frac{G_F^2 s}{\pi}$$

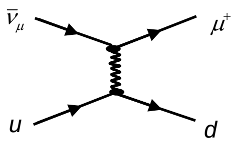


$\nu_\mu \bar{u} \to \mu^- \bar{d}$



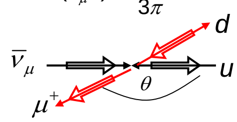
$$\frac{d\sigma}{d\Omega}(\nu_\mu \bar{u}) = \frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u)$$

$\bar{\nu}_\mu u \to \mu^+ d$



$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu u) = \frac{G_F^2 s}{16\pi^2}(1+\cos\theta)^2$$

$$\sigma(\bar{\nu}_\mu u) = \frac{G_F^2 s}{3\pi}$$



$\bar{\nu}_\mu \bar{d} \to \mu^+ \bar{u}$

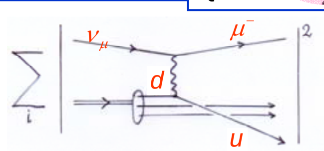
$$\frac{d\sigma}{d\Omega}(\bar{\nu}_\mu \bar{d}) = \frac{d\sigma}{d\Omega}(\nu_\mu d)$$

Neutrinos only interact w/ d and anti-u quarks
Anti-neutrinos only interact w/ u and anti-d quarks

d) Neutrino-nucleon (iso-scalar) scattering

QPM: $x = \frac{Q^2}{2M\nu}$ $y = \frac{\nu}{E}$ $\nu = E - E'$

$$\frac{d\sigma(\nu_\mu N \to \mu^- X)}{dy} = $$



$$\frac{d^2\sigma(\nu N)}{dxdy} = \sum_i f_i(x)\left(\frac{d\sigma_i(\nu q_i)}{dy}\right)_{\hat{s}=xs}$$

$$\frac{d^2\sigma(\nu N)}{dxdy} = \frac{G_F^2 xs}{2\pi}\cdot\left[Q(x) + \bar{Q}(x)(1-y)^2\right]$$

$$\frac{d\sigma(\nu d)}{dy} = \frac{G_F^2 xs}{\pi}$$

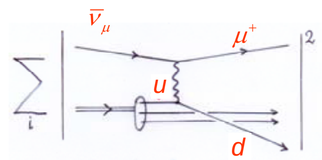
$$\frac{d\sigma(\nu \bar{u})}{dy} = \frac{G_F^2 xs}{\pi}(1-y)^2$$

Unter Vernachlässigung der Massen gilt:

$$y = \frac{1-\cos\theta}{2}$$

$$1-y \approx \frac{1}{2}(1+\cos\theta)$$

$$\frac{d\sigma(\bar{\nu}_\mu N \to \mu^+ X)}{dy} = $$



$$\frac{d^2\sigma(\bar{\nu} N)}{dxdy} = \frac{G_F^2 xs}{2\pi}\cdot\left[\bar{Q}(x) + Q(x)(1-y)^2\right]$$

Total cross section after integration over x and y (0…1):

$$\sigma(\nu N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[Q_l + \frac{1}{3}\overline{Q}_l \right]$$

$$\sigma(\overline{\nu} N) = \frac{G_F^2 M E_\nu}{2\pi} \cdot \left[\overline{Q}_l + \frac{1}{3} Q_l \right]$$

with $Q_l = \int xQ(x)dx$

$$R = \frac{\sigma_{\overline{\nu}N}}{\sigma_{\nu N}} = \frac{1+3\overline{Q}_l/Q_l}{3+\overline{Q}_l/Q_l}$$

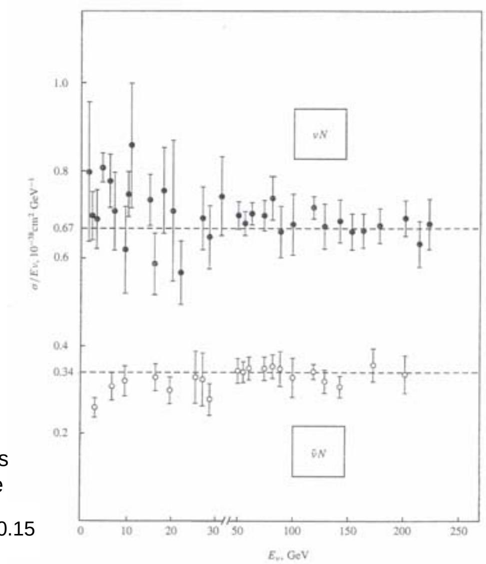


Fig. 5.13. Neutrino and antineutrino cross-sections on nucleons. The ratio σ/E_ν is plotted as a function of energy and is indeed a constant, as predicted in (5.45) and (5.46).

If nucleon consists only of valence quarks ($\overline{Q}$=0): R=1/3 , because of V-A structure

Measurement: $R = \frac{0.34}{0.67} \Rightarrow \overline{Q}_l/Q_l \approx 0.15$

⇒ There are sea quarks !

3.8 Problems with V-A theory

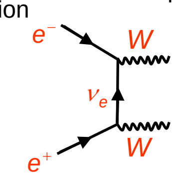
- Cross section for $\nu e^- \to e^- \nu_e$ in 4-fermion ansatz: i.e. cross section goes to infinity if s→∞: violates unitarity $\sigma(\nu e^-) = \frac{G_F^2 s}{\pi}$
- Lee and Wu (1965) introduced a massive exchange boson. Effect of propagator:

$$\frac{G_F}{\sqrt{2}} \mapsto \frac{G_F}{\sqrt{2}} \frac{1}{1-q^2/M_W^2} \qquad \sigma(\nu e^-) \mapsto const.$$

Not trivial, see e.g.: C.Quigg, Gauge Theory of Strong and Weak interaction

This fix leads to a new problem, namely the violation of unitarity of the predicted W pair production !

W pair production:



Violates unitarity

➡ **We need a new theory: Standard Model**

4. Neutral currents (CERN, 1973)

Gargamelle Bubble Chamber

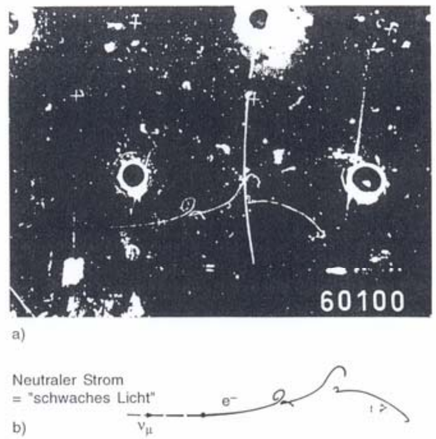
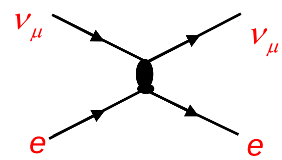


Abb. 9. Dieses erste Ereignis mit einem neutralen schwachen Strom wurde in Aachen entdeckt. Ein Neutrino dringt von links in die Blasenkammer ein (auf dem Bild nicht sichtbar) und wird elastisch an einem Elektron gestreut. Das Elektron ist als rechte Spurkaskade (Bremsstrahlung) zu erkennen. Dieses Bild ist in die Geschichte des CERN eingegangen

One out of three $\nu e \to \nu e$ events

$$\nu_\mu + e^- \to \nu_\mu + e^-$$



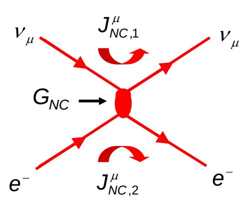
Neutral current νN events appear with a significant rate:

$$R_\nu = \frac{\sigma_{NC}(\nu N \to \nu X)}{\sigma_{CC}(\nu N \to \mu X)} = 0.307 \pm 0.008$$

i.e. approx.1/3 of the νN interactions are neutral current interactions.

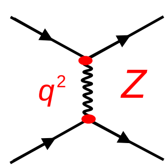
Structure of Neutral currents

Ansatz: four-fermion interaction



$$M = \frac{8G_{NC}}{\sqrt{2}} \cdot J_{NC,1,\mu} \cdot J^\mu_{NC,2}$$

as $q^2 \to 0$ approximation of:



Experimental determination of the structure of the weak neutral currents:

$$J^\mu_{NC} = \bar{u}\,\gamma^\mu \frac{1}{2}(g_V - g_A\gamma^5)u$$

➡ Neutral weak interaction couples to left- and right-handed fermion (chirality) current contributions differently:

$$g_L = \frac{1}{2}(g_V + g_A) \qquad g_R = \frac{1}{2}(g_V - g_A)$$

$$J^\mu_{NC} = \bar{u}\gamma^\mu(g_R\frac{1+\gamma^5}{2} + g_L\frac{1-\gamma^5}{2})u$$

4.1 Vector and axial-vector couplings

Standard Model prediction for g_v and g_A:

	g_V	g_A
ν	$1/2$	$1/2$
ℓ^-	$-1/2 + 2\sin^2\theta_W$	$-1/2$
$u-quark$	$+1/2 - 4/3\sin^2\theta_W$	$1/2$
$d-quark$	$-1/2 + 2/3\sin^2\theta_W$	$-1/2$

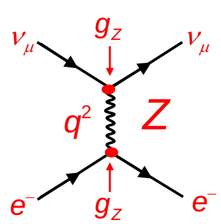
with $\sin^2\theta_W = 1 - \frac{M_W^2}{M_Z^2} \approx 0.223$

In case of the left-handed neutrinos: $J_\nu^\mu = \bar{u}_\nu \gamma^\mu \frac{1}{2}\cdot\underbrace{\frac{1}{2}(1-\gamma^5)}u_\nu$

pure V-A structure

4.2 Effective coupling G_{NC}

(copy of charged current)



$$J_e^\mu = \bar{u}\,\gamma^\mu \frac{1}{2}(g_V - g_A\gamma^5)u$$

$$M = J_{e,\mu}\cdot g_Z \cdot \frac{g_{\mu\nu} - q_\mu q_\nu / M_Z^2}{q^2 - M_Z^2}\cdot g_Z \cdot J_\nu^\nu$$

$$M = \frac{8G_{NC}}{\sqrt{2}}\cdot J_{e,\mu}\cdot J_\nu^\mu$$

As 4-fermion interaction is the $q^2 \rightarrow 0$ approximation of a massive boson exchange:

Comparison of the coupling constants in the $q^2 \rightarrow 0$ limit:

$$\frac{G_{NC}}{\sqrt{2}} = \frac{g_Z^2}{8M_Z^2} = \frac{g_W^2}{8M_W^2}\cdot\underbrace{\frac{g_Z^2 M_W^2}{g_W^2 M_Z^2}}_{\rho=1 \text{ in the SM}} = \frac{g_W^2}{8M_W^2}\cdot\rho = \frac{G_F}{\sqrt{2}}$$