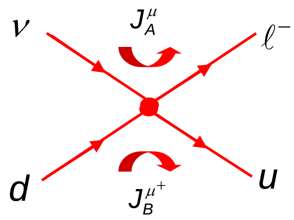
3.2 V-A ansatz for fundamental fermions

Four-Fermion Theory



J_A and J_B are lepton and quark currents

$$J_\ell^\mu = \bar{u}_\ell \gamma^\mu (1-\gamma^5) u_\nu$$

$$J_q^\mu = \bar{u}_u \gamma^\mu (1-\gamma^5) u_d$$

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{A,\mu} \cdot J_B^{\mu +}$$

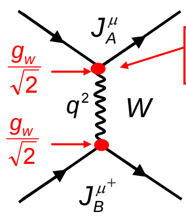
Reminder

$$u_L = \frac{1}{2}(1-\gamma^5)u$$

$$u_R = \frac{1}{2}(1+\gamma^5)u$$

Electroweak Theory

According today's understanding the 4-fermion coupling is the $q^2 \to 0$ limit of W propagator:



g_w = coupling for weak interaction

$$M = \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_{A,\mu} \frac{\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2}\right)}{q^2 - M_W^2} \cdot \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_B^{\nu +}$$

for $q^2 \to 0$: $= \frac{1}{M_W^2}$

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

With $G_F \approx 1.16 \times 10^{-5}\,\text{GeV}^{-2}$ follows w $M_W \approx 80\,\text{GeV}$: $g_w \approx 0.65$

3.3 Helicity and chirality

Dirac spinors: solution spin ↑

i.e. helicity $\lambda = +\frac{1}{2}$

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix}$$

Dirac spinors: solution spin ↓

i.e. helicity $\lambda = -\frac{1}{2}$

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix}$$

Helicity operator $\frac{1}{2} \frac{\vec{\Sigma} \cdot \vec{p}}{|\vec{p}|}$

Measurable quantity ! $\vec{s} = \frac{1}{2}\vec{\sigma}$

Chirality operator:

Projection of left- and right-handed components of spinor u

$$u_L = \frac{1}{2}(1-\gamma^5)u \qquad u_R = \frac{1}{2}(1+\gamma^5)u \qquad \gamma^5 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

Not directly measurable!

$$\frac{1-\gamma^5}{2}u_1 = \underbrace{\frac{1}{2}\sqrt{E+m}\cdot\left(1-\frac{p}{E+m}\right)}_{\approx 0 \text{ for } E \gg m}\cdot\begin{pmatrix}1\\0\\1\\0\end{pmatrix} \rightarrow 0 \quad \text{for } E \gg m$$

$$\frac{1-\gamma^5}{2}u_2 = \underbrace{\frac{1}{2}\sqrt{E+m}\cdot\left(1+\frac{p}{E+m}\right)}_{\approx \sqrt{E} \text{ for } E \gg m}\cdot\begin{pmatrix}0\\1\\0\\-1\end{pmatrix} \rightarrow u_2 \quad \text{for } E \gg m$$

In the relativistic limit helicity states are also eigenstates of the chirality.

Polarization for particles with finite mass

Left handed spinor component $u_1, u_2 \rightarrow u_L, u_R$

$$u_L = \frac{1-\gamma^5}{2}\underbrace{(u_1+u_2)}_{\text{unpolarized}}$$

Helicity polarization of u_L state:

$$Pol = \frac{P(\lambda=+1/2)-P(\lambda=-1/2)}{P(\lambda=+1/2)+P(\lambda=-1/2)} = \frac{(1-p/(E+m))^2-(1+p/(E+m))^2}{(1-p/(E+m))^2+(1+p/(E+m))^2}$$

$$= -\frac{p}{E} = -\frac{v}{c}$$

i.e. the LH spinor component for a particle with finite mass is not fully in the helicity state "spin down" (λ=-½)

Measurable quantity !

For massive particles there is a finite probability to measure the "wrong" helicity state!

3.4 V-A coupling of leptons and quarks

Reminder

$$\bar{u}_\ell \gamma^\mu (1-\gamma^5) u_\nu = \bar{u}_\ell \gamma^\mu u_\nu^L = (\bar{u}_\ell^L + \bar{u}_\ell^R)\gamma^\mu u_\nu^L = \bar{u}_\ell^L \gamma^\mu u_\nu^L$$

In V-A theory the weak interaction couples left-handed lepton/quark currents (right-handed anti-lepton/quark currents) with an universal coupling strength:

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

Weak transition appear only inside weak-isospin doublets:

Not equal to the mass eigenstate

Lepton currents:

1. $\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}$ $\quad j_{e\nu}^\mu = \bar{u}_e \gamma^\mu (1-\gamma^5) u_\nu$
2. $\begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}$ $\quad j_{\mu\nu}^\mu = \bar{u}_\mu \gamma^\mu (1-\gamma^5) u_\nu$
3. $\begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}$ $\quad j_{\tau\nu}^\mu = \bar{u}_\tau \gamma^\mu (1-\gamma^5) u_\nu$

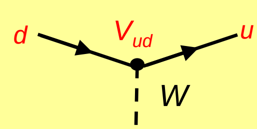
Quark currents:

1. $\begin{pmatrix} u \\ d' \end{pmatrix}$ $\quad j_{du}^\mu = \bar{u}_d \gamma^\mu (1-\gamma^5) u_u$
2. $\begin{pmatrix} c \\ s' \end{pmatrix}$ $\quad j_{sc}^\mu = \bar{u}_s \gamma^\mu (1-\gamma^5) u_c$
3. $\begin{pmatrix} t \\ b' \end{pmatrix}$ $\quad j_{bt}^\mu = \bar{u}_b \gamma^\mu (1-\gamma^5) u_t$

3.5 CKM matrix to describe the quark mixing

One finds that the weak eigenstates of the down type quarks entering the weak isospin doublets are not equal to the their mass/flavor eigenstates:

$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \cdot \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$



Cabibbo-**K**obayashi-**M**askawa mixing matrix

The quark mixing is the origin of the flavor number violation of the weak interaction.

Historical retrospect

Until the early 70s, only 3 quark flavor were known. The weak transition between quarks was described by a quark doublet:

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ \cos\theta_c \cdot d + \sin\theta_c \cdot s \end{pmatrix}$$

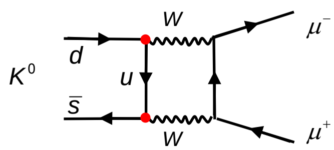
Mixing angle θ_c = Cabibbo-Angle

The mixing described automatically the suppression of $\Delta S=1$ transitions ($\sim\sin^2\theta_c$)

Missing FCNC and GIM mechanism

Historical retrospect

FCNC in the 3 quark model: $K^0 \rightarrow \mu^+ \mu^-$



$M \sim \sin\theta_c \cos\theta_c$

Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

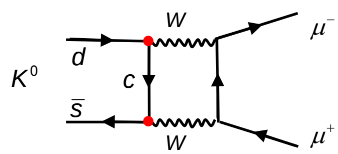
$$\frac{BR(K_L \rightarrow \mu^+ \mu^-)}{BR(K_L \rightarrow all)} = (7.2 \pm 0.5) \cdot 10^{-9}$$

Proposal by Glashow, Iliopoulos, Maiani, 1970:

There exists a fourth quark which builds together with the s quark a second doublet:

GIM

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\theta_c \cdot d + \cos\theta_c \cdot s \end{pmatrix}$$



$M \sim -\sin\theta_c \cos\theta_c$

→ Additional Feynman-Graph for $K^0 \rightarrow \mu\mu$ which compensates the first one:

Prediction of a fourth quark:
Mass prediction BR=f(m_c,…)

Historical retrospect

1964 Discovery of CP violation by J.H. Christenson et al.

$$BR(K_L^0 \rightarrow \pi^+ \pi^-) \neq 0$$

1973 Kobayashi and Maskawa:

CP Violation can be explained through quark mixing if a complete new, third quark generation exists: in this case mixing matrix has **complex elements**.

⬇

Prediction of a 3rd quark generation

1974 Discovery of cc quark state

1977 Discovery of bb quark state

1995 Discovery of top quark

Properties of the CKM-Matrix

$$\mathbf{V_{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Unitary N × N Matrix: $V_{CKM}V_{CKM}^{+} = 1$

	N=3	N=2
→ N^2 Parameters:	9	4
N (N - 1)/2 Euler angles (rotation angles)	3	1
Remaining parameters are phases:	6	3
2N-1 are unmeasurable phase diff	5	3
Observable phases	1	0
$(N-1)^2$ observable parameters	4	1

Parameterization of CKM Matrix: 3 Angles + 1 Phase

PDG choice where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

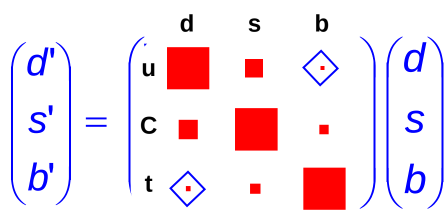
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Wolfenstein Parameterization λ, A, ρ, η

→ hierarchy expressed by orders of λ = **sinθ_c≈ 0.22**

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

Modulus of the matrix elements: $|\mathbf{V}_{ij}|$



PDG 2006

$$|\mathbf{V}_{ij}| = \begin{pmatrix} 0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.0010} & (3.96^{+0.09}_{-0.09}) \times 10^{-3} \\ 0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & (42.21^{+0.10}_{-0.80}) \times 10^{-3} \\ (8.14^{+0.32}_{-0.64}) \times 10^{-3} & (41.61^{+0.12}_{-0.78}) \times 10^{-3} & 0.999100^{+0.000034}_{-0.000004} \end{pmatrix}$$

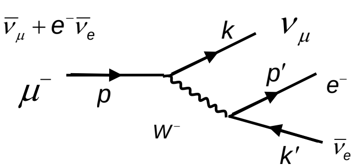
◇ in leading order only the elements $\mathbf{V}_{ub}$ and $\mathbf{V}_{td}$ are complex.

3.6 Test of V-A structure in particle decays

a) Muon decay

Muon lifetime

$\mu^- \rightarrow \nu_\mu + e^- \bar{\nu}_e$



$$M = \frac{G_F}{\sqrt{2}} \left[\bar{u}_\nu(k) \gamma_\alpha (1-\gamma^5) u_\mu(p) \right] \left[\bar{u}_e(p') \gamma^\alpha (1-\gamma^5) v_\nu(k') \right]$$

Analogous to the QED calculations of chapter III one finds after a lengthy calculation:

$$M = \frac{1}{2} \sum_{Spins} |M|^2 = 64 G_F^2 (k \cdot p')(k' \cdot p)$$

Using $d\Gamma = \frac{1}{2E} |M|^2 d\Phi$ ("incoming flux": $2E$) one obtains the electron spectrum in the muon rest frame:

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 \left(3 - \frac{4E'}{m_\mu}\right)$$

with E' = electron energy

$$\frac{1}{\tau} = \Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE'} dE' = \frac{G_F^2 m_\mu^5}{192\pi^3}$$

Measurement of the muon lifetime thus provides a determination of the fundamental coupling $\mathbf{G_F}$

$\tau_\mu = (2.19703 \pm 0.00004) \cdot 10^{-6}\,s$

$G_F = (1.16639 \pm 0.00001) \cdot 10^{-5}\,GeV^{-2}$

Fermi constant measured in muon decays is often called $\mathbf{G_\mu}$