1.3 Names of hadrons

Historically mesons of light quarks are called „unflavored" in contrast to „flavored" mesons of heavy quarks:

Kaons: net-strangeness (s quark ⇒ $S=-1$)
D-mesons: net-charm (c-quark ⇒ $C=+1$)
B-mesons: net-beauty (b-quarks ⇒ $B=-1$)

- Pseudoscalar mesons: $J^{PC}=0^{-+}$: $\pi, \eta, \eta', K, D, B$
- Vector mesons: $J^{P}=1^{--}$: $\rho, \omega, \phi, K^*, D^*, B^*$
- other mesons: $J^{PC}=0^{++}$ mesons: $a_0, a_1, a_2, \dots$
  (~ relative angular momentum $l=1$, Spin $S=1 \rightarrow J=0$)
  $J^{PC}=1^{+-}$ mesons: b_0, b_1, b_2
  ($l=1$, $S=0$ ⇒ $J=1$)

- **Baryons** (half-integer spin):
  - Baryons with 3 light u, d quarks
    $I=\frac{1}{2}$: nucleons N
    $I=\frac{3}{2}$: Δ's
  - Baryons with 2 light (u, d) quarks + s, c, b:
    $I=0$: Λ (Λ_c, Λ_b)
    $I=1$: Σ (Σ_c, Σ_b)
  - Baryons with 1 light quark
    $I=\frac{1}{2}$: $\underset{ss}{\Xi}$ ($\underset{sc}{\Xi_c}$, $\underset{sb}{\Xi_b}$)
  - Baryons with no light quark
    $\Omega\,(sss)$ $\Omega_c\,(ssc)$ $\Omega_b\,(ssb)$

Hyperons

1.4 Weak - decays of hadrons

Theoretically quarks are the fundamental particles participating in IA. Experimentally however one observes / probes hadrons as asymptotic states.

↳ Introduce a clever parametrization to treat the problem.

- Factorize the different physical effects in the transition amplitude. Factorization exploits the idea that one can separate diff. kinds of physics in a factorizable way: e.g. physics at diff. scales decouples.
- Form factors: describes shape corrections to the approximation that the scattering object is not point like (→ familiar concept in non-relativistic Rutherford scattering)
- Decay constant: Absorbes the non-perturbative properties of meson decays.

Example: $\pi \to \ell \nu$: $\mathcal{A} = \langle \ell\nu | \mathcal{O} | \pi \rangle$

↳ asymptotic states

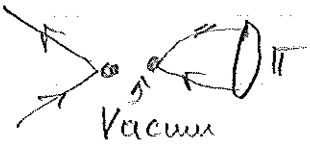
As the decay is a low energy process $m_\ell, m_\pi < M_W$ one can integrate-out the W-boson → effective 4 fermion IA

↳ Effective operator $\mathcal{O}$: $\mathcal{O} \sim \mathcal{O}_\ell \frac{1}{M_W^2} \mathcal{O}_\pi$

"symbolically" $= (\text{"}\bar{\ell} \gamma^5 \gamma^\mu \nu\text{"}) \frac{1}{M_W^2} (\text{"}\bar{u} \gamma^5 \gamma_\mu d\text{"})$

The matrix element should factorize the same way:

$$\langle \ell\nu | \mathcal{O} | \pi \rangle = \langle \ell\nu | \mathcal{O}_e | 0 \rangle \frac{1}{M_W^2} \langle 0 | \mathcal{O}_H | \pi \rangle$$



As we don't know how to calculate $\langle 0 | \mathcal{O}_H | \pi \rangle$ one treats it as a parameter of the theory:

$$\pi\text{- decay const } f_\pi \sim \langle 0 | \mathcal{O}_H | \pi \rangle$$

The decay const. absorbes all the non-perturbative „brown mud" that keep the π together:
The operator $\mathcal{O}_H$ should annihilate the 2 quarks in the pion and must therefore be of the form:

$\bar{u} \Gamma d$ with coupling structure $\Gamma = S, P, A, V, T$

Since the parity of the vacuum is „even" and the parity of π is „odd" the main contribution comes from the axial-vector operator:

$$\langle 0 | \mathcal{O}_H | \pi \rangle = \underbrace{\langle 0 | \bar{u} \gamma^5 \gamma^\mu d | \pi \rangle}_{} = \langle 0 | A^\mu | \pi \rangle = -i f_\pi q^\mu$$

we don't how to calculate

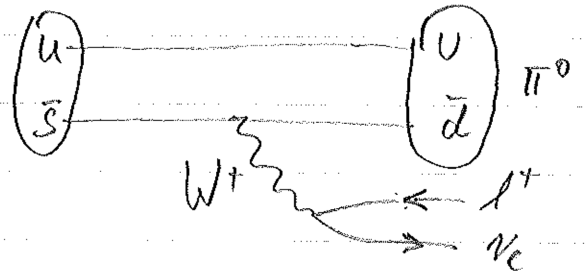
→ we absorb our ignorance in the decay constant (must have mass dimension)

→ As the Lorentz structure should be an (axial) vector and the only vector involved in the decay of the scalar π is the momentum transfer: $\langle 0 | \mathcal{O}_H | \pi \rangle \sim q^\mu$

The decay constant f_π can be measured (π decay): $f_\pi \approx 131$ MeV
One obtains the matrix element for the decay:

$$\mathcal{M}(\pi^- \to \mu^- \bar{\nu}) = \frac{-g^2}{4 M_W^2} \cdot f_\pi \cdot (V_{ud}) \cdot m_\mu (\bar{\mu}_R \nu_L)$$

Example : $K^+ \to \pi^0 l^+ \nu$



$$\mathcal{A} = \langle \pi^0 l \nu | \mathcal{O} | K^+ \rangle$$

$$= \underbrace{\langle l \nu | \mathcal{O}_e | 0 \rangle}_{\text{easy - see above}} \cdot \frac{1}{M_W^2} \cdot \underbrace{\langle \pi | \mathcal{O}_H | K^+ \rangle}_{\substack{\text{QCD binding of quarks} \\ \hookrightarrow \text{difficult}}}$$

→ Absorb the non-perturbative effects into a form-factor

$$F \sim \langle H_2 | \mathcal{O} | H_1 \rangle$$

Form factors can be determined (measured) in semi-leptonic decays.

2. CKM-Matrix

The origine of the CKM-matrix are the non-diagonal Yukawa coupling matrices for u and d quarks. By diagonalizing these matrices the off-diagonal elements appear as CKM matrix in the charged currents. The 3×3 unitary CKM matrix has 3 real parameters and 1 phase which is the origine of CP violation in the quark sector.

→ Nobel price for Kobayashi and Maskawa in 2008.

2.1 Parametrizations of the CKM matrix

A convinient parametrization also adopted by the PDG uses 3 Euler angles and 1 phase parameter:

$$V_{CKM} = \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13} e^{-i\delta_{13}} \\ 0 & 1 & 0 \\ -s_{13} e^{i\delta_{13}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix}$$

where $c_{ij} = \cos\theta_{ij}$ $s_{ij} = \sin\theta_{ij}$ → Fig, with multiplied matrix.

The phase in this convention is chosen to appear in the matrix to describe the mixing between 1st and 3rd generation. Size:

$$|V_{CKM}| = \begin{pmatrix} 0.97428 & 0.2253 & 0.00347 \\ 0.2252 & 0.97345 & 0.0410 \\ 0,00862 & 0.0403 & 0.99915 \end{pmatrix} \quad \text{(see Fig)}$$

The strengths of the couplings V_{ij} exhibit a hierarchy:

$$|V_{CKM}| = \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix}$$

This pattern motivated Wolfenstein to parametrize the CKM in powers of $\underset{\approx 0.22}{\lambda \propto \sin\theta_{12}}$ in the PDG convention.

Wolfenstein parametrization $\rightarrow$ usefull for quantitative discussion of many quark mixing effects

approximate parametrization:

$$\sin\theta_{12} \simeq \lambda$$

$$\sin\theta_{23} \simeq A\lambda^2$$

$$\sin\theta_{12} e^{-i\delta_{13}} \simeq A\lambda^3(\rho - i\eta)$$

CKM matrix in $O(\lambda^3)$:

$$V_{CKM} = \begin{pmatrix} 1-\frac{1}{2}\lambda^2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\frac{1}{2}\lambda^2 & A\lambda^2 \\ \underline{A\lambda^3(1-\rho-i\eta)} & -A\lambda^2 & 1 \end{pmatrix} + \delta V\left(O(\lambda^4)\right)$$

where A, ρ, η are of $O(1)$.
Up to $O(\lambda^3)$ only V_{td} and V_{ub} are complex!
Matrix element V_{ts} becomes complex only at $O(\lambda^4)$

2.2 Unitarity relations and resulting constraints

From the unitarity conditions $V^\dagger V = 1$ or $V V^\dagger = 1$ one obtains to following 12 equations to be fullfilled:

$$(1) \quad \underset{\substack{c\\t}}{V^*_{ud}}\,\underset{\substack{c\\t}}{V_{ud}} + \underset{\substack{c\\t}}{V^*_{us}}\,\underset{\substack{c\\t}}{V_{us}} + \underset{\substack{c\\t}}{V^*_{ub}}\,\underset{\substack{c\\t}}{V_{ub}} = 1$$

$$\underset{\substack{s\\b}}{V^*_{ud}}\,\underset{\substack{s\\b}}{V_{ud}} + \underset{\substack{s\\b}}{V^*_{cd}}\,\underset{\substack{s\\b}}{V_{cd}} + \underset{\substack{s\\b}}{V^*_{td}}\,\underset{\substack{s\\b}}{V_{td}} = 1$$

- Weak universality : u couples to the sum of $\sum_i d_i$ the same way as c and t as well as leptons!
- the sum of $u \to d + u \to s + u \to b$ add up to 1 → no space for another decay!

(2) orthogonality relations:

$$V_{ud}^* V_{cd} + V_{us}^* V_{cs} + V_{ub}^* V_{cb} = 0$$

$$B \longrightarrow \quad ud \;\; td \qquad us \;\; ts \qquad ub \;\; tb \approx 0$$

$$cd \;\; td \qquad cs \;\; ts \qquad cb \;\; tb \approx 1$$

$$V_{ud} V_{us}^* + V_{cd} V_{cs}^* + V_{td} V_{ts}^* = 0$$

$$A \longrightarrow \quad ud \;\; ub \qquad cd \;\; cb \qquad td \;\; tb =$$

$$us \;\; ub \qquad cs \;\; cb \qquad ts \;\; tb = 0$$

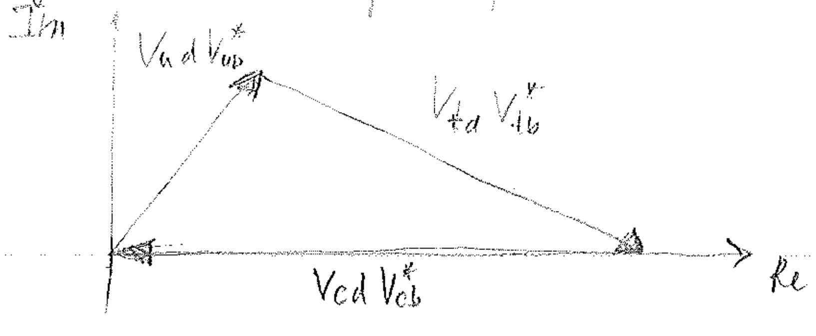
These relations describe triangular relations in the complex plane
It is very instructive to study 2 of these relations in more detail:

$$A_1 \qquad \underset{\mathcal{O}(\lambda^3)}{V_{ud} V_{ub}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{cd} V_{cb}^*} + \underset{\mathcal{O}(\lambda^3)}{V_{td} V_{tb}^*} = 0$$

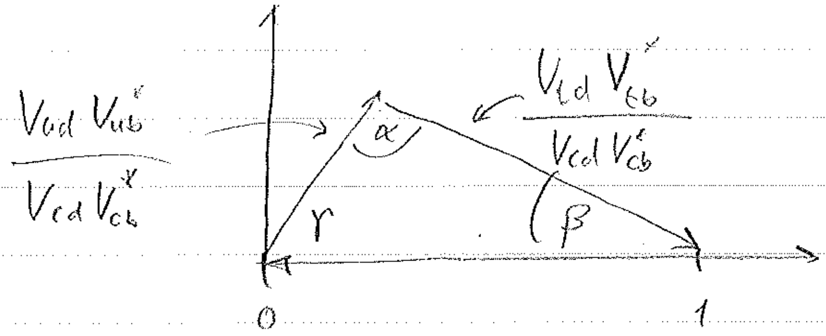
$$B \qquad \underset{\mathcal{O}(\lambda^3)}{V_{ud}^* V_{td}} + \underset{\mathcal{O}(\lambda^3)}{V_{us}^* V_{ts}} + \underset{\mathcal{O}(\lambda^3)}{V_{ub}^* V_{tb}} = 0$$

These are the only two „tri-angle" relations where all sides have approx. the same size $\mathcal{O}(\lambda^3)$. One can draw the relations as triangles in the complex plane:

A:



The other relations also describe triangles which are, however squashed. By dividing in A all sides by $V_{cd}^{*}V_{cb}$ (~ real) one obtains the Unitarity Triangle (renormalizing & rephasing)



The apex of the triangle $(\bar{\rho},\bar{\eta})$ is given by:

$$\bar{\rho} + i\bar{\eta} = \frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}$$

In terms of the Wolfenstein Parameters ρ and η the apex is given by:

$$\bar{\rho} = \rho\left(1-\tfrac{1}{2}\lambda^{2}\right)+\mathcal{O}(\lambda^{4}) \quad \bar{\eta} = \eta\left(1-\tfrac{1}{2}\lambda^{2}\right)+\mathcal{O}(\lambda^{4})$$

The angles α, β, γ of the unitarity triangle are defined as:

$$(*) \quad \alpha \equiv \arg\left[-\frac{V_{td}V_{tb}^{*}}{V_{ud}V_{ub}^{*}}\right] \quad \beta \equiv \arg\left[-\frac{V_{cd}V_{cb}^{*}}{V_{td}V_{tb}^{*}}\right] \quad \gamma \equiv \arg\left[-\frac{V_{ud}V_{ub}^{*}}{V_{cd}V_{cb}^{*}}\right]$$

In the Wolfenstein Parametrization a phase convention is used such that V_{td}, V_{ub} and (V_{ts}) have an imaginary component (to Order $\mathcal{O}(\lambda^{4})$) and $V_{cd}V_{cb}^{*}$ is real!

Therefore one finds: $\beta \simeq -\arg(V_{td})$

$$\gamma = -\arg(V_{ub})$$

(for the unitarity triangle B_s $\beta_s \simeq \arg(V_{ts}) + \pi$)

With these phases one can rewrite the CKM matrix using the Wolfenstein phase convention:

$$V_{CKM} = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}|e^{-i\gamma} \\ -|V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}|e^{-i\beta} & -|V_{ts}|e^{i\beta_s} & |V_{tb}| \end{pmatrix} + \mathcal{O}(\lambda^5)$$

While the specific appearance of the phases is a convention the parameters β, γ and β_s (as defined in (*)) are in principle rephasing invariant parameters, independent of the CKM parametrization.

Another „rephasing" invariant variable is the Jarlskog Invariant

$$J = \pm \mathrm{Im}\left(V_{ij} V_{kl} V_{il}^* V_{kj}^* \right) \qquad i \neq k,\ j \neq l$$

$$[\; J = \mathrm{Im}\left(V_{ud} V_{cs} V_{us}^* V_{cd}^* \right) = -\mathrm{Im}\left(V_{ud} V_{cb} V_{ub}^* V_{cd}^* \right) = \dots \;]$$

Using the Wolfenstein parametrization:

$$J \approx A^2 \lambda^6 \eta \quad \approx 3.03 \cdot 10^{-5} \qquad \text{CKM-Fitter 2013}$$

, or with the PDG parametrization

$$J = c_{12}\, c_{13}^2\, c_{23}\, s_{12}\, s_{13}\, s_{23} \sin\delta_{13}$$

Jarlskog Invariant describes the CP Violation in the SM. It appears in all CP violating effects. It is zero if one of the mixing angle vanishes ($\theta_{ij} = 0$) or the phase vanishes ($\delta_{23} = 0$).

All the Jarlskog terms are products of the type

$$\mathrm{Im}\, A B^* = |A|\,|B|\, \mathrm{Im}\left(e^{i\underbrace{(\arg(A)-\arg(B))}_{\delta}}\right)$$

$$= |A|\,|B| \sin(\delta)$$

$\rightarrow$ = twice the area of a triangle in the $\mathbb{C}$-plane

Since the A and B are the sides of the unitarity triangle

$$\Rightarrow \boxed{J = 2 \cdot A_{UT}}$$

Status of the Unitarity Triangle:

Fig: CKM Triangle

$\lambda = 0{,}22457^{+0.00186}_{-0.00014}$

$A = 0.823^{+0{,}012}_{-0{,}033}$

$\bar{\rho} = 0{,}11289^{+0{,}0176}_{-0{,}0094}$

$\bar{\eta} = 0{,}348 \pm 0{,}012$

CKM-Fitter 2013