1.6 Solutions for negative energies $E = -\sqrt{p^2 + m^2}$

$\Rightarrow \phi = \dfrac{\vec{\sigma}\cdot\vec{p}}{\mathrm{E-m}}\chi$ and using $\chi_1 = \mathrm{N}\cdot\begin{pmatrix}1\\0\end{pmatrix}$ and $\chi_2 = \mathrm{N}\cdot\begin{pmatrix}0\\1\end{pmatrix}$

solution spin $\uparrow$

$$u_3(p) = N\cdot\begin{pmatrix}-\dfrac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi_1\\ \chi_1\end{pmatrix} = N\cdot\begin{pmatrix}\dfrac{-p_z}{|E|+m}\\ \dfrac{-p_x - ip_y}{|E|+m}\\ 1\\ 0\end{pmatrix}$$

solution spin $\downarrow$

$$u_4(p) = N\cdot\begin{pmatrix}-\dfrac{\vec{\sigma}\cdot\vec{p}}{|E|+m}\chi_2\\ \chi_2\end{pmatrix} = N\cdot\begin{pmatrix}\dfrac{-p_x + ip_y}{|E|+m}\\ \dfrac{+p_z}{|E|+m}\\ 0\\ 1\end{pmatrix}$$

Particles: $E<0, \vec{p}$

$\mathrm{u}_3^{\uparrow}, \mathrm{u}_4^{\downarrow}$

Anti-particles: $E>0, -\vec{p}$

$\mathrm{v}_2^{\downarrow}, \mathrm{v}_1^{\uparrow}$

$$v_2(p) = N\cdot\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_1\\ \chi_1\end{pmatrix} = N\cdot\begin{pmatrix}\dfrac{p_z}{E+m}\\ \dfrac{p_x + ip_y}{E+m}\\ 1\\ 0\end{pmatrix}$$

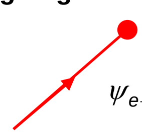
solution spin $\downarrow$

$$v_1(p) = N\cdot\begin{pmatrix}\dfrac{\vec{\sigma}\cdot\vec{p}}{E+m}\chi_2\\ \chi_2\end{pmatrix} = N\cdot\begin{pmatrix}\dfrac{p_x - ip_y}{E+m}\\ \dfrac{-p_z}{E+m}\\ 0\\ 1\end{pmatrix}$$

solution spin $\uparrow$ $N = \sqrt{E+m}$

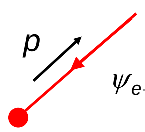
In- and out-going (anti)-particles

In-going electron:



$$\psi_{e-}(x) = u_{1,2}(p)\cdot\exp(-ip\cdot x) = u_{1,2}(p)\cdot\exp(-iEt)\exp(+i\vec{p}\cdot\vec{x})$$

Out-going positron:



$$\psi_{e+}(x) = v_{1,2}(p)\cdot\exp(+ip\cdot x) = v_{1,2}(p)\cdot\exp(+iEt)\exp(-i\vec{p}\cdot\vec{x})$$

To describe **out-going electrons** or **in-going positrons** the adjoint spinors $\overline{\psi}_{e-} = \psi_{e-}\gamma^0$ or $\overline{\psi}_{e+} = \psi_{e+}\gamma^0$ and $\overline{u}_{1,2}$ or $\overline{v}_{1,2}$ are used.

2. Quantum Electrodynamics

Lagrangian for free spin ½ particle:

$$L(\vec{x},t) = i\overline{\psi}(\vec{x},t)\gamma^{\mu}\partial_{\mu}\psi(\vec{x},t) - m\overline{\psi}(\vec{x},t)\psi(\vec{x},t)$$

Applying the Euler-Lagrange formalism leads to the Dirac equation.

Invariance under Local Gauge Transformation

Demanding invariance under local phase transformation of the free Langrangian (local gauge invariance):

$$\psi(x) \rightarrow \psi(x) = e^{i\alpha(x)}\psi(x)$$

requires the substitution:

$$i\partial_{\mu} \rightarrow i\partial_{\mu} + eA_{\mu}(x)$$

If one defines the tansformation of A under local gauge transformation as

$$A_{\mu}(x) \rightarrow A_{\mu}(x) + \partial_{\mu}\alpha(x)$$

one finds invariance of L:

$$L(x) \xrightarrow{\psi \rightarrow \psi\, e^{i\alpha(x)}} L(x)$$

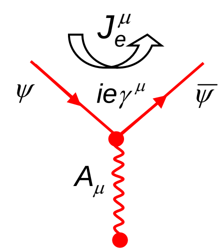
To interpret the introduced field A_{μ} as photon field requires to complete the Langrangian by the corresponding field energy:

$$\boxed{L = \overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi + e\overline{\psi}\gamma^{\mu}\psi\, A_{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}}$$

$$F_{\mu\nu} = \partial_{\mu}A_{\nu} - \partial_{\nu}A_{\mu}$$

The requirement of local gauge invariance has automatically led to the interaction of the free electron with a field.

$$L = \underbrace{\overline{\psi}(i\gamma^{\mu}\partial_{\mu} - m)\psi}_{\text{free electron}} + \underbrace{e\overline{\psi}\gamma^{\mu}\psi A_{\mu}}_{\text{Interaction between electron and photon}} - \underbrace{\frac{1}{4}F_{\mu\nu}F^{\mu\nu}}_{\text{Photon field energy}}$$



Lagrangian defines the Feynman rules of a theory.

Perturbation Theory

$$L(x) = L_0(x) + \underbrace{L'(x)}_{\text{Interaction}}$$

Interaction Hamiltonian:

$$H'(t) = -\int d^3x\, L'(x) = -e\int d^3x\, \overline{\psi}\gamma^{\mu}\psi\, A_{\mu}$$

Time dependent state | t > should satifie the Schrödinger eq.:

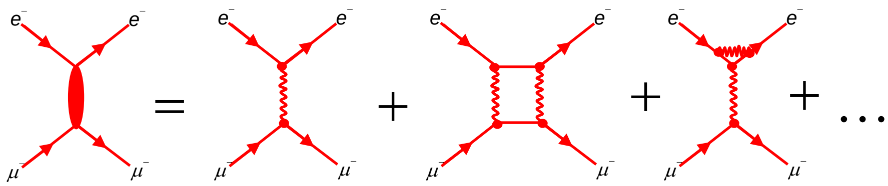
$$i\frac{\partial}{\partial t}\left|t\right\rangle = H'(t)\left|t\right\rangle$$

A solution is given by (can be checked by insertion):

$$\underset{\uparrow \atop t\to\infty:\left|f\right\rangle}{\left|t\right\rangle} = \{1 + \underbrace{(-i)\int_{-\infty}^{t} dt' H'(t') + (-i)^2 \int_{-\infty}^{t} dt' \int_{-\infty}^{t'} dt'' H'(t')H'(t'') + \ldots\}}_{t\to\infty:\,T_{fi}} \underbrace{\left|t = -\infty\right\rangle}_{=\left|i\right\rangle}$$

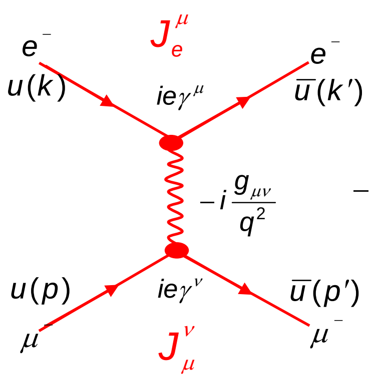
Perturbative expansion in powers of coupling strength e

Example: $e^- \mu^- \to e^- \mu^-$



$$\underbrace{\sim e^2}_{\text{Born Approximation}} \qquad \sim e^4 \qquad \sim e^4$$

Feynman rules for scattering processes



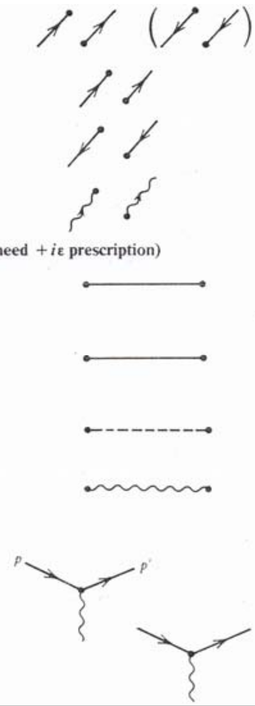
$$e^- \mu^- \to e^- \mu^-$$

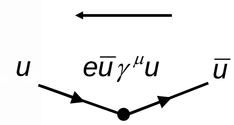
$$-iM = \underbrace{\bar{u}(k')(ie\gamma^\mu)u(k)}_{ie\cdot J_e^\mu} \cdot \underbrace{\frac{-ig_{\mu\nu}}{q^2}}_{\text{Propagator}} \cdot \underbrace{\bar{u}(p')(ie\gamma^\nu)u(p)}_{ie\cdot J_\mu^\nu}$$

$$M = -\frac{e^2}{q^2}\bar{u}(k')\gamma_\mu u(k)\cdot\bar{u}(p')\gamma^\mu u(p)$$

There are similar rules for other Feynman diagrams

• External Lines	
Spin 0 boson (or antiboson)	1
Spin $\frac{1}{2}$ fermion (in, out)	$u, \bar{u}$
antifermion (in, out)	$\bar{v}, v$
Spin 1 photon (in, out)	$\epsilon_\mu, \epsilon^*_\mu$
• Internal Lines—Propagators (need $+i\varepsilon$ prescription)	
Spin 0 boson	$\frac{i}{p^2 - m^2}$
Spin $\frac{1}{2}$ fermion	$\frac{i(\not{p} + m)}{p^2 - m^2}$
Massive spin 1 boson	$\frac{-i(g_{\mu\nu} - p_\mu p_\nu / M^2)}{p^2 - M^2}$
Massless spin 1 photon (Feynman gauge)	$\frac{-ig_{\mu\nu}}{p^2}$
• Vertex Factors	
Photon—spin 0 (charge $-e$)	$ie(p + p')^\mu$
Photon—spin $\frac{1}{2}$ (charge $-e$)	$ie\gamma^\mu$





Halzen, Martin: Quarks&Leptons

3. Fermion-fermion scattering

3.1 Process $e^- \mu^- \rightarrow e^- \mu^-$

Sect. II.5 ➡

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |M_{fi}|^2$$

Sect. III.2 ➡

$$M_{fi} = -\frac{e^2}{q^2} \underbrace{\bar{u}(k')\gamma_\mu u(k) \cdot \bar{u}(p')\gamma^\mu u(p)}_{}$$

Measurements often ignores specific spin states ⬅ Spinors decribe a specific spin state of the fermions

For non-polarized ingoing particles and for non-observation of final state spin one observes unpolarized cross sections $\Rightarrow$ need to **average over possible initial spin states** and **sum over all final spin states**.

$$\overline{|M|^2} = \frac{1}{(2s_e + 1)(2s_\mu + 1)} \cdot \sum_{s_e, s_\mu} \sum_{s'_e, s'_\mu} |M|^2$$

$$\boxed{\overline{|M|^2} =} \frac{1}{(2s_e+1)(2s_\mu+1)} \cdot \sum_{s_e,s_\mu} \sum_{s'_e,s'_\mu} |M|^2$$

$$= \frac{1}{4} \cdot \frac{e^4}{q^4} \sum_{s_e,s_\mu} \sum_{s'_e,s'_\mu} [\bar{u}_{s'_e}(k')\gamma^\mu u_{s_e}(k)][\bar{u}_{s'_e}(k')\gamma^\nu u_{s_e}(k)]^* \cdot$$

$$[\bar{u}_{s'_\mu}(p')\gamma_\mu u_{s_\mu}(p)][\bar{u}_{s'_\mu}(p')\gamma_\nu u_{s_\mu}(p)]^*$$

$$\boxed{= \frac{e^4}{q^4} L_e^{\mu\nu} \cdot L_{muon,\mu\nu}}$$

Electron tensor $\quad L_e^{\mu\nu} = \frac{1}{2} \sum_{s_e,s'_e} [\bar{u}(k')\gamma^\mu u(k)][\bar{u}(k')\gamma^\nu u(k)]^*$

Muon tensor $\quad L_{muon,\mu\nu} = \frac{1}{2} \sum_{s_\mu,s'_\mu} [\bar{u}(p')\gamma_\mu u(p)][\bar{u}(p')\gamma_\nu u(p)]^*$

<u>Useful relations I:</u>

Completeness relation:

$$\sum_{s=1,2} u_s(p)\bar{u}_s(p) = \not{p} + m$$

$$\sum_{s=1,2} v_s(p)\bar{v}_s(p) = \not{p} - m$$

$$\not{a} = \gamma^\mu a_\mu$$

$$\underbrace{\sum_{s_i,s_f} \bar{u}_{s_f}(p')\gamma^\mu u_{s_i}(p)\bar{u}_{s_i}(p)\gamma^\mu u_{s_f}(p')}_{} = \sum_{s_f} \bar{u}_{s_f} \underbrace{\gamma^\mu(\not{p}_i + m)\gamma^\mu}_{\text{Matrix A}} u_{s_f}$$

$$= \sum_{j,k=1}^{4} \left\{ \sum_{s_f} (\bar{u}_{s_f})_j \mathbf{A}_{jk} (u_{s_f})_k \right\}$$

$$=$$

$$\sum_{s_i,s_f} \left| \overbrace{\bar{u}_f \gamma^\mu u_i} \right|^2 = \sum_{j,k=1}^{4} (\not{p}_f + m)_{kj} \mathbf{A}_{jk} = \sum_j \mathbf{B}_{jj} = \text{Trace}\,\mathbf{B}$$

Matrix $\mathbf{B} = (\not{p}_f + m)\mathbf{A}$

Useful relations II:

$$\sum_{s_i, s_f} \left| \bar{u}_f \gamma^\mu u_i \right|^2 = \mathrm{Trace}\left((\not{p}_f + m)\gamma^\mu (\not{p}_i + m)\gamma^\mu \right)$$

Averaging over spins $\rightarrow$ trace of matrix element

Trace theorems:

$$\mathrm{Trace}(\mathrm{I}) = 4$$

$$\mathrm{Trace}(\text{odd number of } \gamma^\mu) = 0$$

$$\mathrm{Trace}(\not{a}\not{b}) = 4(ab) = 4a_\mu b^\mu$$

$$\mathrm{Trace}(\not{a}\not{b}\not{c}\not{d}) = 4(ab)(cd) + 4(ad)(bc) - 4(ac)(bd)$$

Electron tensor $\quad L_e^{\mu\nu} = \frac{1}{2} \sum_{s_e, s'_e} [\bar{u}(k')\gamma^\mu u(k)][\bar{u}(k')\gamma^\nu u(k)]^*$

Muon tensor $\quad L_{\mu\nu}^{\mathrm{Muon}} = \frac{1}{2} \sum_{s_\mu, s'_\mu} [\bar{u}(p')\gamma_\mu u(p)][\bar{u}(p')\gamma_\nu u(p)]^*$

With this relations one obtains after a lengthy calculation

Berechnung von $L_e^{\mu\nu}$:

$$\left(\bar{u}(k')\gamma^\mu u(k)\right)^* = 1\times 1 \text{ Matrix, deswegen } (\;)^* = (\;)$$

$$= (\bar{u}\gamma^\mu u)^\dagger = \left(u^\dagger(k')\gamma^0\gamma^\mu u(k)\right)^\dagger$$

$$= u^\dagger(k)\gamma^{\mu\dagger}\gamma^0 u(k') \leftarrow \gamma^{0\dagger} = \gamma^0$$

$$= u^\dagger(k)\gamma^0\gamma^\mu u(k')$$

$$= \bar{u}(k)\gamma^\mu u(k')$$

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{s,s'} \bar{u}(k')\gamma^\mu u(k)\cdot \bar{u}(k)\gamma^\nu u(k')$$

mit $(A\cdot B\cdot C)_{\alpha\beta} = A_{\alpha\gamma} B_{\gamma\delta} C_{\delta\beta}$

$$= \frac{1}{2}\sum_{s,s'} \bar{u}_\alpha(k')\gamma^\mu_{\alpha\beta} u_\beta(k)\bar{u}_x(k)\gamma^\nu_{x\lambda} u_\lambda(k')$$

$$= \frac{1}{2}\sum_{s'} u_\lambda(k')\bar{u}_\alpha(k')\gamma^\mu_{\alpha\beta}\cdot\sum_s u_\beta(k)\bar{u}_x(k)\gamma^\nu_{x\lambda}$$

mit Vollständigk. Relation $\sum_s u\bar{u} = \not{k}+m$

$$= \frac{1}{2}(\not{k}'+m)_{\lambda\alpha}\gamma^\mu_{\alpha\beta}(\not{k}+m)_{\beta x}\gamma^\nu_{x\lambda}$$

$$= \frac{1}{2}\left[(\not{k}'+m)\gamma^\mu(\not{k}+m)\gamma^\nu\right]_{\lambda\lambda} =$$

$$= \frac{1}{2}\mathrm{Sp}\left[(\not{k}'+m)\gamma^\mu(\not{k}+m)\gamma^\nu\right]$$

Bem.: Über Indices μ und ν bisher nicht sammiert

$$= \frac{1}{2}\mathrm{Sp}\left[\not{k}'\gamma^\mu\not{k}\gamma^\nu + \not{k}'\gamma^\mu m\gamma^\nu + m\gamma^\mu\not{k}\gamma^\nu + m\gamma^\mu m\gamma^\nu\right]$$

Spur ungerader Zahl von γ's $= 0$: $\not{a} = a_\mu\gamma^\mu$!

$$= \frac{1}{2}\mathrm{Sp}\left[\not{k}'\gamma^\mu\not{k}\gamma^\nu + m^2\gamma^\mu\gamma^\nu\right]$$

$$= \frac{1}{2}\mathrm{Sp}\left[k'_\alpha\gamma^\alpha\gamma^\mu k_\beta\gamma^\beta\gamma^\nu + m^2\gamma^\mu\gamma^\nu\right]$$

$$= \frac{1}{2}k'_\alpha k_\beta\cdot\mathrm{Sp}\left[\gamma^\alpha\gamma^\mu\gamma^\beta\gamma^\nu\right] + \frac{1}{2}m^2\,\mathrm{Sp}\left[\gamma^\mu\gamma^\nu\right]$$

$$= \frac{1}{2}k'_\alpha k_\beta\cdot 4\cdot\left[g^{\alpha\mu}g^{\beta\nu} + g^{\alpha\nu}g^{\mu\beta} - g^{\alpha\beta}g^{\mu\nu}\right] + \frac{1}{2}m^2\cdot 4g^{\mu\nu}$$

$$= 2\left[k'^\mu k^\nu + k'^\nu k^\mu - [k'\cdot k - m^2]g^{\mu\nu}\right]$$

also

$$\boxed{\begin{aligned} L_e^{\mu\nu} &= 2\left[k'^\mu k^\nu + k'^\nu k^\mu - [k'\cdot k - m^2]g^{\mu\nu}\right] \\ L_{\mu\nu}^{\text{Myon}} &= 2\left[p'_\mu p_\nu + p'_\nu p_\mu - [p'\cdot p - M^2]g_{\mu\nu}\right]\end{aligned}}$$

$$L_e^{\mu\nu} = \frac{1}{2}\sum_{s_e,s'_e}[\bar{u}(k')\gamma^\mu u(k)][\bar{u}(k')\gamma^\nu u(k)]^*$$

$$L_{\mu\nu}^{\text{Muon}} = \frac{1}{2}\sum_{s_\mu,s'_\mu}[\bar{u}(p')\gamma_\mu u(p)][\bar{u}(p')\gamma_\nu u(p)]^*$$

$$\Downarrow$$

$$\boxed{\begin{aligned} L_e^{\mu\nu} &= 2\left(k'^\mu k^\nu + k'^\nu k^\mu - (k'\cdot k - m^2)g^{\mu\nu}\right) \\ L_{\mu\nu}^{\text{Muon}} &= 2\left(p'_\mu p_\nu + p'_\nu p_\mu - (p'\cdot p - M^2)g^{\mu\nu}\right)\end{aligned}}$$

Remember: Matrix^2 element can only depend scalar products of 4-momenta

m electron mass
M muon mass

Spin averaged matrix element for $e^-\mu^- \to e^-\mu^-$

$$\overline{|M|^2} = \frac{1}{(2s_e+1)(2s_\mu+1)} \cdot \sum_{s_e, s_\mu} \sum_{s'_e, s'_\mu} |M|^2 \quad = \frac{e^4}{q^4} L_e^{\mu\nu} \cdot L_{\mu\nu}^{\text{Muon}}$$

$$= 8\frac{e^4}{q^4}\left[(k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p') - m^2 p'\cdot p - M^2 k'\cdot k + 2m^2M^2\right]$$

exact 1st order result for $e^-\mu^- \to e^-\mu^-$

Relativistic limit ➡ neglect masses m and M

$$\overline{|M|^2} = 8\frac{e^4}{(k-k')^4}\left[(k'\cdot p')(k\cdot p) + (k'\cdot p)(k\cdot p')\right] = 2e^4\frac{s^2+u^2}{t^2}$$

By using the Mandelstam variables in the relativistic limit

$$\left.\begin{aligned} s &= (k+p)^2 = m^2 + M^2 + 2kp \approx 2kp \approx 2k'p' \\ t &= (k-k')^2 = m^2 + M^2 - 2kk' \approx -2kk' \approx -2pp' \\ u &= (k-p')^2 = m^2 + M^2 - 2kp' \approx -2kp' \approx -2k'p \end{aligned}\right\} \text{if masses neglected}$$

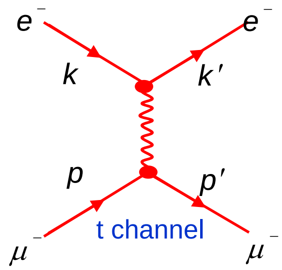
Scattering cross section for any two non-identical spin ½ particles:

$$e^-\mu^- \to e^-\mu^-$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2} \cdot \frac{1}{s} \cdot \frac{|\vec{p}_f|}{|\vec{p}_i|} \cdot |M_{fi}|^2$$

$$= \frac{\alpha^2}{2s} \cdot \left(\frac{s^2+u^2}{t^2}\right)$$

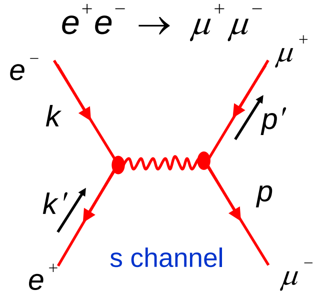
3.2 Process $e^+e^- \rightarrow \mu^+\mu^-$



Crossing

$k' \rightarrow -k' \quad p \rightarrow -p$

$e^+e^- \rightarrow \mu^+\mu^-$



$$t = (k-k')^2 \quad \rightarrow \quad \tilde{s} = (k+k')^2$$
$$s = (k+p)^2 \quad \rightarrow \quad \tilde{t} = (k-p)^2$$
$$u = (k-p')^2 \quad \rightarrow \quad \tilde{u} = (k-p')^2 = u$$

$$\boxed{\overline{|M|^2}_{e^-\mu^-\rightarrow e^-\mu^-}(s,t,u) \quad = \quad \overline{|M|^2}_{e^+e^-\rightarrow \mu^+\mu^-}(\tilde{t},\tilde{s},\tilde{u})}$$

$$\boxed{\overline{|M|^2}_{e^-\mu^-\rightarrow e^-\mu^-}(s,t,u) = 2e^4\frac{s^2+u^2}{t^2} \quad \Rightarrow \quad \overline{|M|^2}_{e^+e^-\rightarrow \mu^+\mu^-}(\tilde{t},\tilde{s},\tilde{u}) = 2e^4\frac{\tilde{t}^2+\tilde{u}^2}{\tilde{s}^2}}$$

Differential cross section for $e^+e^- \rightarrow \mu^+\mu^-$ (CMS)

Reminder:

$$\frac{d\sigma}{d\Omega} = \frac{1}{64\pi^2}\cdot\frac{1}{s}\cdot\frac{|\vec{p}_f|}{|\vec{p}_i|}\cdot|M_{fi}|^2$$

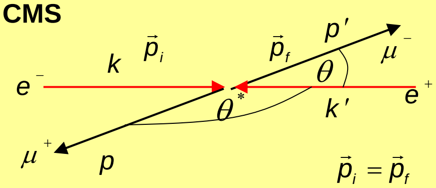
$$\frac{d\sigma}{d\Omega} = \frac{e^4}{32\pi^2}\cdot\frac{1}{s}\cdot\frac{t^2+u^2}{s^2}$$
$$= \frac{e^4}{64\pi^2}\cdot\frac{1}{s}\cdot(1+\cos^2\theta)$$

$e^2 = 4\pi\alpha$

$$\boxed{\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)}$$

Kinematics for high-relativistic particles

CMS



$$s = (k+k')^2 \approx 4E_i^2$$
$$t = (k-p)^2 \approx -2kp \approx -2E_i^2(1-\cos\theta^*)$$
$$\approx -\frac{s}{2}(1+\cos\theta)$$
$$u = (k-p')^2 \approx -2kp' \approx -2E_i^2(1-\cos\theta^*)$$
$$\approx -\frac{s}{2}(1-\cos\theta)$$

←1/s dependence from flux factor

$$\left.\frac{d\sigma}{d\Omega}\right|_{CMS} = \frac{\alpha^2}{4s}\cdot(1+\cos^2\theta)$$

$$\sigma_{tot} = \frac{4\pi\alpha^2}{3s} = \frac{86.86\,\mathrm{nb\,GeV}^2}{s}$$

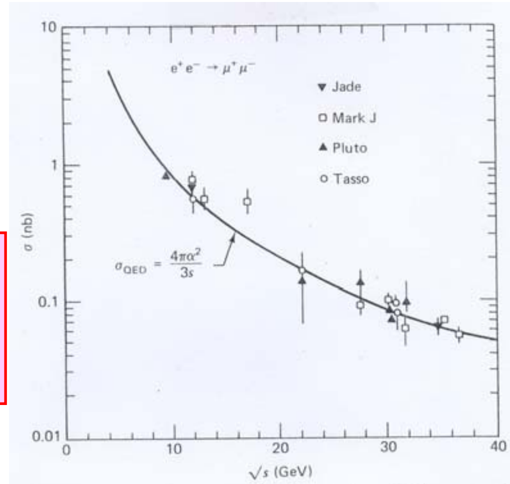
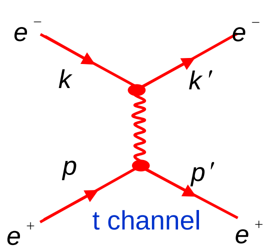


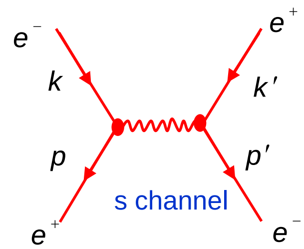
Fig. 6.6 The total cross section for $e^-e^+ \to \mu^-\mu^+$ measured at PETRA versus the center-of-mass energy.

3.3 Bhabha scattering $e^+e^- \to e^+e^-$

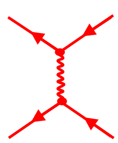
$M =$



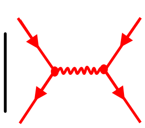
$+$



$\overline{|M|^2} =$

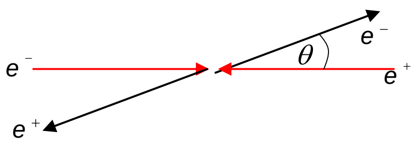


$\underbrace{\left|\ \cdot\ \right|^2}_{e^-\mu^- \to e^-\mu^-} +$ interference $+ \underbrace{\left|\ \cdot\ \right|^2}_{e^+e^- \to \mu^+\mu^-}$



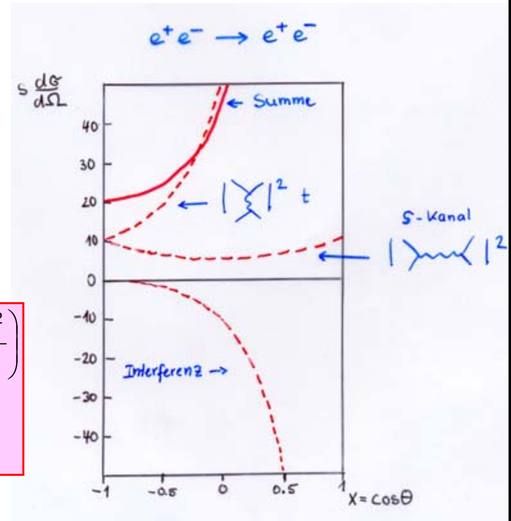
$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{s^2+u^2}{t^2} + \frac{2u^2}{ts} + \frac{t^2+u^2}{s^2}\right)$$

CM system:



with x=cosθ

$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{4+(1+x)^2}{(1-x)^2} - \frac{(1+x)^2}{1-x} + \frac{1+x^2}{2}\right)$$
$$= \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$



$$\frac{d\sigma}{d\Omega} = \frac{\alpha^2}{2s}\left(\frac{3+\cos^2\theta}{1-\cos\theta}\right)^2$$

divergent for
cosθ→1