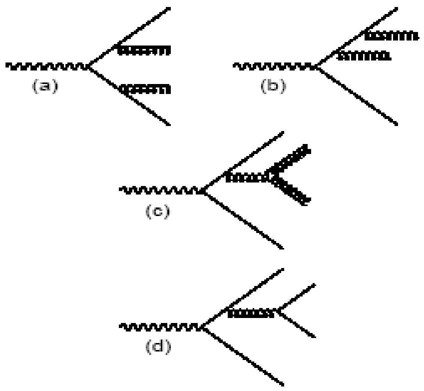
2.3 Multi-jet events and gluon self coupling

Non-abelian gauge theory (SU(3))

4-jet events



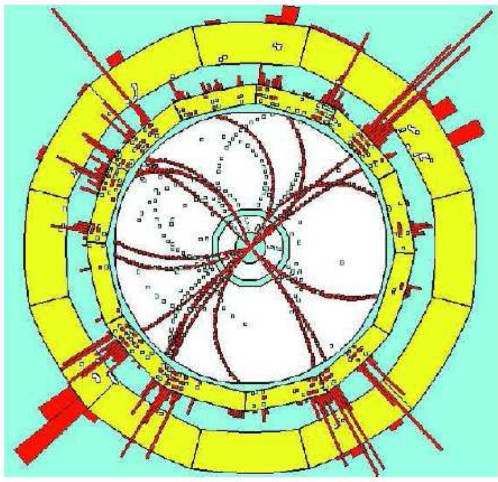


Figure 1: Hadronic event of the type $e^+e^- \to 4$ jets recorded with the ALEPH detector at LEP-I.

Multiple jets and jet algorithm

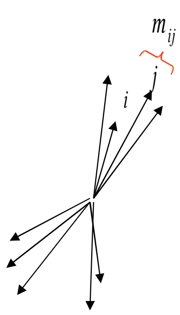
Durham Jet Algorithm

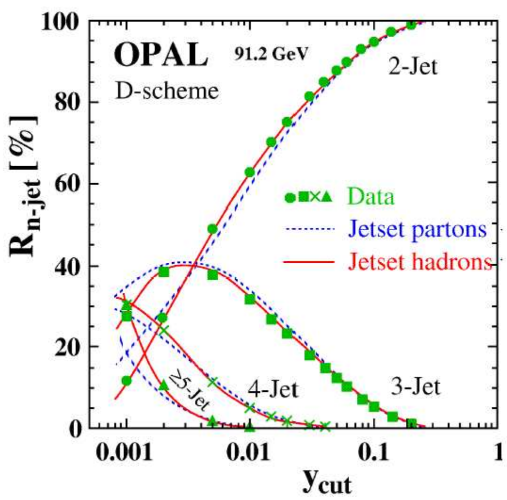
Hadronic particles are i and j grouped to a pseudo particle k as long as the invariant mass is smaller than the **jet resolution parameter**:

$$\frac{m_{ij}^2}{s} < y_{cut}$$

$_{ij}$ is the invariant mass of i and j.

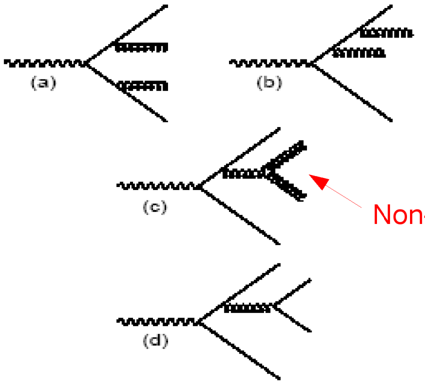
Remaining pseudo particles are **jets.**



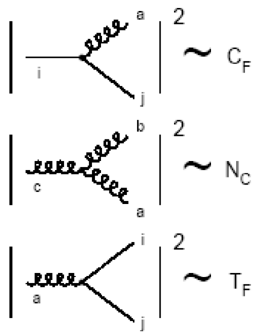


Note: small hadronisation corrections down to y ~ 0.001

-jet events



Color factors:



et cross section:

$$d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[F_A + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_B + \frac{N_C}{C_F} F_C\right]$$

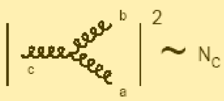
$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2 \left[\frac{T_F}{C_F} N_f F_D + \left(1 - \frac{1}{2}\frac{N_C}{C_F}\right) F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions

Group	N_C	C_F	T_F
U(1)	0	1	1
$U(1)_3$	0	1	3
SU(N)	N	$(N^2-1)/2N$	1/2
SU(3)	3	4/3	1/2

ngular correlations of jets in 4-jet events

$N_C/C_F \neq 0$ → contribution from gluon
elf-coupling in the 4-jet events



his should lead to the enhacement in
tal cross section. But it is difficult to compare
ith theory due to renormalisation uncertainty.

stead, exploiting angular distributions:

engtsson-Zerwas angle

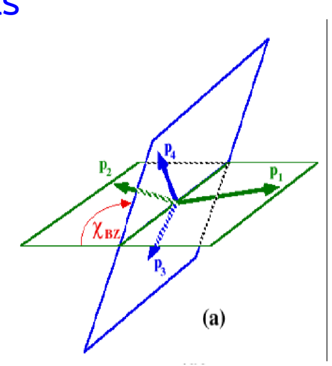
$$\cos\chi_{BZ} \propto (\vec{p}_1 \times \vec{p}_2)\cdot(\vec{p}_3 \times \vec{p}_4)$$

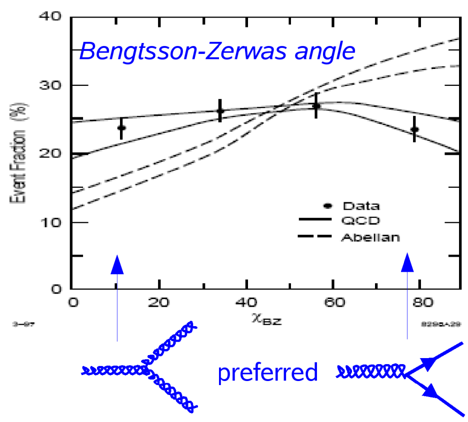
Nachtmann-Reiter angle

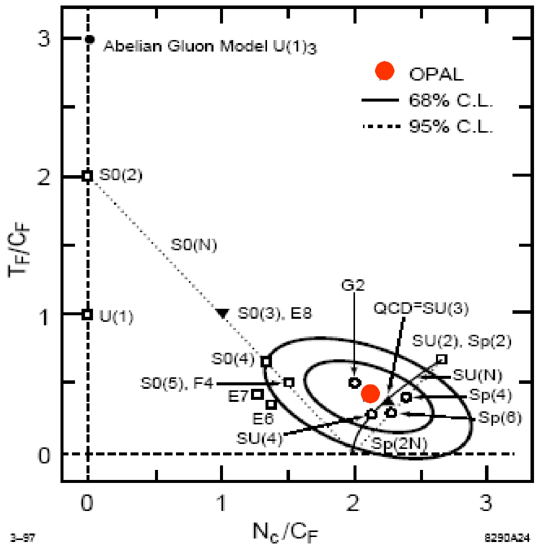
$$\cos\theta_{NR} \propto (\vec{p}_1 - \vec{p}_2)\cdot(\vec{p}_3 - \vec{p}_4)$$

Körner-Schierholz-Willrodt angle ...

ow us to measure the ratios T_F/C_F and N_C/C_F





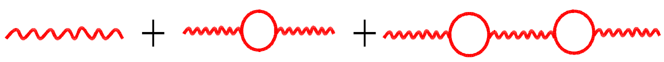


Confirms QCD prediction (SU(3)) and gluon self-coupling:

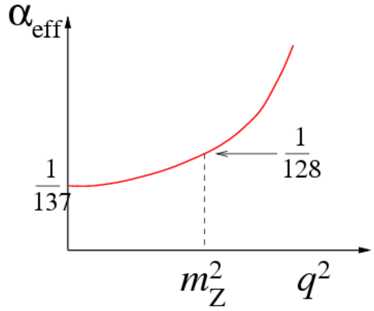
$T_F/C_F = 0.375$ and $N_C/C_F = 2.25$

2.4 Strong coupling constant α_s

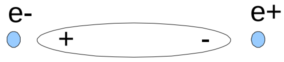
QED: Running coupling constant



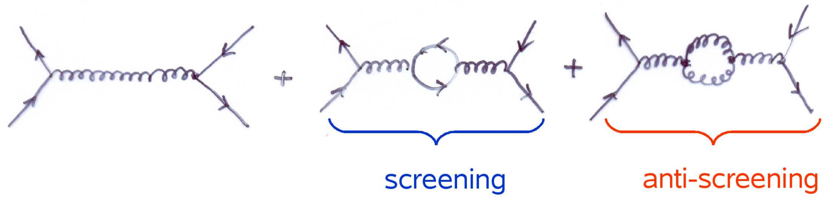
$$\alpha(q^2) = \frac{\alpha}{\underbrace{1 - \frac{\alpha}{3\pi} \cdot \sum_f Q_f^2 \cdot \log\frac{q^2}{m_f^2}}_{-\alpha\, \beta_0^{QED} \log\frac{Q^2}{\mu^2}}}$$



QED vacuum works like dielectric



QCD propagator corrections:



Running strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{1}{12\pi}(33 - 2n_f)\log\frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{1}{12\pi}(33 - 2n_f)$$

n_f = active quark flavors

μ^2 = renormalization scale

conventionally $\mu^2 = M_Z^2$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \log\frac{Q^2}{\mu^2}}$$

"+" is possible only in non-abelian theories!

Here "+" remains for $n_f < 17$

QCD vacuum works like "paramagnetic"

Introduce scale Λ_{QCD} at which perturbative solutions diverge:

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\beta_0 \log\frac{Q^2}{\mu^2}} \quad \longleftarrow \mu^2 = \Lambda_{QCD}^2$$

$$\Lambda_{QCD} \approx 200 \text{ MeV}$$

(parameter, must be determined experimentally)

$$\Rightarrow \frac{1}{\alpha_s(Q^2)} = \underbrace{\frac{1}{\alpha_s(\Lambda_{QCD}^2)}}_{=0} + \beta_0 \ln(Q^2/\Lambda_{QCD}^2)$$

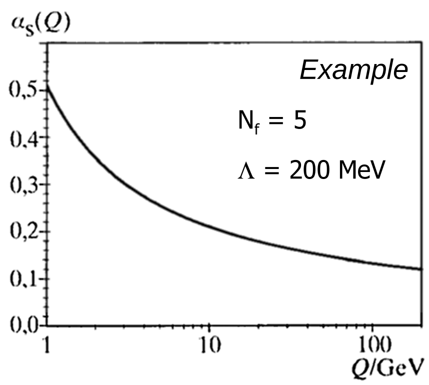
$$\Rightarrow \alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda_{QCD}^2)}$$

→ Confinement for small Q^2

→ Asymptotic freedom for large Q^2

For large Q^2 quarks can be treated as free particles: **→ Quark Parton Model**

Gross&Wilczek (1973), Politzer (1974)



The Nobel Prize in Physics 2004



David J. Gross	**H. David Politzer**	**Frank Wilczek**

"for the discovery of asymptotic freedom in the theory of the strong interaction"

2.5 Measurement of strong coupling α_s

➡ α_s measurements are done at given scale Q^2: $\alpha_s(Q^2)$

a) α_s from total hadronic cross section

$$\sigma_{had}(s)=\sigma_{had}^{QED}(s)\left[1+\frac{\alpha_s(s)}{\pi}+1.411\cdot\frac{\alpha_s(s)^2}{\pi^2}+\ldots\right]$$

$$R_{had}=\frac{\sigma(ee\rightarrow hadrons)}{\sigma(ee\rightarrow\mu\mu)}=3\sum Q_q^2\left\{1+\frac{\alpha_s}{\pi}+1.411\frac{\alpha_s^2}{\pi^2}+\ldots\right\}$$

Not very precise. Got first indication on running $\alpha_s(s)$

b) α_s from hadronic event shape variables

3-jet rate: $R_3\equiv\frac{\sigma_{3-jet}}{\sigma_{had}}$ depends on α_s

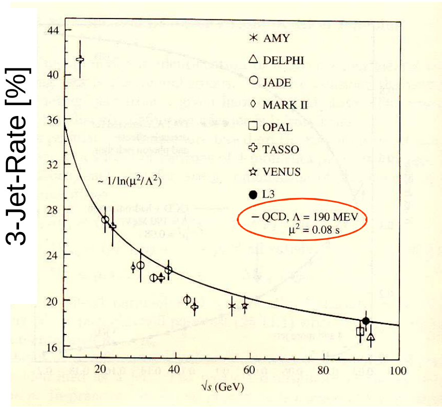
3-jet rate is measured as function of a jet resolution parameter y_{cut}

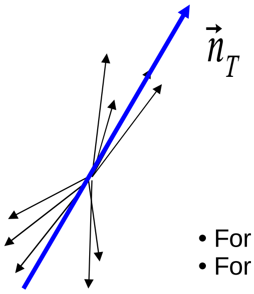
QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s, y_{cut})$

$\rightarrow$ fit $R_3^{theo}(\alpha_s, y_{cut})$ to the data to determine α_s

Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for α_s

➡ $\alpha_s(s)$

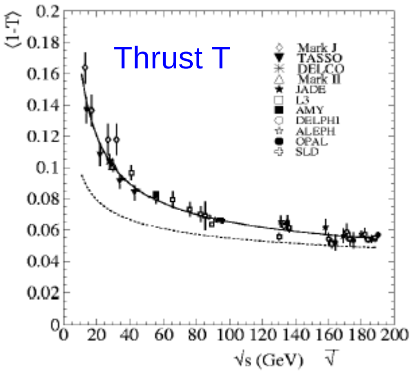




$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$

thrust axis
maximizes longitudinal momentum

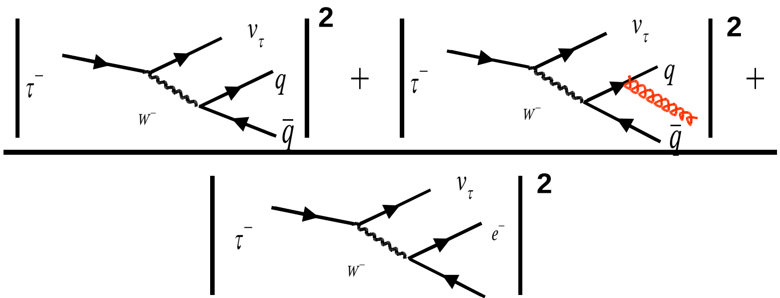
- For 2 back-to-back jets: $T = 1$
- For spherical events: $T = 1/2$



c) α_s from hadronic τ decays

$$R_{had}^{\tau} = \frac{\Gamma(\tau \rightarrow \nu_\tau + Hadrons)}{\Gamma(\tau \rightarrow \nu_\tau + e\,\bar{\nu}_e)} \sim f(\alpha_s)$$

$R_{had}^{\tau} =$



at τ mass

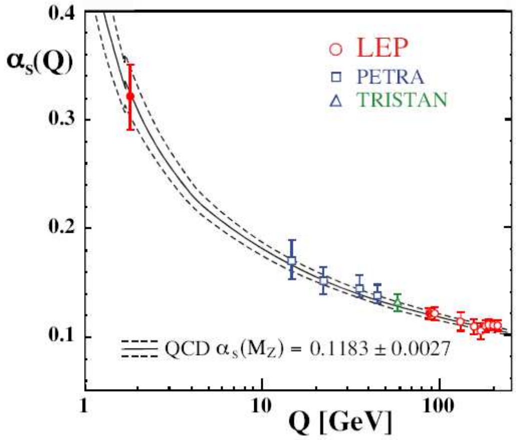
$$R_{had}^{\tau} = R_{had}^{\tau,0}\left(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots\right)$$

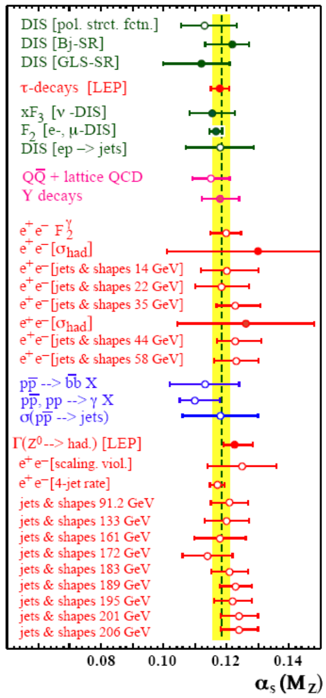
d) α_s from DIS (deep inelastic scattering)

will be discussed later

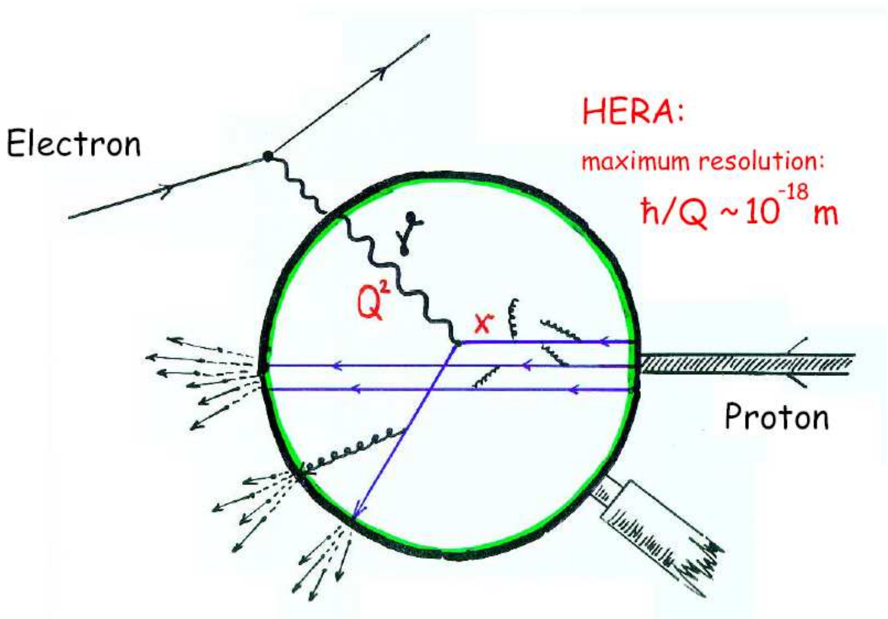
Running of α_s and asymptotic freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2 / \Lambda_{QCD}^2)}$$

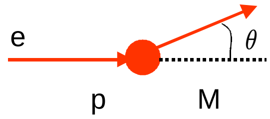




3. Study of QCD in deep inelastic scattering (DIS)



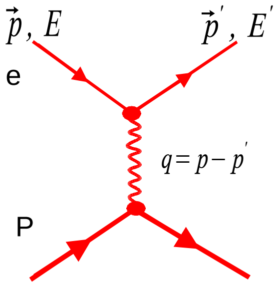
3.1 Elastic electron-proton scattering



General form of differential cross section

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4EE'\sin^4\frac{\theta}{2}}}_{\text{Rutherford}} \{\ldots\ldots\}$$

$\{\ldots\ldots\}$ ← non pointlike scattering or partners w/ spin



$$-q^2 = 4EE'\sin^4\frac{\theta}{2}$$

Spin ½ electron +

Pointlike target w/o spin
Mott scattering

$$\{\ldots\ldots\}_{Mott}^{elastic} = \left(\cos^2\frac{\theta}{2}\right)$$

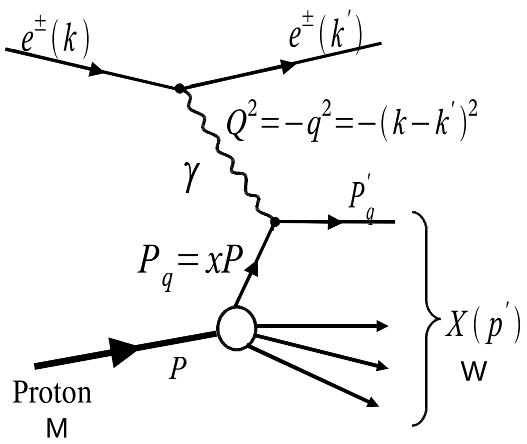
Pointlike target w/ spin
and mass M

$$\{\ldots\ldots\}_{e\mu\to e\mu}^{elastic} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

Extended proton w/ spin

$$\{\ldots\ldots\}_{ep\to ep}^{elastic} = \left(\frac{G_E^2 + \tau\, G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau\, G_M^2 \sin^2\frac{\theta}{2}\right) \quad \text{with} \quad \tau = \frac{Q^2}{4M^2}$$

3.2 DIS in the quark parton model (QPM)



- Elastic scattering: W = M

  ⇒ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W ≠ M

  ⇒ scattering described by 2 independent variables

$$(E, \nu),\ (Q^2, x),\ (x, y),\ \ldots$$

x = fractional momentum of struck quark

y = Pq/Pk = fractional energy transfer in proton rest frame

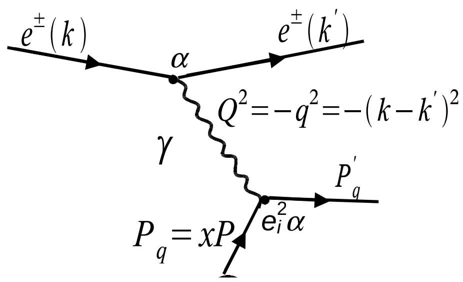
ν = E - E′ = energy transfer in lab

$$Q^2 = sxy \qquad \text{s = CMS energy}$$

$$x = \frac{Q^2}{2M\nu} \qquad \text{(Bjorken x)}$$

Cross section in quark parton model (QPM)

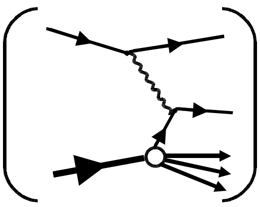
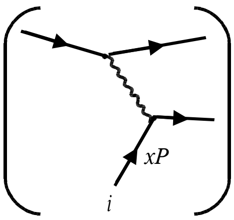
Elastic scattering on single quark



$$\{\ldots\ldots\}_{e\mu\to e\mu}^{elastic}=\left(\cos^2\frac{\theta}{2}+\frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

charge ↓

$$\frac{d\sigma}{dQ^2}=\left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot e_i^2\left(\cos^2\frac{\theta}{2}+\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

σ  $=\sum_i q_i(x)\,\sigma_i$ 

Parton density $q_i(x)dx$: Probability to find parton i in momentum interval [x, x+dx]

$$\frac{d^2\sigma}{dQ^2\,dx}=\left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\sum_i q_i(x)\left(\cos^2\frac{\theta}{2}+\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

Using structure functions

$$F_2(x) = x \sum_i e_i^2 q_i(x)$$

$$F_1(x) = \frac{1}{2} \sum_i e_i^2 q_i(x)$$

$$\frac{d^2\sigma}{dQ^2\,dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\left(\frac{F_2(x)}{x}\cos^2\frac{\theta}{2} + 2F_1(x)\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

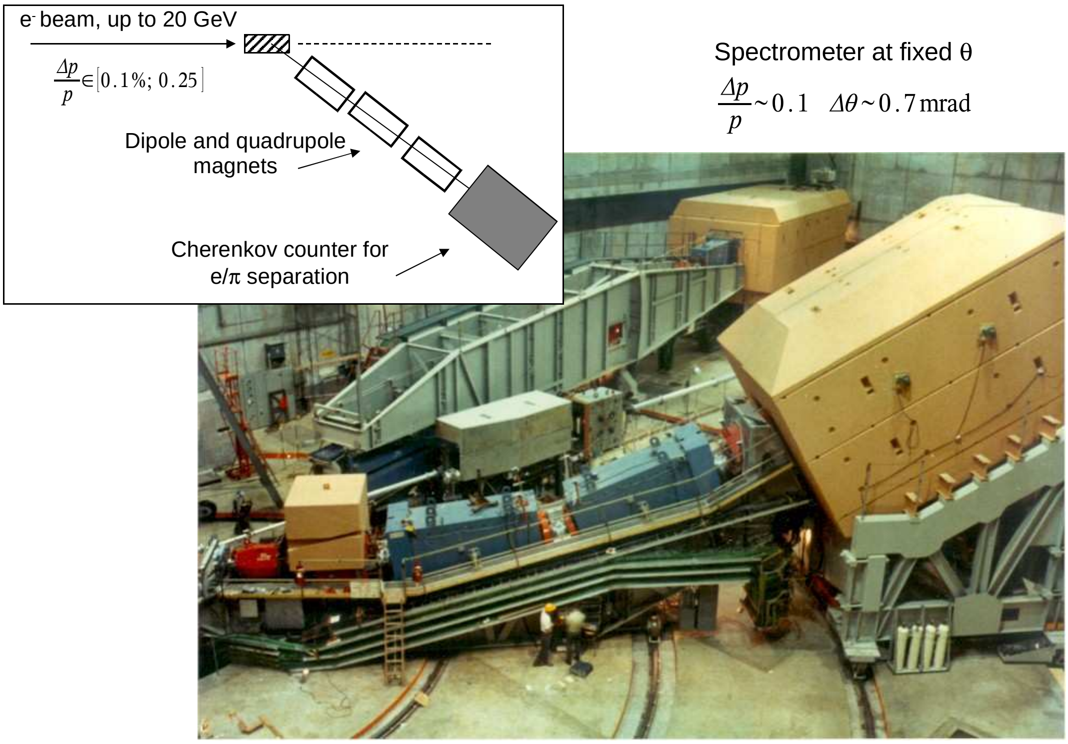
Using kinematical relations transform to Lorenz-invariant variables

$$\frac{d^2\sigma}{dQ^2\,dx} = \left(\frac{4\pi\alpha^2}{Q^4 x}\right)\cdot\left((1-y)F_2(x) + xy^2F_1(x)\right)$$

Predictions of Quark – Parton model

- Free partons: $F_2 = F_2(x)$ ⇔ "scaling"
- Spin ½ partons: $2xF_1(x) = F_2(x)$ (Callan-Gross relation)

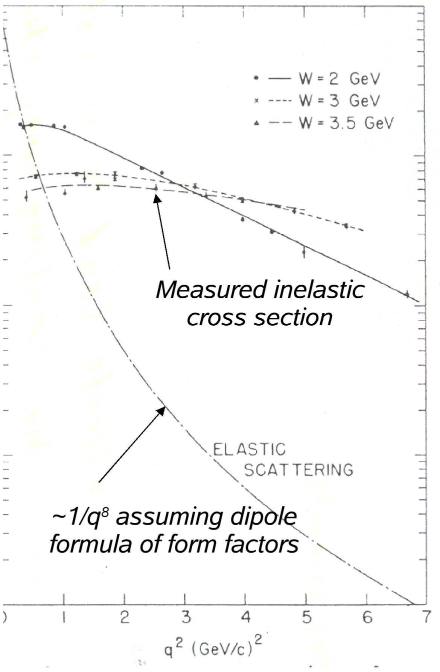
SLAC/MIT Experiment (1969)



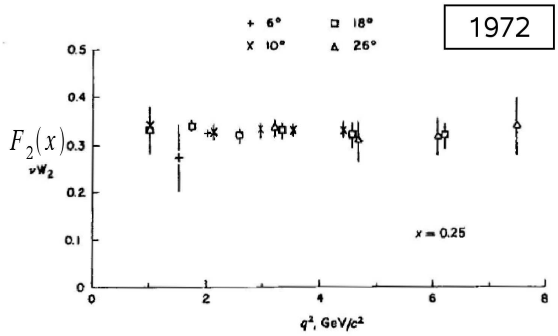
Spectrometer at fixed θ

$$\frac{\Delta p}{p} \sim 0.1 \quad \Delta\theta \sim 0.7\,\text{mrad}$$

1969



1972



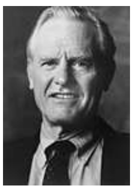
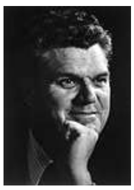
Structure function F_2 depends only on the dimensionless variable x

$$x = \frac{Q^2}{2M\nu}$$

→ **Scale invariance: "scaling"**

Indicates elastic scattering at point-like free constituents of the proton: partons

The Nobel Prize in Physics 1990

Jerome I. Friedman	**Henry W. Kendall**	**Richard E. Taylor**

"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

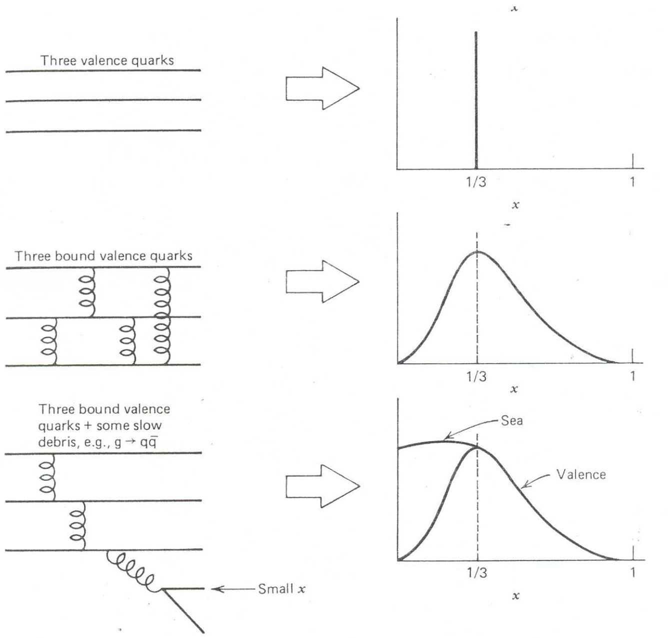
3.3 Structure functions – Nucleon structure

In infinite momentum frame, x - fractional momentum of struck quark

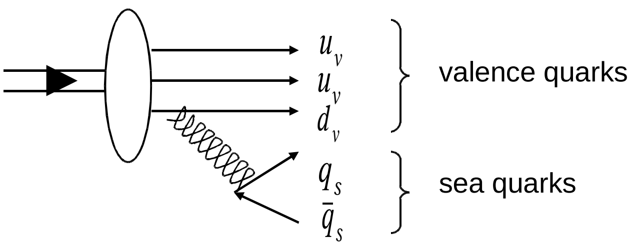
$$p = xP$$

Proton model

$$F_2(x) = x \sum_i e_i^2 q_i(x)$$



Sea and valence quarks in the proton



Quark composition of the proton

$$u_v + u_v + d_v + \underbrace{(u_s + \bar{u}_s) + (d_s + \bar{d}_s) + (s_s + \bar{s}_s)}$$

Heavy sea quarks are strongly suppressed

$$\frac{F_2^{ep}(x)}{x} = \sum_i e_i^2 \cdot q_i(x)$$

$$= \frac{4}{9}(u^p(x) + \bar{u}^p(x)) + \frac{1}{9}(d^p(x) + \bar{d}^p(x)) + \frac{1}{9}(s^p(x) + \bar{s}^p(x))$$

Quark composition of the neutron

$$\frac{F_2^{en}(x)}{x} = \sum_i e_i^2 \cdot q_i(x)$$
$$= \frac{4}{9}(u^n(x)+\bar{u}^n(x)) + \frac{1}{9}(d^n(x)+\bar{d}^n(x)) + \frac{1}{9}(s^n(x)+\bar{s}^n(x))$$
$$\frac{F_2^{en}(x)}{x} = \frac{4}{9}(d(x)+\bar{d}(x)) + \frac{1}{9}(u(x)+\bar{u}(x)+s(x)+\bar{s}(x))$$

Iso-spin symmetry

$$u^n(x) = d^p(x) = d(x)$$
$$d^n(x) = u^p(x) = u(x)$$
$$s^n(x) = s^p(x) = s(x)$$
$$\bar{q}^n(x) = \bar{q}^p(x) = \bar{q}(x)$$

In total 6 unknown quark distributions ⬅

Sum rules

$$q^i(x) = q_v^i(x) + q_s^i(x)$$
$$\bar{q}^i(x) = \bar{q}_s^i(x)$$

$$\int_0^1 u(x) - \bar{u}(x)\,dx = \int_0^1 u_v(x)\,dx = 2$$
$$\int_0^1 d(x) - \bar{d}(x)\,dx = \int_0^1 d_v(x)\,dx = 1$$
valence in proton

$$\int_0^1 (q_s(x) - \bar{q}_s(x))\,dx = 0$$
sea

Sum of quark momenta

Scattering at an iso-scalar target N: #p = #n (e.g. Deuteron, C, Ca)

$$F_2^{eN} = \frac{1}{2}\left[F_2^{ep} + F_2^{en}\right] = \frac{5}{18} x \cdot [u + \bar{u} + d + \bar{d}] + \frac{1}{9} x \cdot [s + \bar{s}]$$
$$\approx \underbrace{\frac{5}{18} x \cdot [u + \bar{u} + d + \bar{d}]}_{} = \frac{5}{18}[\text{Sum of all quark momenta}]$$

Small s quark distribution neglected

Naively one expects: $\frac{18}{5} \cdot \int_0^1 F_2^{eN}(x)\,dx \approx 1$

Experimental observation: $\frac{18}{5} \cdot \int_0^1 F_2^{eN}(x)\,dx \approx 0.5$

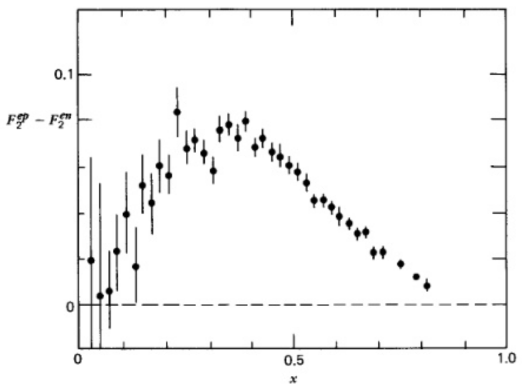
- Probed quarks and anti-quarks carry only 50% of nucleon momentum
- Remaining momentum is carried by gluons (see later)

Valence Quark Distribution

After subtracting

$$\frac{1}{x}\left[F_2^{ep}-F_2^{en}\right]=\frac{1}{3}\cdot[u_v(x)-d_v(x)]$$

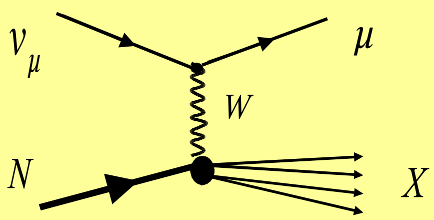
Shape of valence quark distribution. It peaks really at 1/3



3.4 Neutrino nucleon scattering

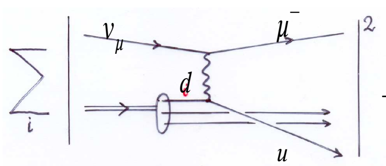
$$\nu_\mu N \rightarrow \mu^\pm X$$

- More information on quark distribution
- Separation between quarks / anti-quarks



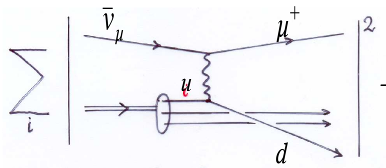
QPM: $x=\frac{Q^2}{2M\nu}$ $y=\frac{\nu}{E}$ $\nu=E-E'$

$$\frac{d\sigma(\nu_\mu N\rightarrow\mu^- X)}{dy}=$$



$$\rightarrow F_i^{\nu p}(x) \text{ and } F_i^{\nu n}(x)$$

$$\frac{d\sigma(\bar{\nu}_\mu N\rightarrow\mu^+ X)}{dy}=$$



$$\rightarrow F_i^{\bar{\nu} p}(x) \text{ and } F_i^{\bar{\nu} n}(x)$$

Structure functions for neutrino scattering

$$F_i^{\nu n} = F_i^{\bar{\nu} p}$$
$$F_i^{\nu p} = F_i^{\bar{\nu} n}$$

Equal because of Charge symmetry

$$F_i^{\nu N} = \frac{1}{2}(F_i^{\nu p} + F_i^{\nu n}) = \frac{1}{2}(F_i^{\bar{\nu} n} + F_i^{\bar{\nu} p}) = F_i^{\bar{\nu} N} \quad \text{for } i = 1,2$$

$$F_3^{\bar{\nu} N} = -F_3^{\nu N}$$

Additional structure function to account for parity violation

Double differential cross section: Scattering at iso-scalar target

$$\frac{d^2\sigma(\nu N, \bar{\nu} N)}{dxdy} = 2ME\frac{G_F^2}{2\pi}\left[(1-y)F_2^{\nu N}(x) + \frac{y^2}{2}2xF_1^{\nu N}(x) \pm y\left(1-\frac{y}{2}\right)xF_3^{\nu N}(x)\right]$$

$$\frac{4\pi\alpha^2}{Q^4} \qquad \frac{G_F^2}{2\pi}$$

$\nu = +$

$\bar{\nu} = -$

to account for parity violation

Structure functions in QPM

$$F_1^{\nu N} = \frac{1}{2x} F_2^{\nu N}$$

$$F_2^{\nu p} = 2x[d + \bar{u}] \qquad xF_3^{\nu p} = 2x[d - \bar{u}]$$

$$F_2^{\nu n} = 2x[d^n + \bar{u}^n] \qquad xF_3^{\nu n} = 2x[d^n - \bar{u}^n]$$

$$= 2x[u + \bar{d}] \qquad = 2x[u - \bar{d}]$$

Iso-scalar target →

$$F_2^{\nu N} = x[u + \bar{u} + d + \bar{d}] \qquad xF_3^{\nu N} = x[(u + d) - (\bar{u} + \bar{d})]$$

$$F_2^{\nu N} = x[Q(x) + \bar{Q}(x)] \qquad xF_3^{\nu N} = x[Q(x) - \bar{Q}(x)]$$

Measures sum of quarks and anti-quarks

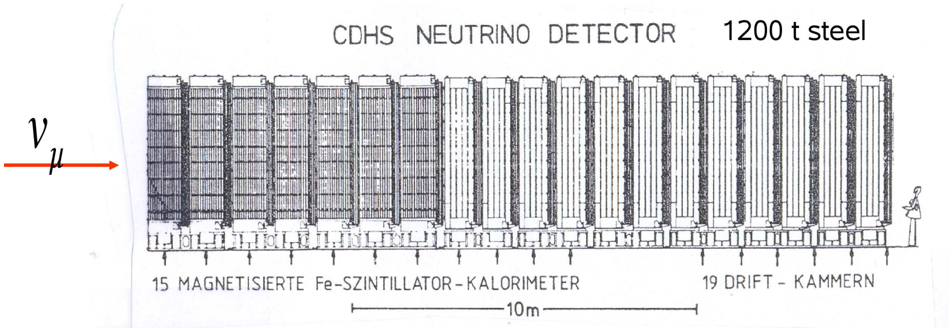
Measures valence quarks

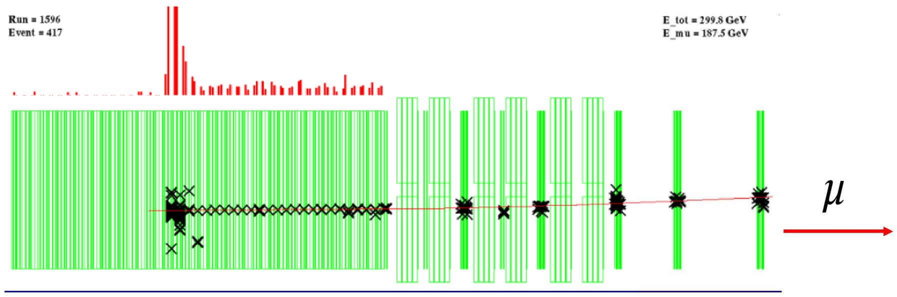
Measurement:

$$F_2^{\nu N} + xF_3^{\nu N} = 2xQ(x)$$ ➡ Sea and valence quarks

$$F_2^{\nu N} - xF_3^{\nu N} = 2x\bar{Q}(x)$$ ➡ Sea quarks

CDHS (CERN-Dortmund-Heidelberg-Saclay) Experiment





Note: Fe is not iso-scalar: 26 *p*, 30 *n*. Iso-scalar correction is applied.

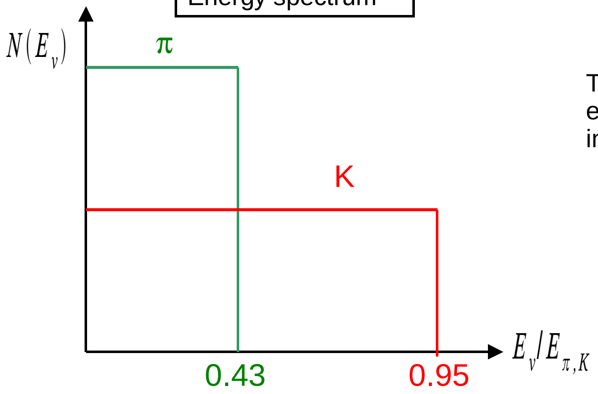
Neutrino beams

Sources of intense neutrino beams are 2-body decays of intense hadron beams

$$\pi^{\pm} \rightarrow \mu^{\pm} \nu_{\mu} (\bar{\nu}_{\mu}) \quad K^{\pm} \rightarrow \mu^{\pm} \nu_{\mu} (\bar{\nu}_{\mu})$$

where the pions/kaons are generated in proton-nucleon interactions: p+N→π, K

Energy spectrum

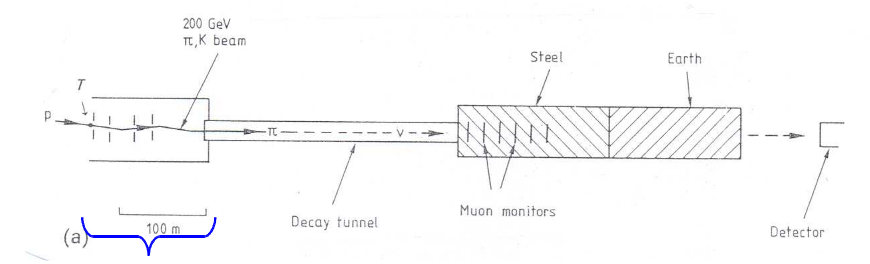


The Lorentz boost transforms the mono-energetic neutrino of the two-body decay into a flat energy spectrum:

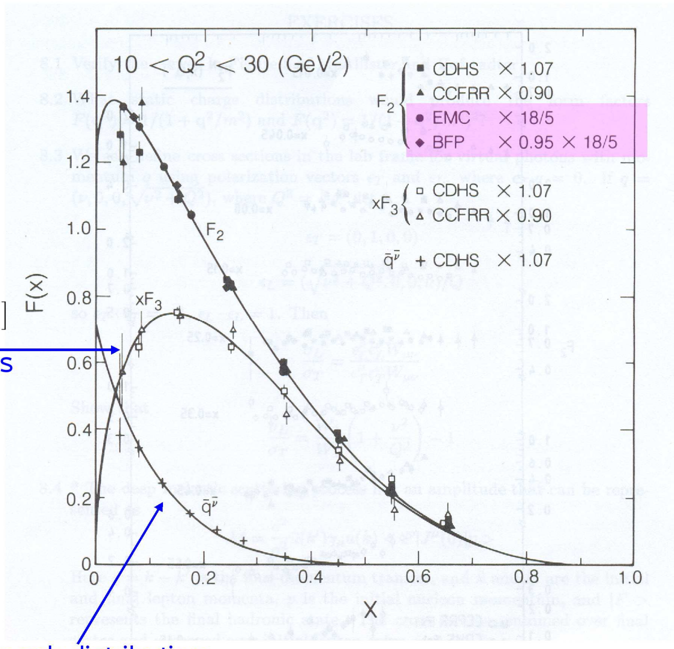
$$0 < \frac{E_{\nu}}{E_{\pi,K}} < 1 - \frac{m_{\mu}^2}{m_{\pi,K}^2}$$

Beams contain small admixture (0.5%) of electron neutrinos !

Generation of neutrino beams



1. ~400 GeV proton beam on a (Be) target: secondary hadrons π, K
2. Momentum and charge selection of π's and K's using a focusing system
3. Selected π's and K's enter a decay tunnel: $\pi^{\pm}, K^{\pm} \rightarrow \mu^{\pm} \nu_{\mu} (\bar{\nu}_{\mu})$
4. Remaining hadrons and decay muons are filtered by a massive absorber (~400 m iron, concrete, earth): only neutrinos after absorber



$=[Q(x)-\bar{Q}(x)]$

lence quarks

Anti-quark distribution:
a measure of sea quarks

Parton distribution in eN and νN scattering

Question:

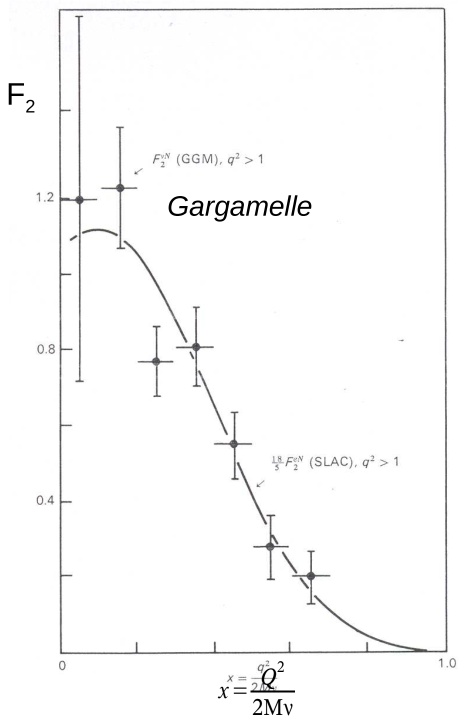
Do the parton distribution seen in electro-magnetic (F_2^{eN}) and in weak interaction F_2^{eN}) agree?

$$\Rightarrow \frac{F_2^{\nu N}(x)}{F_2^{eN}(x)} = \frac{x[Q(x)+\bar{Q}(x)]}{\frac{5}{18}\cdot x[Q(x)+\bar{Q}(x)]} = \frac{18}{5}$$

↑ Factor from fractional charge

Answer:

- e.m. and weak quark structure is the same
- Factor 18/5 → fractional quark charge



Summary: eN and νN scattering (N=iso-scalar target)

eN scattering

$$\frac{d^2\sigma^{eN}}{dxdy}=\frac{2\pi\alpha^2}{Q^4}xs\left[1+(1-y)^2\right]\cdot\frac{5}{18}\left[Q(x)+\bar{Q}(x)\right]$$

$$F_2^{eN}(x)=\frac{5}{18}x\left[Q(x)+\bar{Q}(x)\right]$$

$\nu N+\bar{\nu}N$ scattering

$$\frac{d^2\sigma^{\nu N}}{dxdy}=\frac{G_F^2}{2\pi}xs\left[Q(x)+(1-y)^2\cdot\bar{Q}(x)\right]$$

$$\frac{d^2\sigma^{\bar{\nu}N}}{dxdy}=\frac{G_F^2}{2\pi}xs\left[\bar{Q}(x)+(1-y)^2\cdot Q(x)\right]$$

$$F_2^{\nu N}(x)=x\left[Q(x)+\bar{Q}(x)\right] \quad F_3^{\nu N}(x)=x\left[Q(x)-\bar{Q}(x)\right]$$

$$F_2^{\bar{\nu}N}(x)=x\left[Q(x)+\bar{Q}(x)\right] \quad F_3^{\bar{\nu}N}(x)=x\left[\bar{Q}(x)-Q(x)\right]$$