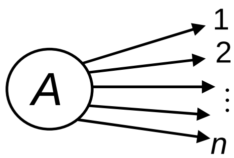
5. Decay width, lifetime and Dalitz plots

5.1 Decay width



$$\tau = \frac{1}{\Gamma} \qquad \Gamma = \sum \Gamma_i$$

Differential decay width (rate):

$$d\Gamma_i(A \to 1+2+...+n) = \frac{W_{fi}}{n_A} d\rho_f$$

$$d\Gamma_i = \frac{|M_{fi}|^2}{2E_A} \cdot (2\pi)^4 \delta^4(p_A - p_1 - p_2 - ... - p_n) \cdot \frac{d^3p_1}{2E_1(2\pi)^3} \cdot \frac{d^3p_2}{2E_2(2\pi)^3} \cdot ... \cdot \frac{d^3p_n}{2E_n(2\pi)^3}$$

Two-body decay:

$$A \to 1+2$$

$$d\Gamma_i(A \to 1+2) = \frac{|M_{fi}|^2}{2E_A} d\Phi_2 = \frac{|M_{fi}|^2}{2E_A} \frac{1}{16\pi^2} \frac{|\vec{p}_f|}{\sqrt{s}} d\Omega$$

CMS: $d\Phi_2 = \frac{1}{16\pi^2} \frac{|\vec{p}_f|}{\sqrt{s}} d\Omega_f$

$\sqrt{s} = E_A = m_A$

$$d\Gamma_i(A \to 1+2) = \frac{|\vec{p}_f|}{32\pi^2 m_A^2} |M_{fi}|^2 d\Omega$$

Three-body decay:

$$A \to 1+2+3$$

$$\int d\Phi_3 = \frac{1}{8(2\pi)^5} \int dE_1 dE_2 \, d\alpha \, d(\cos\beta) \, d\gamma$$

flat in E_1 and E_2

for scalar A or averaged over spins ➡

$$d\Gamma_i(E_1, E_2) = \frac{1}{64\pi^3} \frac{1}{m_A} |M_{fi}|^2 dE_1 dE_2$$

<u>Remark:</u> Instead of variables E_1 and E_2 one can use variables m_{12}^2 and m_{23}^2 = invariant mass of pairs (i,j)

$$m_{ij}^2 = (p_i + p_j)^2$$

$$dE_1 dE_1 = C \cdot dm_{12}^2 dm_{23}^2$$

If phase space is flat in E_i then it is also flat in m_{ij}

$$d\Gamma_i(m_{12}{}^2, m_{23}{}^2) = \frac{1}{256\pi^3} \frac{1}{m_A^3} |M_{fi}|^2 dm_{12}{}^2 dm_{23}{}^2$$

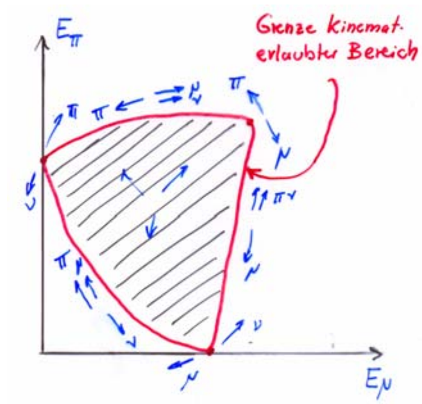
(for A being a scalar, or average over all spin states)

Experimental method to explore behavior of $\boldsymbol{M_{fi}}$: **Dalitz Analysis**

5.2 Dalitz Plots

Method:

Put every measured decay into a 2-dim,. (E_1,E_2) or (m_1^2, m_2^2) distribution. A flat distribution over the allowed region corresponds to a "flat matrix element". Structures in the distribution point to a varying matrix element

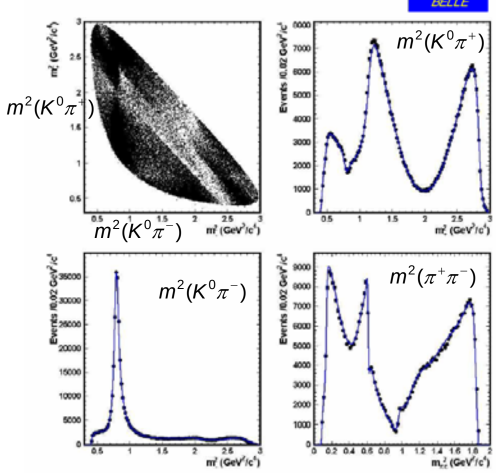


Ex.: $K^0 \to \pi^{\pm}\mu^{\mp}\nu$

Dalitz-Plot at Work:

$D^0 \to K_S^0 \pi^+ \pi^-$

M. Staric, HEP 2007 (Manchester)

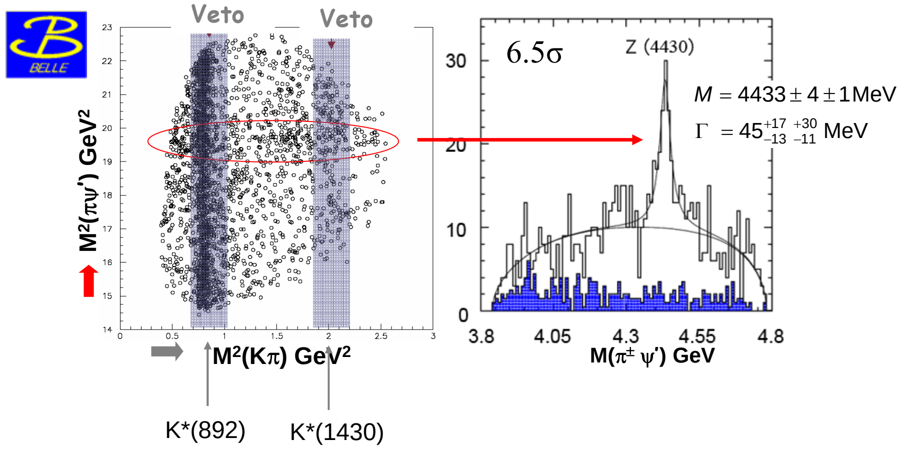


Resonance	Amplitude	Phase (deg)	Fit fraction
$K^*(892)^-$	1.629 ± 0.005	134.3 ± 0.3	0.6227
$K_0^*(1430)^-$	2.12 ± 0.02	-0.9 ± 0.5	0.0724
$K_2^*(1430)^-$	0.87 ± 0.01	-47.3 ± 0.7	0.0133
$K^*(1410)^-$	0.65 ± 0.02	111 ± 2	0.0048
$K^*(1680)^-$	0.60 ± 0.05	147 ± 5	0.0002
$K^*(892)^+$	0.152 ± 0.003	-37.5 ± 1.1	0.0054
$K_0^*(1430)^+$	0.541 ± 0.013	91.8 ± 1.5	0.0047
$K_2^*(1430)^+$	0.276 ± 0.010	-106 ± 3	0.0013
$K^*(1410)^+$	0.333 ± 0.016	-102 ± 2	0.0013
$K^*(1680)^+$	0.73 ± 0.10	103 ± 6	0.0004
$\rho(770)$	1 (fixed)	0 (fixed)	0.2111
$\omega(782)$	0.0380 ± 0.0006	115.1 ± 0.9	0.0063
$f_0(980)$	0.380 ± 0.002	-147.1 ± 0.9	0.0452
$f_0(1370)$	1.46 ± 0.04	98.6 ± 1.4	0.0162
$f_2(1270)$	1.43 ± 0.02	-13.6 ± 1.1	0.0180
$\rho(1450)$	0.72 ± 0.02	40.9 ± 1.9	0.0024
σ_1	1.387 ± 0.018	-147 ± 1	0.0914
σ_2	0.267 ± 0.009	-157 ± 3	0.0088
NR	2.36 ± 0.05	155 ± 2	0.0615

Recent discovery of a new exotic particle Z(4430)

$$B \rightarrow K \pi^{\pm} \psi'$$

arXiv:0708.1790 subm. to PRL



6. Unitarity of S-Matrix and optical theorem

From the normalization of an arbitrary state $\langle i | i \rangle = 1$

one finds the unitarity of the S matrix $\langle i' | i' \rangle = \langle i | \mathbf{S}^{+}\mathbf{S} | i \rangle = 1$

$$\mathbf{S}\mathbf{S}^{+} = \mathbf{S}^{+}\mathbf{S} = \mathbf{1}$$

$$\mathbf{S} = \mathbf{S}^{-1}$$

Where the matrix elements of the adjoint operator are defined in the usual way $S_{if}^{+} = S_{fi}^{*}$

Unitarity

⇔ Conservation of the probability in the scattering process.

What goes in, should also go out !!

Optical theorem follows from unitarity of the S matrix (Nachtmann):

$$\sigma_{tot}(1+2\rightarrow X) = \frac{-1}{w(s,m_1,m_2)}\,\mathrm{Im}\left(M_{fi}(1+2\rightarrow 1+2)^{el}_{\theta=0}\right)$$

Total cross section of two interacting particles

Flux normalization

Transition amplitude for elastic forward scattering

$$w(s,m_1,m_2) = \left((s-(m_1+m_2)^2)(s-(m_1-m_2)^2)\right)^{1/2}$$