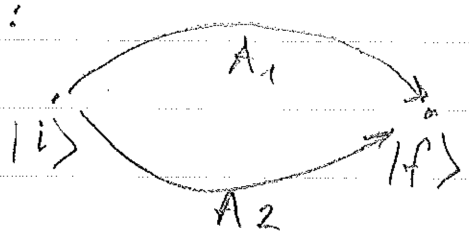
3.3 CP violation in meson decays – general remarks

CP violation is linked to the CKM phases in the transition amplitudes. But: all observable qualities are in general „squares" of matrix-elements
→ phases do not lead easily to measurable effects
(absolute phases are not observable)

Phase differences are observable via interference effects:
At least two interfering amplitudes are required to observe a phase related effect and to study CP violation:



$$A(i \to f) = |A_1| e^{i\phi_1} + |A_2| e^{i\phi_2}$$

$$(*) \quad |A(i \to f)|^2 = |A_1|^2 + |A_2|^2 + 2|A_1||A_2| \cos(\phi_1 - \phi_2)$$

However this is also not sufficient to observe CPV:
CP conjugation is changing the sign of both weak phases ϕ_1 and ϕ_2 such that one obtains for the CP conjugated version of (*):

$$|A(\bar{i} \to \bar{f})|^2 = |A_1|^2 + |A_2|^2 + 2|A_1||A_2| \cos(\phi_1 - \phi_2)$$

(exactly the same expression!)

In order to observe CP violation additional „strong" phases δ_1 and δ_2 which do not change sign, must be present:

$$A(i \to f) = |A_1| e^{i\phi_1} e^{i\delta_1} + |A_2| e^{i\phi_2} e^{i\delta_2}$$

$$A(\bar{i} \to \bar{f}) = |A_1| e^{-i\phi_1} e^{i\delta_1} + |A_2| e^{-i\phi_2} e^{i\delta_2}$$

For the diff. of the transition rate one now finds:

$$|A(\bar{i} \to \bar{f})|^2 - |A(i \to f)|^2 = 2|A_1||A_2| \sin(\phi_1 - \phi_2) \sin(\delta_1 - \delta_2)$$

Problem: The strong phases are a result of the interaction between the hadronic final state particles → difficult to calculate

→ The observation of CP violation in many cases does not allow to conclude on the weak phases! (only special cases lead to information on the weak phases)

Before coming back and explaining the origin of the observed CPV in kaon decays I like to introduce the phenomenologie of neutral meson mixing.

3.4. Neutral meson mixing and CP violation

In the following neutral flavored meson systems (K^0, D^0, B^0_d, B^0_s) in which particle and anti-particle are distinguished by the flavor quantum number will be discussed in a general way: P^0 and $\overline{P^0}$.

A.) Flavor and CP eigenstates

A generic flavored meson P^0 and $\overline{P^0}$ is described by:

$$F|P^0\rangle = +|P^0\rangle \qquad F|\overline{P^0}\rangle = -|\overline{P^0}\rangle$$

CP conjugation transforms particle into anti-particle but introduces in general a phase η_{CP}:

$$CP|P^0\rangle = \eta_{CP}|\overline{P^0}\rangle \qquad CP|\overline{P^0}\rangle = \eta^*_{CP}|P^0\rangle$$

Besides the flavor states there exists the physical states which are mixtures of the flavor states and which, in case CP is conserved, could be chosen to be CP eigenstates:

$$\underset{+}{P_1} = \frac{1}{\sqrt{2}}\left(|P^0\rangle + |\overline{P^0}\rangle\right) \qquad \underset{-}{P_2} = \frac{1}{\sqrt{2}}\left(|P^0\rangle - |\overline{P^0}\rangle\right)$$

with $CP|P_1\rangle = +|P_1\rangle \qquad CP|P_2\rangle = -|P_2\rangle$

in case the phase η_{CP} is chosen to be $+1$.
(convention)

B) Effective Lagrangian and the physical states

In the most general case the time dependence of a physical state can be described by the flavor states:

$$\psi(t) = a(t)\,|P^0\rangle + b(t)\,|\bar{P}^0\rangle$$

$\psi(t)$ should fulfill the Schrödinger Eq with the non-hermitian effective Hamiltonian $\mathcal{H}$ (non-hermitian, because ψ decays outside the $(P^0, \bar{P}^0)$ subspace):

$$i\hbar \frac{d}{dt}\psi(t) = \mathcal{H}\,\psi(t)$$

One usually splits $\mathcal{H}$, which consists of a flavor conserving part $\mathcal{H}_0$ and a weak flavor-violating part $\mathcal{H}_W$ ($\mathcal{H} = \mathcal{H}_0 + \mathcal{H}_W$), into an hermitian and an anti-hermitian part:

$$\mathcal{H} = M - \frac{i}{2}\Gamma$$

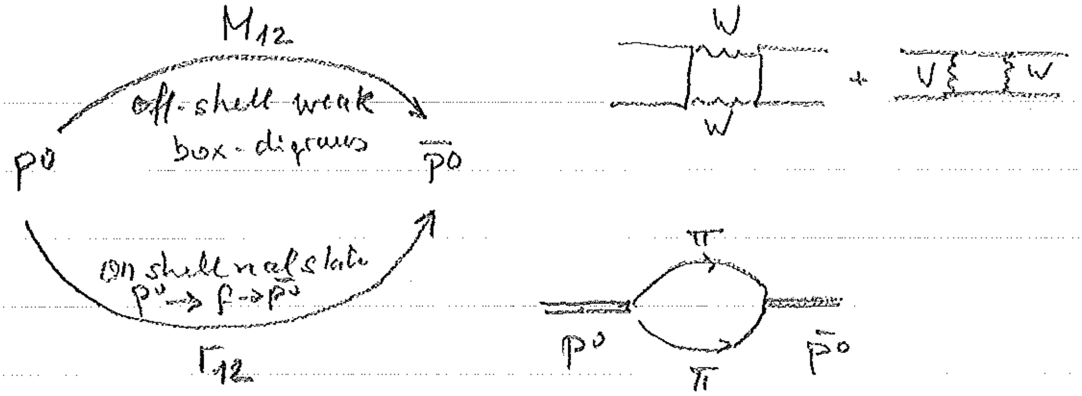
where both M (mass matrix) and Γ (decay matrix) are hermitian.

With the representation $|\psi\rangle = \begin{pmatrix} a \\ b \end{pmatrix}$ one can write $\mathcal{H}$ a 2dim matrix

$$\mathcal{H} = \begin{pmatrix} M_{11} - \frac{i}{2}\Gamma_{11} & M_{12} - \frac{i}{2}\Gamma_{12} \\ M_{21} - \frac{i}{2}\Gamma_{21} & M_{22} - \frac{i}{2}\Gamma_{22} \end{pmatrix}$$

Hermiticity of M and Γ: $M_{12} = M_{21}^*$, $\Gamma_{12} = \Gamma_{21}^*$ CPT: $M_{11} = M_{22}$ $\Gamma_{11} = \Gamma_{22}$

- The states P^0 and $\bar{P}^0$ are eigenstates to $\mathcal{H}_0$ $\rightarrow$ no mixing, no decay
  $\mathcal{H}_0$ therefore contributes only to diag(M)
- M_{12} dispersive part of the $P^0 \leftrightarrow \bar{P}^0$ transition describes mixing via virtual intermediate states
- Γ_{12} absorptive part of $P^0 \leftrightarrow \bar{P}^0$ transition describe the mixing via real intermediate states
- Γ_{11}, Γ_{22} describe the decay to real final states



The physical states $|P_a\rangle$ and $|P_b\rangle$ are obtained by diagonality the matrix $\to$ eigenvalues $\lambda_{a,b}$ & eigenstates $P_{a,b}$:

$$\mathcal{H}\,|P_{a,b}\rangle = \lambda_{a,b}\,|P_{a,b}\rangle$$

with $\lambda_{a,b} = m_{a,b} - \frac{i}{2}\Gamma_{a,b}$

where $m_{a,b}$ are the masses and $\Gamma_{a,b}$ are the decay widths:

$$|P_{a,b}(t)\rangle = e^{-i m_{a,b}\cdot t}\, e^{-\Gamma_{a,b}\cdot t/2}\,|P_{a,b}(t=0)\rangle$$

(a,b)

The physical states are usually labelled by the properties which distinguish them the best:

lifetime for kaons: K_S, K_L

masses for Bmesons: B_H, B_L

CP values for Dmesons: D_1, D_2 (assuming no direct CPV)

The physical states can be written as

$$|P_a\rangle = p\,|P^0\rangle + q\,|\bar{P}^0\rangle$$
$$|P_b\rangle = p\,|P^0\rangle - q\,|\bar{P}^0\rangle$$

with $|p|^2 + |q|^2 = 1$

(Remark: we used CPT invariance: $q_a/p_a \stackrel{!}{=} q_b/p_b =: q/p$)

One further defines the following quantities:

$$\Delta m = m_b - m_a \qquad \Delta\Gamma = \Gamma_b - \Gamma_a$$

$$\Gamma = \frac{1}{2}(\Gamma_a + \Gamma_b) \qquad m = \frac{1}{2}(m_a + m_b)$$

For q/p one finds:

$$q/p = \pm\sqrt{\frac{\mathcal{H}_{21}}{\mathcal{H}_{12}}} = \pm\sqrt{\frac{M_{12}^* - \frac{i}{2}\Gamma_{12}^*}{M_{12} - \frac{i}{2}\Gamma_{12}}}$$

The sign $\pm$ of q/p determines whether m_a or m_b is heavier;
The usual choice is $\Delta m > 0$: $q/p > 0 \Leftrightarrow$ „+" sign.
Attention: This choice <u>is not</u> fixing the sign of $\Delta\Gamma$
(experiment has to tell whether $\Delta\Gamma \gtrless 0$; i.e. CP even/odd lives longer)

<u>Remark:</u> While P^0 and $\overline{P^0}$ as well as P_1 and P_2 are orthogonal P_a and P_b are in general <u>not</u> orthogonal:

$$\xi = \langle P_a | P_b \rangle = |p|^2 - |q|^2 \neq 0$$

If CP symmetry is conserved and P_a, P_b are CP eigenstates: $|q/p| = 1 \rightarrow \xi = 0$

C.) Time evolution of flavor states:

Physical states: $|P_a\rangle = p\,|P^0\rangle + q\,|\bar{P}^0\rangle$

(forget about the norm) $|P_b\rangle = p\,|P^0\rangle - q\,|\bar{P}^0\rangle$

[Depending on the properties which distinguish the states best one could label them (P_H, P_L), (P_S, P_L) or (P_1, P_2)]

For the flavor states one obtains correspondingly:

$$|P^0\rangle = \frac{1}{2p}\left[\,|P_a\rangle + |P_b\rangle\right]$$
$$|\bar{P}^0\rangle = \frac{1}{2q}\left[\,|P_a\rangle - |P_b\rangle\right]$$

Using the time dependence of $|P_a\rangle$ and $|P_b\rangle$ as above:

$$|P^0(t)\rangle = \frac{1}{2p}\left\{ e^{-im_a t}\, e^{-\Gamma_a t/2}\, \underbrace{|P_a(0)\rangle}_{=p|P^0\rangle + q|\bar{P}^0\rangle} + e^{-im_b t}\, e^{-\Gamma_b t/2}\, \underbrace{|P_b(0)\rangle}_{=p|P^0\rangle - q|\bar{P}^0\rangle} \right\}$$

$$= g_+(t)\,|P^0\rangle + \left(\frac{q}{p}\right) g_-(t)\,|\bar{P}^0\rangle$$

$$|\bar{P}^0(t)\rangle = g_+(t)\,|\bar{P}\rangle + \left(\frac{p}{q}\right) g_-(t)\,|P^0\rangle$$

with: $g_\pm(t) = \frac{1}{2}\left(e^{-im_a t - \Gamma_a t/2} \pm e^{-im_b t - \Gamma_b t/2}\right)$

$$= \frac{1}{2}\, e^{-imt}\left(e^{-\frac{1}{2}\Delta m t - \Gamma_a t/2} \pm e^{+\frac{1}{2}\Delta m t - \Gamma_b t/2}\right)$$

$$g_-(t) = \quad " \quad \left(\quad " \quad - \quad " \quad \right)$$

with $m = \frac{1}{2}(m_a + m_b)$ as above.

Starting from a pure sample of P^0 mesons (e.g. produced in strong interaction) the probability to measure the flavor state $\overline{P^0}$ at time t is given by:

$$|\langle \overline{P^0}|P^0(t)\rangle|^2 = |g_-(t)|^2 \left|\frac{q}{p}\right|^2$$

Possibility for CPV in mixing

and correspondingly for a pure $\overline{P^0}$ sample at $t=0$:

$$|\langle P^0|\overline{P^0}(t)\rangle|^2 = |g_-(t)|^2 \left|\frac{p}{q}\right|^2$$

With the above expressions for $g_\pm(t)$ one finds:

$$|g_\pm(t)|^2 = \frac{1}{4}\left(e^{-\Gamma_a t} + e^{-\Gamma_b t} \pm e^{-\Gamma t}\left(e^{-i\Delta m t} + e^{+i\Delta m t}\right)\right)$$

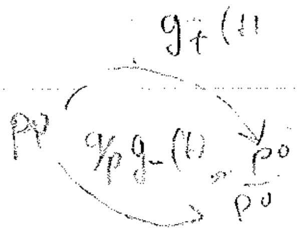
$$= \frac{e^{-\Gamma t}}{2}\left(\cosh\left(\tfrac{1}{2}\Delta\Gamma t\right) \pm \cos(\Delta m t)\right)$$

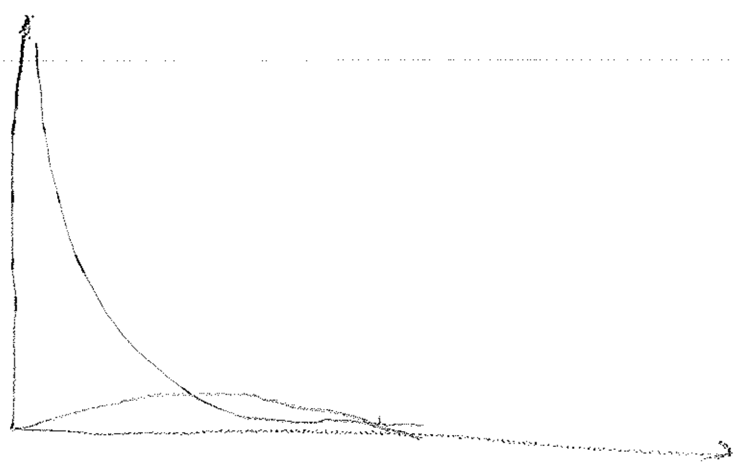
See Chap. 3.1 $K^0 \to \overline{K^0}$ Mixing

For $\Delta\Gamma \approx 0$ (in B-mesons) and $|p/q| \approx 1$ (fullfilled for all P^0):

$$\mathrm{Prob}(P^0 \to P^0)(t) = \frac{e^{-\Gamma t}}{2}\left(1 + \cos(\Delta m t)\right)$$

$$\mathrm{Prob}(P^0 \to \overline{P^0})(t) = \frac{e^{-\Gamma t}}{2}\left(1 - \cos(\Delta m t)\right)$$





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