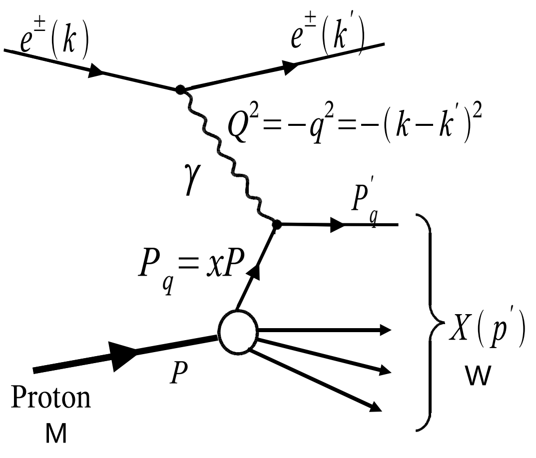
Experimental studies of QCD

1. Elements of QCD
2. Tests of QCD in e^+e^- annihilation
3. Studies of QCD in DIS
4. QCD in $pp\,(p\bar{p})$ collisions

3.2 DIS in the quark parton model (QPM)



- Elastic scattering: W = M

  ⇒ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W ≠ M

  ⇒ scattering described by 2 independent variables

$$(E,\nu),\ (Q^2,x),\ (x,y),\ \ldots$$

x = fractional momentum of struck quark

y = Pq/Pk = fractional energy transfer in proton rest frame

ν = E - E′ = energy transfer in lab

$Q^2 = sxy$ s = CMS energy

$x = \frac{Q^2}{2M\nu}$ (Bjorken x)

3.5 Further DIS Experiments

- To test Bjorken scaling: go to higher Q^2
- To study sea quarks: go to small Bjorken x

$Q^2 = x\,y\,s$ ⟶ go to higher s

- Fixed target μN scattering.

Higher beam energies from muons produced in pN –> π, K –> μ

Today, most precise data (reaching 1-2%) from

			muon energy
BCDMS (Bologna-Cern-Dubna-Munich-Saclay)	CERN SPS	1978–85	120 – 280 GeV
NMC (New Muon Collaboration)	CERN SPS	1986–89	90 – 280 GeV
E665	FNAL Tevatron	1987–92	470 GeV

+ newer dedicated experiments on polarized structure functions

- Fixed target νN scattering

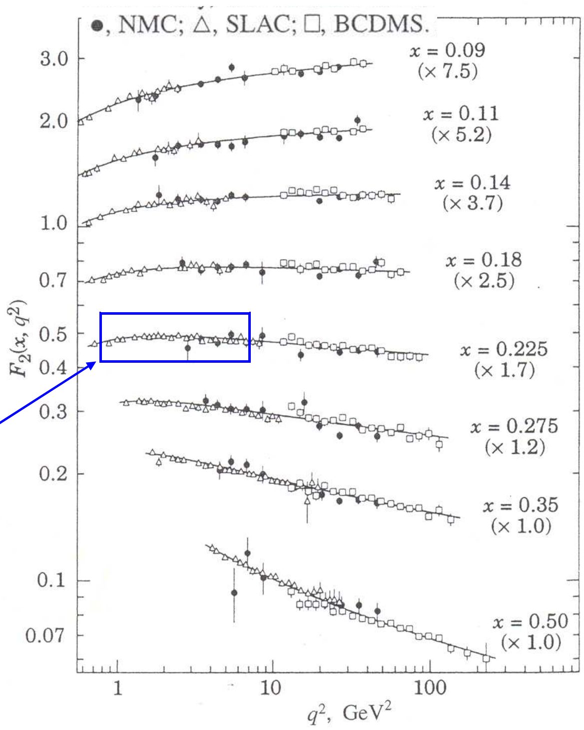
			proton energy
CDHSW (CERN-Dortmund-Heidelberg-Saclay-Warsaw)	CERN	1976–84	400 GeV
CCFR (Chicago-Columbia-Fermilab-Rochester)	FNAL	1984–88	400-600 GeV
NuTeV (based on CCFR detector)	FNAL	1996–97	

- HERA ep collider — DESY — 1992-2007

Scaling violation

$$F_2 = F_2(x, Q^2) = x \sum_i e_i^2 q_i(x)$$

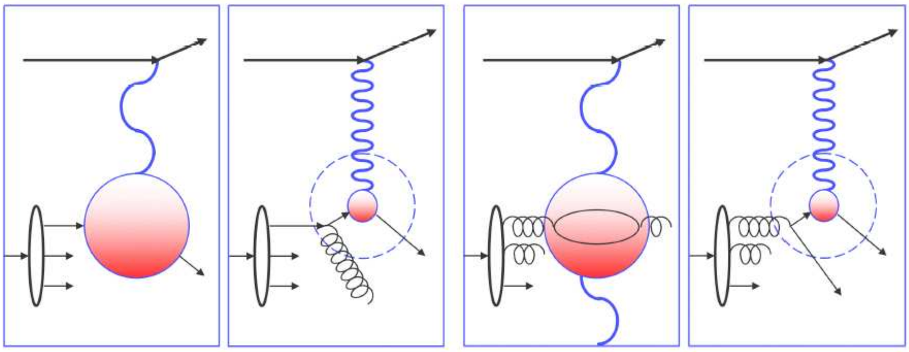
Region of 1st SLAC measurement (1972)



QCD explains observed scaling violation

Large x: valence quarks

Small x: Gluons, sea quarks

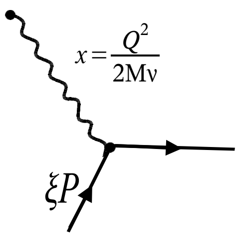


$Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x

$Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed x

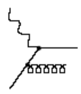
Quantitative description of scaling violation

Quark Parton Model

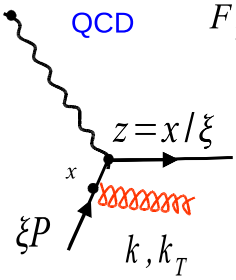


$$F_2(x) = x \sum_i e_i^2 \int_0^1 q_i(\xi) \cdot \delta(x - \xi)\, d\xi = x \sum_i e_i^2 q_i(x)$$

The photon "catches" a quark with the "right" x



QCD



$$F_2(x, Q^2) = x \sum_i e_i^2 \int_0^1 \frac{d\xi}{\xi} q_i(\xi) \cdot \left[\delta\left(1 - \frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi} P_{qq}\left(\frac{x}{\xi}\right) \log \frac{Q^2}{\mu_0^2} \right]$$

$$\hat{\sigma}(\gamma^{¿} q \rightarrow qg) \sim \frac{\alpha_s}{2\pi} P_{qq}(z) \int_{\mu_0^2}^{Q^2} \frac{dk_T^2}{k_T^2}$$

$$\sim \frac{\alpha_s}{2\pi} P_{qq}(z) \log\left(\frac{Q^2}{\mu_0^2}\right)$$

P_{qq} - probability of a quark to emit a gluon and thus to become a quark with momentum reduced by fraction z.

μ^0 cutoff parameter

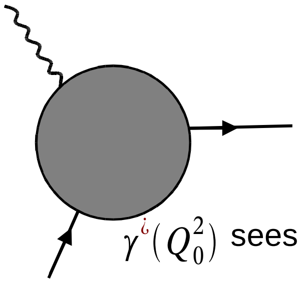
Changing to the quark densities:

$$q_i(x,Q^2)=q_i(x)+\underbrace{\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu_0^2}\int_0^1\frac{d\xi}{\xi}q_i(\xi)\,P_{qq}\left(\frac{x}{\xi}\right)}_{\Delta q(x,Q^2)}$$

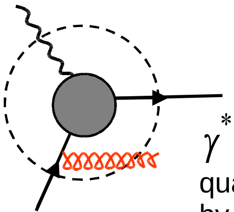
Integro-differential equation for $q(x,Q^2)$:

$$\frac{d}{d\log Q^2}q(x,Q^2)=\frac{\alpha_s}{2\pi}\int_0^1\frac{d\xi}{\xi}q(\xi,Q^2)\,P_{qq}\left(\frac{x}{\xi}\right)$$

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **Al**tarelli, **P**arisi, 1972 – 1977)



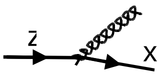
$\gamma^{*}(Q_0^2)$ sees q(x)

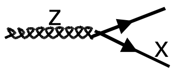


$\gamma^{*}(Q^2>Q_0^2)$ sees that quarks q(x) are surrounded by softer quarks

Evolution of parton densities (quarks and gluons)

evolution of quark density with $\ln Q^2$

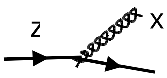


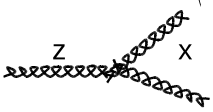


$$\frac{\partial q(x,Q^2)}{\partial\ln Q^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{dz}{z}\left[q(z,Q^2)P_{qq}\left(\frac{x}{z}\right)+g(z,Q^2)P_{qg}\left(\frac{x}{z}\right)\right]$$

$$\frac{\partial g(x,Q^2)}{\partial\ln Q^2}=\frac{\alpha_s}{2\pi}\int_x^1\frac{dz}{z}\left[q(z,Q^2)P_{gq}\left(\frac{x}{z}\right)+g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

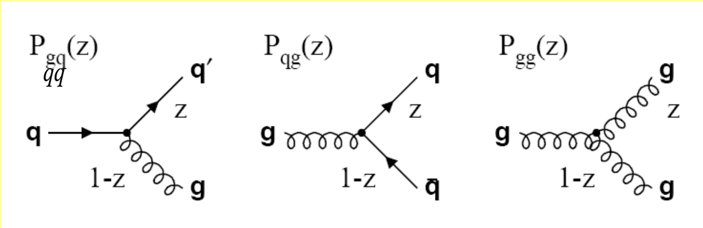
evolution of gluon density with $\ln Q^2$





Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction ε=x/z of the parent parton.

Splitting functions are calculated as power series in α_s up to a given order:



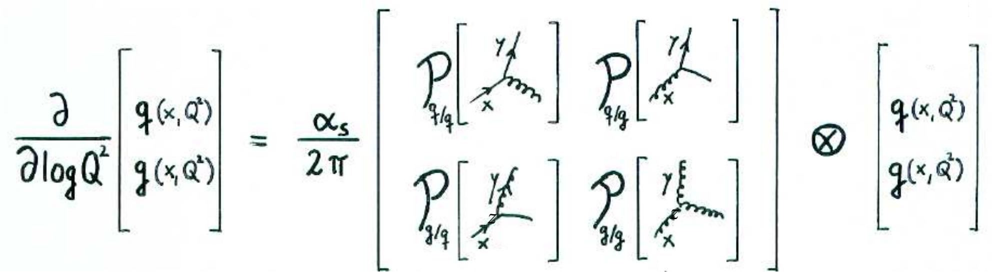
$$P_{ij}(z,\alpha_s)=P_{ij}^0(z)+\frac{\alpha_s}{2\pi}P_{ij}^1(z)+\ldots$$

In leading order: $P_{ij}(z,\alpha_s)\equiv P_{ij}^0(z)$

$$P_{qq}(z)=\frac{4}{3}\frac{1+z^2}{1-z} \qquad P_{gq}(z)=\frac{4}{3}\frac{1+(1-z)^2}{z}$$

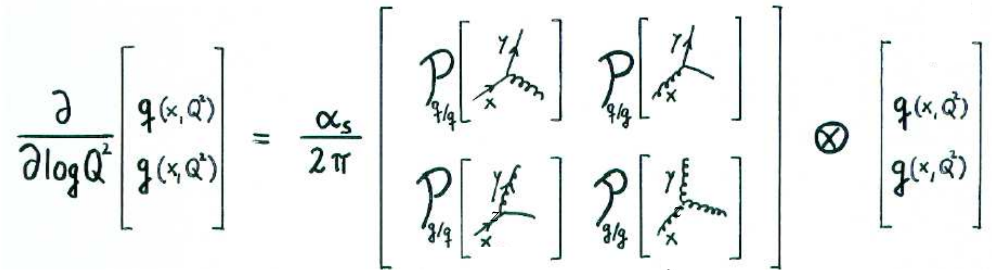
$$P_{qg}(z)=\frac{z^2+(1-z)^2}{2} \qquad P_{gg}(z)=6\left(\frac{z}{1-z}+\frac{1-z}{z}+z(1-z)\right)$$

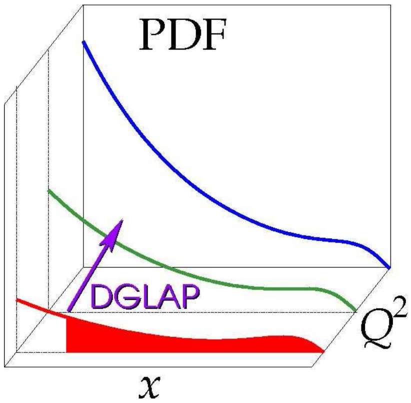
DGLAP Evolution ("symbolic"):



$$P\otimes f(x,Q^2)=\int_x^1\frac{dz}{z}P\left(\frac{x}{z}\right)f(z,Q^2)$$

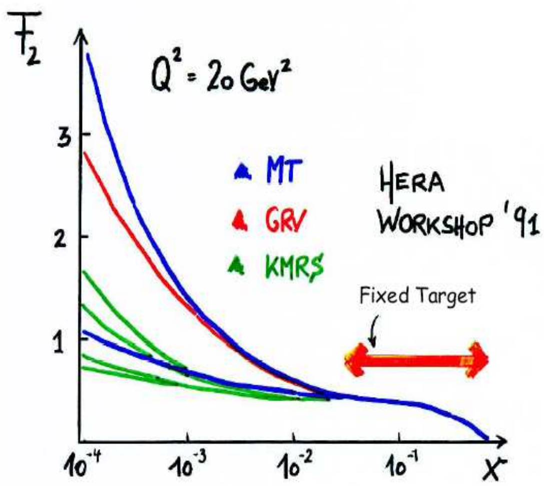
DGLAP Evolution ("symbolic"):





$$P \otimes f(x, Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z, Q^2)$$

Bjorken x dependence of parton densities:



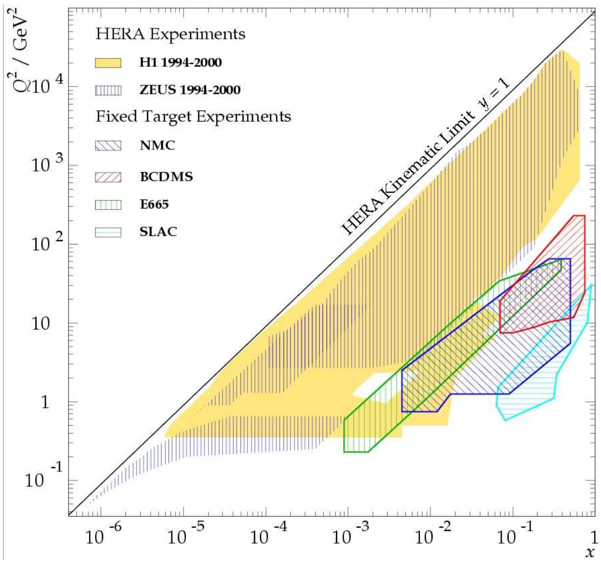
DGLAP:

Q^2 dependence at given x but no prediction for the x dependence of the parton densities.

Status in 1991 (pre HERA): Data limited to a not very small x region. Models to extrapolate to smaller x differ significantly.

Measure structure functions (parton densities) at low x.

Accessing the low x region: HERA ep collider



HERA: y = 1

s~10^5 GeV^2

Fixed Target: y = 1

s<10^3 GeV^2

At HERA:

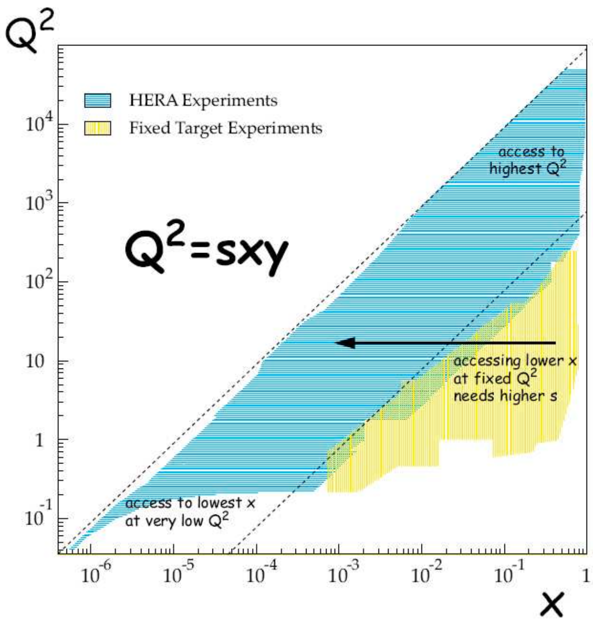
$e^{\pm}$ → 30 GeV ← p 900 GeV

$$s = 4 E_e E_p \approx 10^5 \mathrm{GeV}^2$$

Fixed target equivalent:

$$s = 2 E_e M_p \rightarrow E_e = 50 \mathrm{TeV}$$

Accessing the low x region: HERA ep collider

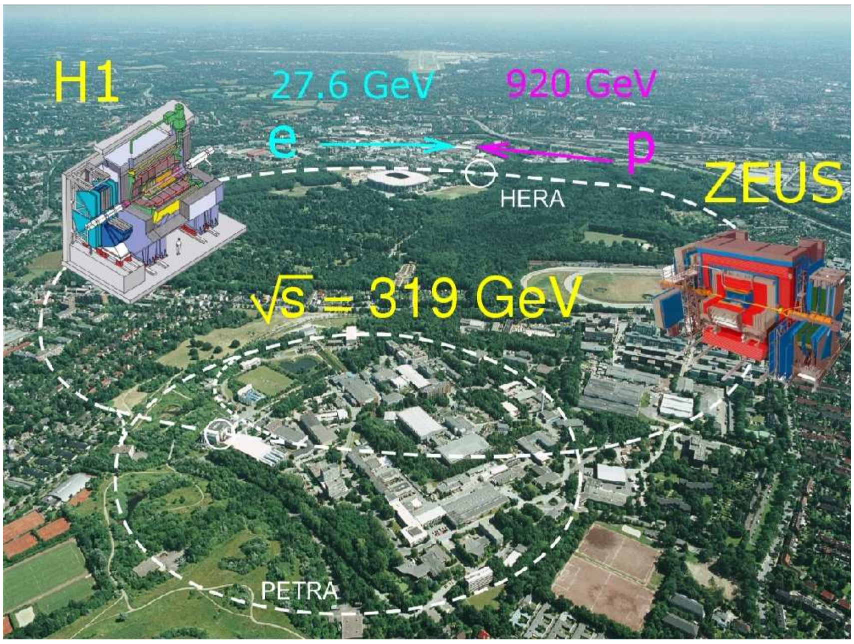


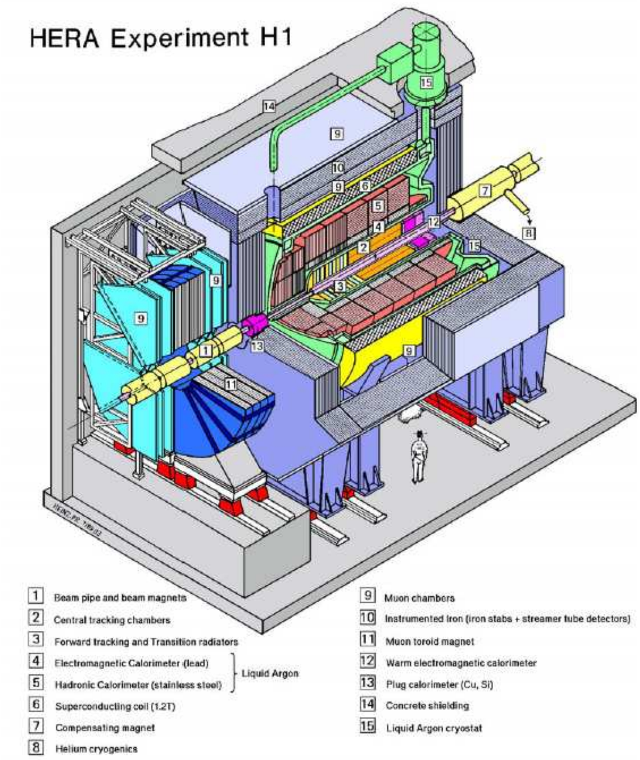
HERA: y = 1

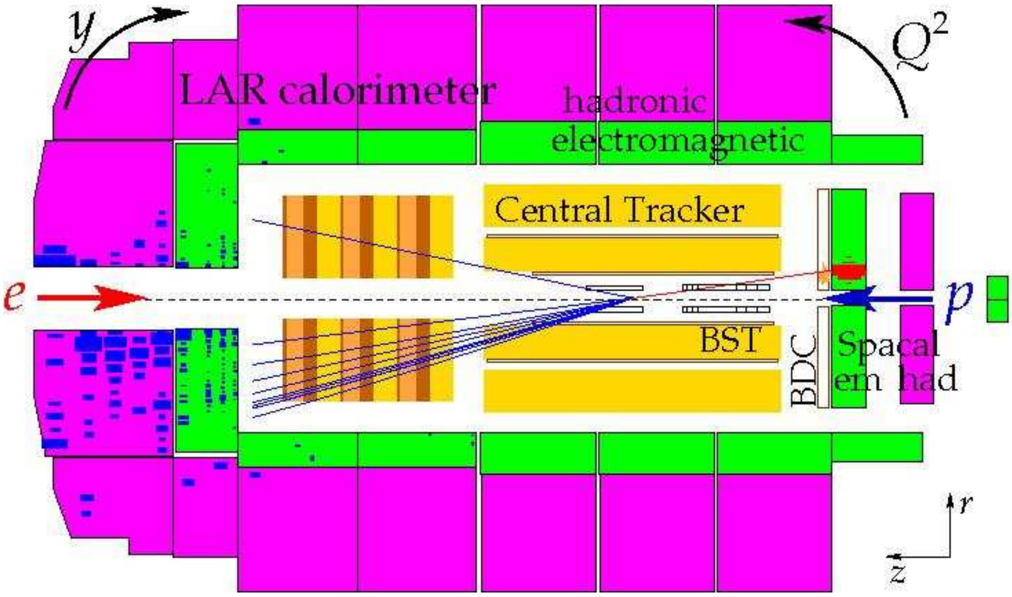
s~10^5 GeV^2

Fixed Target: y = 1

s<10^3 GeV^2

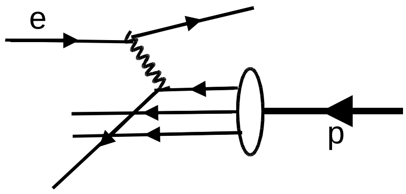

H1
27.6 GeV
920 GeV
e
p
ZEUS
HERA
$\sqrt{s}$ = 319 GeV
PETRA

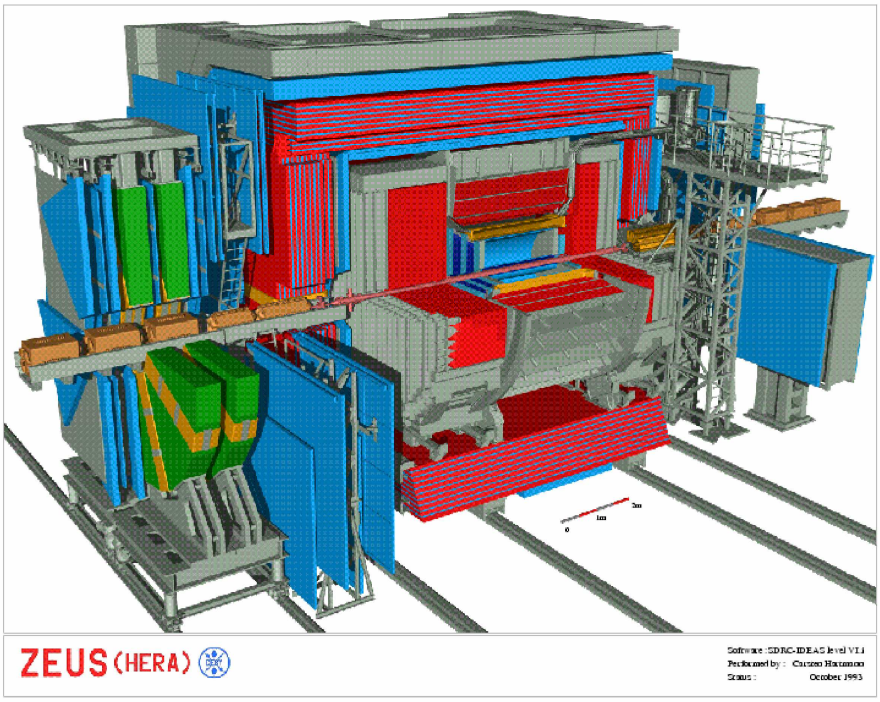

HERA Experiment H1
1 Beam pipe and beam magnets
2 Central tracking chambers
3 Forward tracking and Transition radiators
4 Electromagnetic Calorimeter (lead)
5 Hadronic Calorimeter (stainless steel)
Liquid Argon
6 Superconducting coil (1.2T)
7 Compensating magnet
8 Helium cryogenics
9 Muon chambers
10 Instrumented Iron (iron slabs + streamer tube detectors)
11 Muon toroid magnet
12 Warm electromagnetic calorimeter
13 Plug calorimeter (Cu, Si)
14 Concrete shielding
15 Liquid Argon cryostat

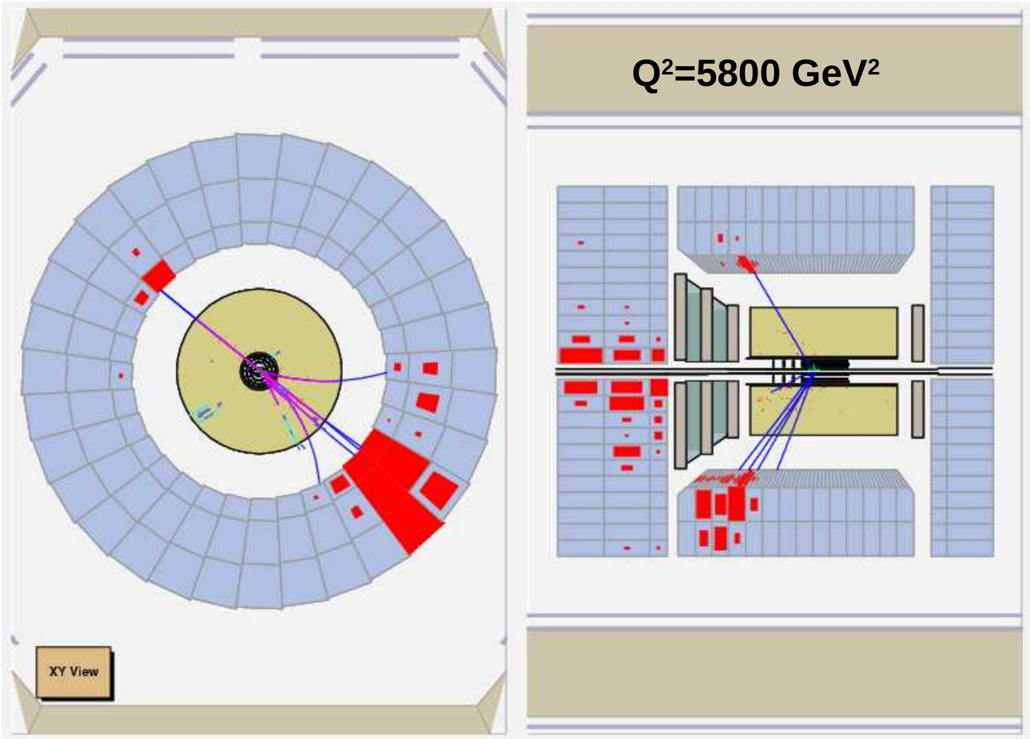


Both scattered electron and hadronic final state are reconstructed:

- Extended kinematic range is covered
- More cross checks -> reduced systematics

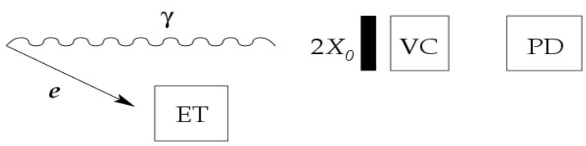


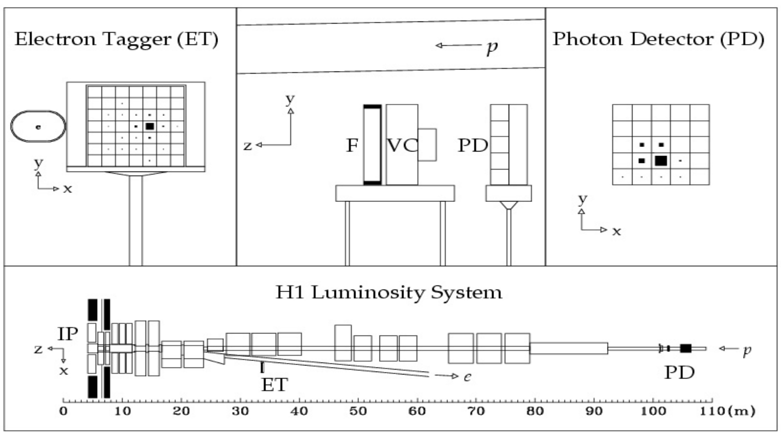


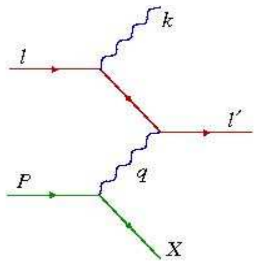


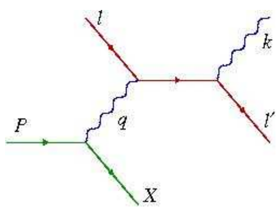
Luminosity measurement

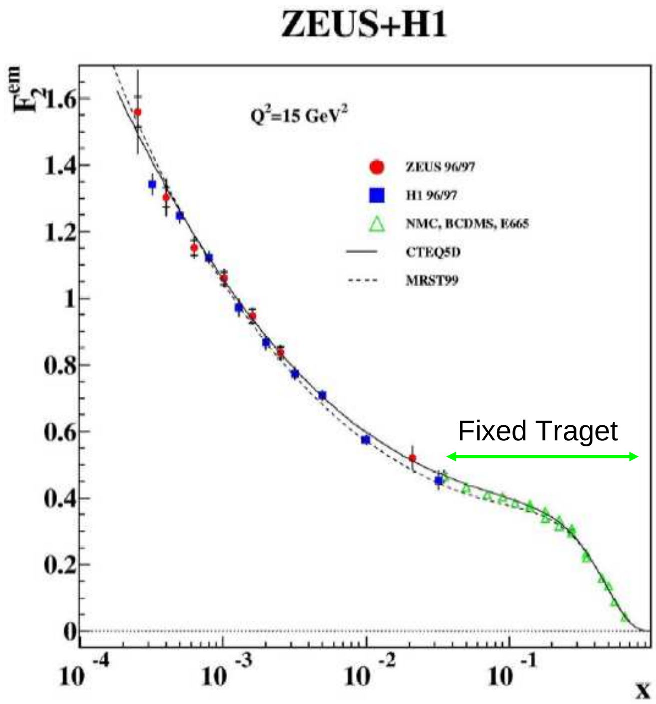
Using low angle elastic *ep* scattering (Bethe-Heitler process)
~1.5% precision









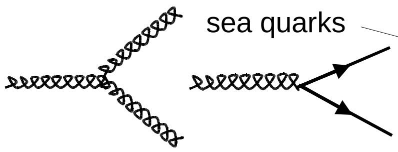


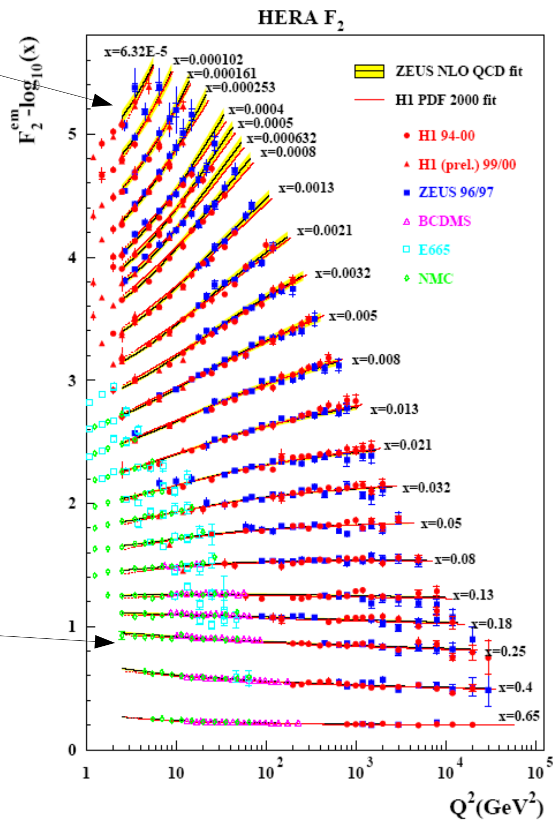
2-3% precision for F_2
at low-medium Q^2

Large increase of $F_2(x)$ for
very small x

When does the rise stop?
will be discussed later

sea quarks



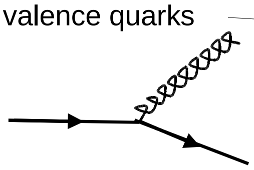


5 orders of magnitude
in x and Q^2

$$F_2(x, Q^2)$$

described by QCD evolution

valence quarks



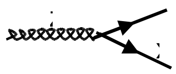
Experimental determination of the gluon density

Using the DGLAP evolution eq. one finds for $F_2(x,Q^2)$:

$$\frac{dF_2(x,Q^2)}{d\ln Q^2} = x\sum_i e_i^2 \frac{\alpha_s(Q^2)}{2\pi}\cdot\int_x^1 \frac{dz}{z}\left[P_{qq}\left(\frac{x}{z}\right)q_i(z,Q^2)+P_{qg}\left(\frac{x}{z}\right)g(z,Q^2)\right]$$

For small x ($x<10^{-2}$):

quark pair production through gluon splitting dominant (1/x gluon spectrum):



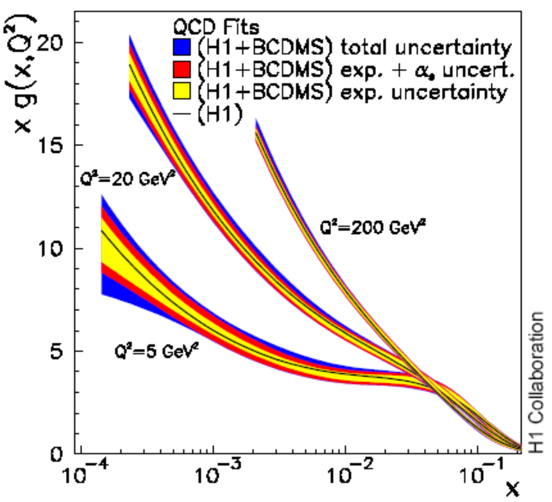
$$\rightarrow\ P_{qg}\left(\frac{x}{z}\right)g(z,Q^2)\ \text{dominant}$$

As an approximation one finds:

$$x\cdot g(x,Q^2)\approx\frac{27\pi}{10\,\alpha_s(Q^2)}\cdot\frac{dF_2(x,Q^2)}{d\ln Q^2}$$

i.e. scaling violation of F_2 at small x measures the gluon density.

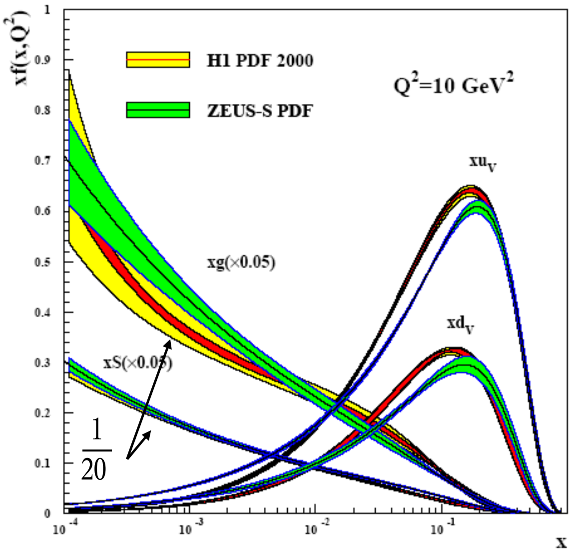
Gluon density $g(x,Q^2)$:



In practice one makes a global DGLAP fit to the measured cross section

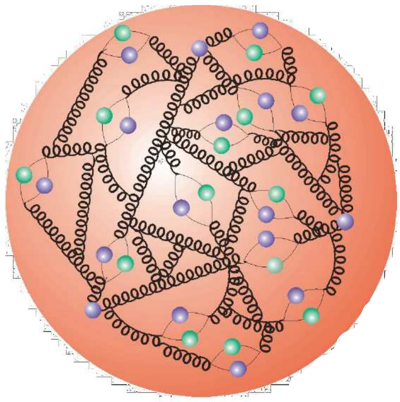
One starts with some *q(x)* and *g(x)* parametrizations at Q^2_0 and evolves to higher Q^2

Structure of the proton as seen by HERA



Valenzquarks = $\int u_v(x) + d_v(x)\,dx = 3$

Gluonen = $\int g(x)\,dx > 30$



PDF fits

Many options - uncertainties:

- Which datasets? [HERA only? Also some fixed target? Also pp data?]
- Which order of perturbation theory [LO, NLO, NNLO]?
- Form of parameterization q(x), g(x) [How many parameters?]

$$x\,p(x,Q^2) = A_p x^{a_p} (1-x)^{b_p} P(x, c_p, \dots)$$

characterizes at x -> 0
sea: a < 0, valence a>0

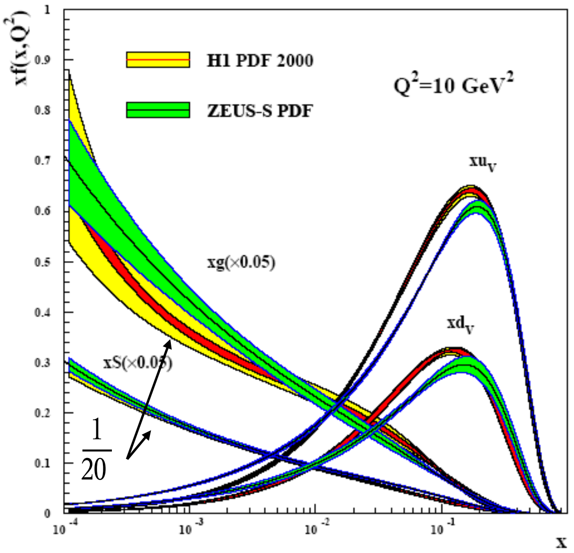
characterizes at x -> 1
always b > 0

"fine tuning"
weakly x-dependent function

- Which PDFs? For each flavour? Some combination?
- Pure DGLAP or some extention/alternative?
- Start-up scale Q^2
- Sum rules
- Heavy quark treatment [What to do with c(x), b(x) at low Q^2?]

H1 and ZEUS do their own fits based mostly on their own data.
Theor. groups (e.g. CTEQ, MRST/MSTW,...) do combined fits of many datasets

Current knowledge of PDFs



Uncertainties:
- *u*-density: ~3%
- *d*-density: ~10%
- *g*-density: 10-20% and more

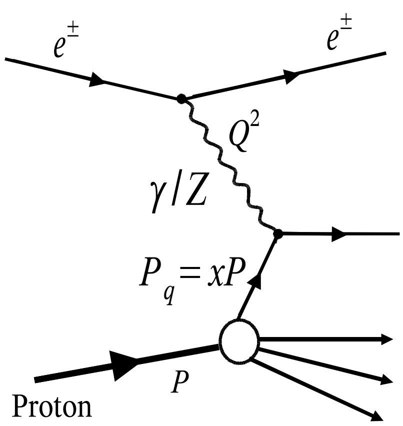
u is better known than *d*
due to el. charge (squared):
$F_2 = x(8/9\ u + 1/9\ d + ...)$

gluon is known worse,
as it is determined from
scaling violations (derivatives)

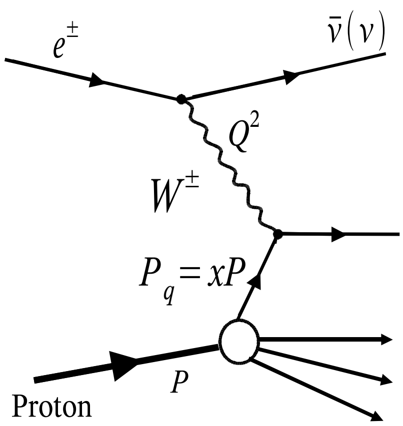
How to separate valence, sea, flavours from HERA only?

Electroweak effects at high Q²

Neutral Current

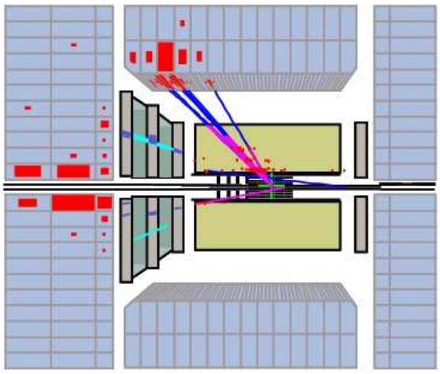


Charge Current



Charge current event

ZEUS detector



Neutrino escapes

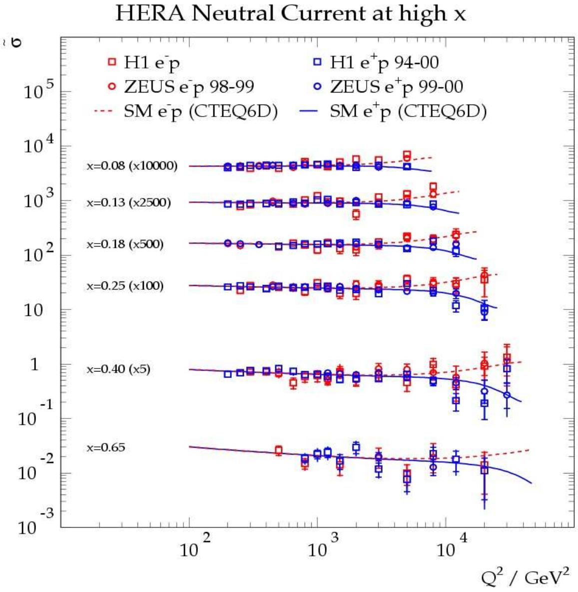
- Missing transverse momentum
- E-Pz < 2 Ee-beam

Q^2 and x are reconstructed from hadronic final state

Neutral current cross section at high Q^2

$$\frac{d^2\sigma^\pm}{dQ^2\,dx} = \left(\frac{2\pi\alpha^2}{Q^4 x}\right)\cdot\left[Y^+ F_2(x) \mp Y^- x F_3(x) - \ldots\right]$$

HERA Neutral Current at high x

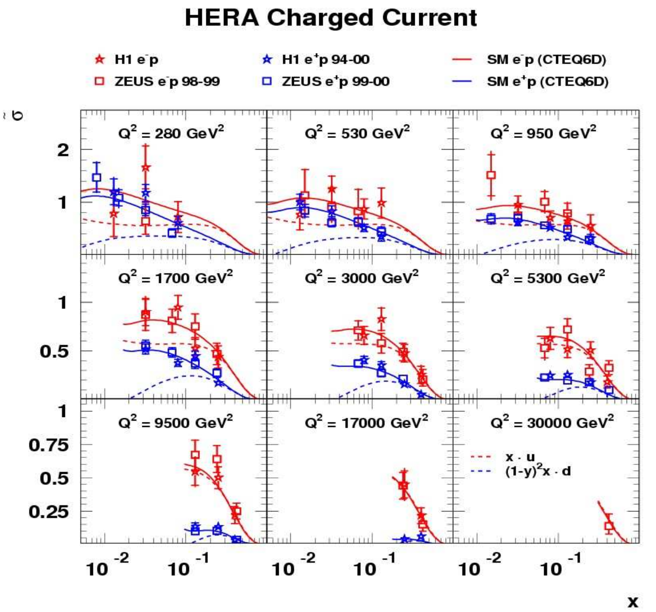


Parity violating function (similar to νN) constraints the valence PDFs

$$x F_3^{\gamma Z} \sim \frac{Q^2}{Q^2 + M_Z^2} \sum e_q a_q (q - \bar{q})$$

Charge current cross section

Is similarly expressed via the structure functions
but with the Fermi constant G_F instead of α



Charge current e⁻p:

$$\sigma^{-}_{cc} \sim xu + (1-y)^2 x\bar{d}$$

At high x: u-quarks dominate

Charge current e⁺p:

$$\sigma^{+}_{cc} \sim (1-y)^2 xd + x\bar{u}$$

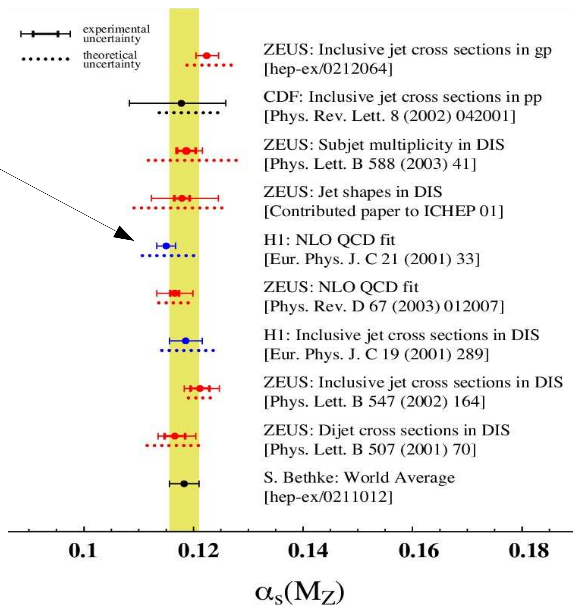
At high x: d-quarks dominate

Because high x
corresponds to high Q²

-> Extract u and d densities

Strong coupling constant $\alpha_s(Q^2)$

- α_s is simultaneously extracted from the fits – most precise!
- Theory error (NLO QCD) is larger than experimental.
- After more than decade of work, NNLO calculations were finished. >1000 diagrams to calculate splitting functions!



$$\alpha_s(M_Z^2) = 0.1186 \pm 0.0011\,(\mathrm{exp.}) \pm 0.005\,(\mathrm{theor.})$$