

# Flavor Physics – Exercise Sheet 5 – SomSem 2015

Discussion: 05/06 during the tutorial

## Exercise 1: $K^0 - \bar{K}^0$ oscillation probability

Assuming CP invariance the observed  $K_S$  and  $K_L$  states are given by the following linear combinations of the flavor states,

$$|K_S\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle + |\bar{K}^0\rangle),$$
$$|K_L\rangle = \frac{1}{\sqrt{2}} (|K^0\rangle - |\bar{K}^0\rangle).$$

The physical states exhibit the time-dependence  $|K_{S,L}(t)\rangle = e^{-im_{S,L}t} e^{-\Gamma_{S,L}t/2} |K_{S,L}\rangle$ , where  $m_{S,L}$  and  $\Gamma_{S,L}$  are the mass and the total decay width of the state.

Derive the time dependent probability  $P(K^0(t=0) \rightarrow K^0(t))$  to observe an initial  $K^0$  after time  $t$  as  $K^0$  and the probability  $P(K^0(t=0) \rightarrow \bar{K}^0(t))$  to observe it in the flavor-mixed  $\bar{K}^0$  state. The formulae were given in the lecture.

## Exercise 2: $K_S$ - $K_L$ interference as confirmation for CP violation

In presence of CP violation the physical states  $K_S$  and  $K_L$  decaying to CP eigenstates can interfere. For a neutral kaon which is produced at  $t = 0$  as a  $K^0$  ( $\bar{K}^0$ ) and propagates freely in vacuum, the time-dependent decay rate to  $\pi^+\pi^-$  is given by

$$\Gamma [K^0 (\bar{K}^0) (t=0)] (t) \propto e^{-\Gamma_S t} + |\eta_{\pi\pi}|^2 e^{-\Gamma_L t} \pm 2|\eta_{\pi\pi}| e^{-(\Gamma_S + \Gamma_L)t/2} \cos(\Delta m t - \phi_{\pi\pi}),$$

where the  $+$  ( $-$ ) sign applies for the  $K^0$  ( $\bar{K}^0$ ). The complex number  $\eta_{\pi\pi} = |\eta_{\pi\pi}| e^{i\phi_{\pi\pi}}$  describes the CP violating amplitude ratio

$$\eta_{\pi\pi} = \frac{\mathcal{A}(K_L \rightarrow \pi\pi)}{\mathcal{A}(K_S \rightarrow \pi\pi)}.$$

- a) Motivate the above formula for the time dependent decay-rate.
- b) Read the attached paper: *C. Geweniger et al., Phys. Lett. 48B (1974) 487.*
  - Explain the selection of the  $K^0 \rightarrow \pi\pi$  events.
  - How is the proper-time distribution in Figure 4 obtained.
  - How do the authors finally obtain  $|\eta_{\pi\pi}|$  and the phase  $\phi_{\pi\pi}$ .

To better understand the detector layout, a second paper describing the apparatus is also added *C. Geweniger et al., Phys. Lett. 48B (1974) 483.*

## A NEW DETERMINATION OF THE $K^0 \rightarrow \pi^+ \pi^-$ DECAY PARAMETERS

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Received 4 February 1974

In a short neutral beam we have measured the proper time-dependence of the decay  $K^0 \rightarrow \pi^+ \pi^-$ . This time structure exhibits the interference between the short- and long-lived states and is in agreement with the general expectations of the  $CP$  violation phenomenology.

This experiment gives new and more precise measurements of the following three parameters:

- i) the decay width of the short-lived  $K_S$  component:  $\Gamma_S = (1.119 \pm 0.006) \times 10^{10} \text{ sec}^{-1}$ ;
- ii) the modulus of the  $CP$  violating parameter  $\eta_{+-}$ :  $|\eta_{+-}| = (2.30 \pm 0.035) \times 10^{-3}$ ;
- iii) the phase of  $\eta_{+-}$  as a function of the  $K_S - K_L$  mass difference  $\Delta m$ :  $\phi_{+-} = (49.4 \pm 1.0)^\circ + [(\Delta m - 0.540)/0.540] \times 305^\circ$ .

The result of  $|\eta_{+-}|$  may be compared with the result of the foregoing letter on  $\text{Re } \epsilon$  in the frame of the superweak model. Good agreement is observed.

The experiment presented here studies the time dependence of the  $\pi^+ \pi^-$  decay mode of the  $K^0$  meson in a short neutral beam. This time dependence has been previously studied [1, 2] and the main purpose of this experiment is to improve the precision, statistically as well as systematically, in order to provide a more accurate measure of the amplitude ratio

$$\eta_{+-} = \frac{\langle \pi^+ \pi^- | T | K_L \rangle}{\langle \pi^+ \pi^- | T | K_S \rangle} = |\eta_{+-}| \exp(i\phi_{+-}).$$

There is a substantial interest in experimental precision, because of the predictions of the superweak [3] and other models. If  $CPT$  is assumed to be conserved and if  $\epsilon$  is the admixture of the  $CP$  odd ( $CP$  even) state in the dominantly  $CP$  even ( $CP$  odd) decay state, then the superweak model predicts

$$\eta_{+-} = \epsilon. \quad (1)$$

Together with unitarity the model further specifies the phase of both parameters:

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$$\phi = \tan^{-1} \frac{2\Delta m}{\Gamma_S - \Gamma_L}.$$

A whole class of other models is less specific in its predictions, but deviations from eq. (1) are expected to be of the order of the admixture of the  $I = 2$  isospin state in the dominantly  $I = 0$ ,  $K_S \rightarrow \pi^+ \pi^-$  decay. The smallness of this admixture ( $\sim 4\%$ ) explains the interest in precision in the experimental verification of eq. (1).

This experiment presents a very substantial effort over a number of years, and all the relevant details of apparatus and analyses unfortunately cannot be included in this letter. The interested reader must be referred to a future, more detailed publication elsewhere.

The apparatus has been presented in the preceding letter [4]. Without going into details we point out its important properties.

The decay region which extends from 2.2 m to 11.6 m after the target permits detection in the proper time interval

$$3.5 \times 10^{-10} \text{ sec} < \tau < 30 \times 10^{-10} \text{ sec}.$$

The use of multiwire proportional chambers allows

a high data-taking rate and limits the amount of matter in the beam.

The spectrometer section is followed by a trigger plane of twelve thin (1.6 mm) counters. A right-left coincidence in this plane is required to initiate an event, and the final read-out system requires two – and only two – hits in each plane of the multiwire proportional chambers.

The following requirements are imposed on the selection of events:

- i) each chamber must have exactly two vertical and two horizontal wires hit;
- ii) a  $\chi^2$  deviation, formed of the vertical kink angle of each track (after correction for vertical focusing) and of the skewness of the vertex, must be less than 12;
- iii) there must be no signal in the Čerenkov counter and no coincidence between the two muon counter planes;
- iv) the longitudinal distance target-decay vertex must be greater than 2.2 m;
- v) the momenta of both charged secondaries must lie in the interval 1.50 GeV/c to 8.50 GeV/c (the minimum range for traversal of the muon detector is 1.45 GeV/c and the threshold for pion detection in the Čerenkov is 8.40 GeV/c);

vi) only events with inwards bending in the magnet are retained;

vii) events for which simultaneously  $m_{p\pi}$  is within 10 MeV of the  $\Lambda^0$  mass, and  $p_+/(p_+ + p_-)$  is greater than 0.74, are withdrawn from the sample to reduce the  $\Lambda$  contamination.

These criteria are designed to select  $K^0 \rightarrow \pi^+\pi^-$  decays and reject other decays as cleanly as necessary.

The remaining sample is plotted in the histograms of figs. 1a and 1b as a function of the invariant mass  $m_{\pi\pi}$  and of  $\rho_T^2$ , the squared distance of the reprojected momenta in the target plane, from the target centre.

A two-dimensional linear background subtraction in  $m_{\pi\pi}$  and  $\rho_T^2$  was performed in each momentum bin ( $\Delta p = 0.5$  GeV/c) and each proper time bin ( $\Delta\tau = 0.5 \times 10^{-10}$  sec). The amount of subtracted events varies from 2.2% for  $K_S$  to 7.3% for  $K_L$  of the accepted events.

The final data sample is accumulated in a two-dimensional histogram in the kaon proper time and momentum. There are 6 million events in total and perhaps more significantly  $\sim 5000$  events per  $10^{-10}$  sec time bin at long times. The momentum distribution of the observed events is shown in fig. 2.

Extraction of the information from the experi-

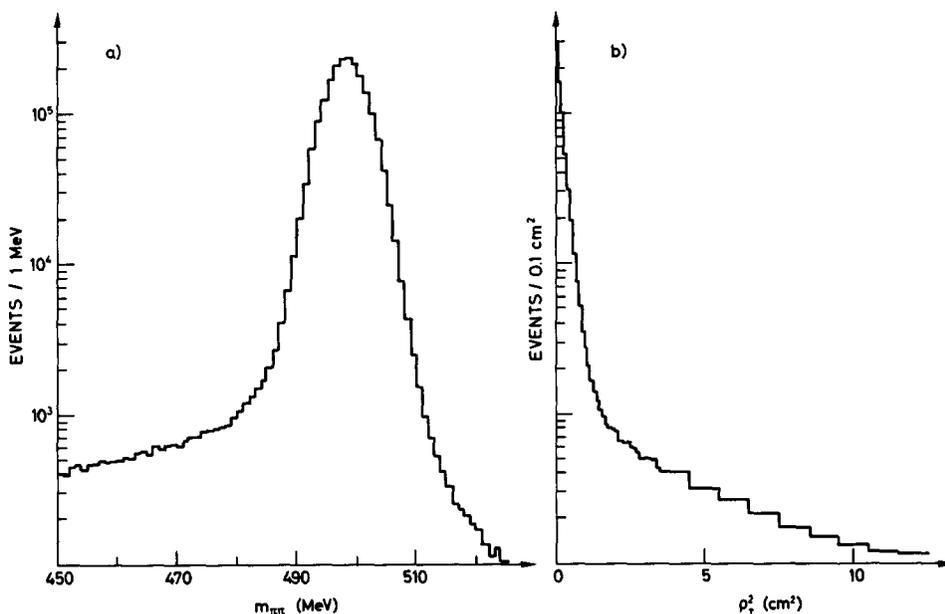


Fig. 1. (a)  $m_{\pi\pi}$  distribution. (b)  $\rho_T^2$  is the squared distance of the reprojected momenta in the target plane, from the target centre.

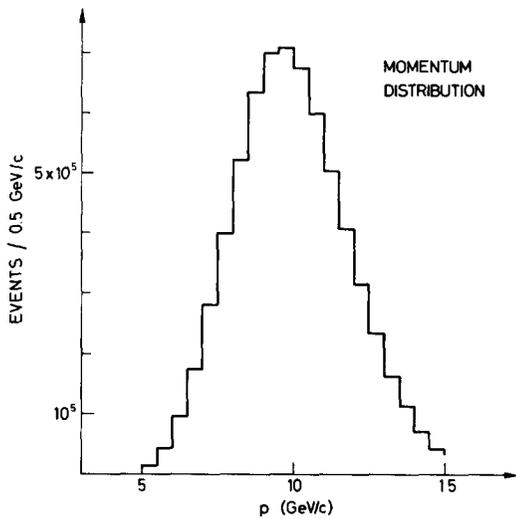


Fig. 2. Kaon momentum spectrum of the accepted events.

mental curve demands a good knowledge of the acceptance of the apparatus. This was achieved by simulating 6.6 million decays with a Monte Carlo program. These events were treated in the same way as the data. Fig. 3 shows the acceptance curves for different momenta as well as a weighted acceptance for events with momentum between 5 and 12.5 GeV/c.

The time distribution of events summed over the momentum interval 5–12.5 GeV/c with limits im-

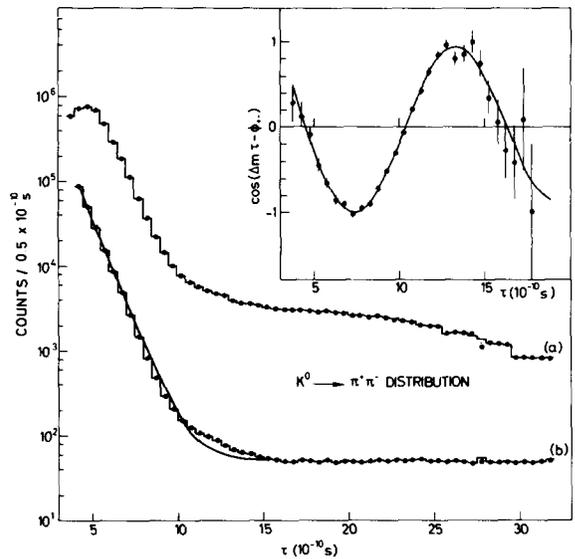


Fig. 4. Time distribution of  $K \rightarrow \pi^+ \pi^-$  events. a) Events (histogram) and fitted distribution (dots). b) Events corrected for detection efficiency (histogram), fitted distribution with interference term (dots) and fitted distribution without interference term (solid line). Insert: Interference term as extracted from data (dots) and fitted term (line).

posed by the decay volume is shown as the histogram a in fig. 4. The curve b of fig. 4 shows the data corrected for the detection efficiency.

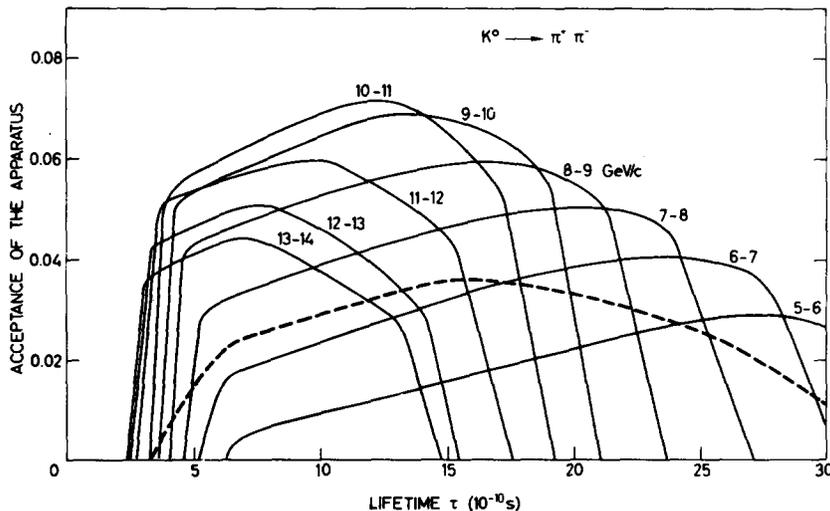


Fig. 3. Acceptance curves for different kaon momenta. The dashed line represents the average acceptance of the apparatus.

The theoretically expected distribution in proper time is:

$$I_{2\pi}(\tau) = [S(p) + \bar{S}(p)] \{ \exp(-\Gamma_S \tau) + 2A(p)|\eta_{+-}| \exp[-(\Gamma_L + \Gamma_S)\tau/2] \cos(\Delta m \tau - \phi_{+-}) + |\eta_{+-}|^2 \exp(-\Gamma_L \tau) \},$$

where  $S(p)$  and  $\bar{S}(p)$  are the production intensities of  $K^0$  and  $\bar{K}^0$  and  $A(p)$  measures the initial admixture of  $K^0$  and  $\bar{K}^0$ :

$$A(p) = \frac{S(p) - \bar{S}(p)}{S(p) + \bar{S}(p)}.$$

This expression is fitted to the data in 0.5 GeV/c momentum bins to find  $\Gamma_S$ ,  $|\eta_{+-}|$ ,  $\phi_{+-}$  and unparameterized  $S(p)$  and  $\bar{S}(p)$  assuming  $\Delta m$  and  $\Gamma_L$  to be known. The experimentally-determined phase  $\phi_{+-}$  is a linear function of  $\Delta m$ .

The  $\chi^2$  of the fit is 421 for 444 degrees of freedom. The result of the fit is shown as the dots in figs. 4a and 4b. The cosine part of the interference term is extracted from the full curve as presented in the insert of fig. 4.

The final results are the following:

$$\Gamma_S = (1.119 \pm 0.006) \times 10^{10} \text{ sec}^{-1}$$

$$|\eta_{+-}| = (2.30 \pm 0.035) \times 10^{-3}$$

$$\phi_{+-} = (49.4 \pm 1.0) + \left( \frac{\Delta m - 0.540}{0.540} \right) \times 305^\circ.$$

The stability of the results was checked by varying the momentum range and the time interval used in the fit, as well as by changing the positions of the cuts mentioned previously. Also different background subtractions gave consistent results. The stated errors include estimated systematic uncertainties. The correction on the phase  $\phi_{+-}$  amounts to  $(0.4 \pm 0.3)^\circ$  for the effects of a  $\gamma$ -ray absorber following the target, the scattering on the collimator walls, the  $K^0$ 's produced by the nucleons absorbed in the collimator and the regeneration in the helium. A 0.5% uncertainty in the magnetic field is also taken into account.

The value of  $\Gamma_S$ , although in disagreement with earlier results [5], agrees with the most recently reported measurement [6].

The value of  $|\eta_{+-}|$ , in disagreement with earlier re-

sults [7] has been confirmed [8] since it was first reported by this group [9]. A check measurement was made in order to confirm the result on  $|\eta_{+-}|$  by comparing the rates of the processes  $K_L \rightarrow \pi^+ \pi^-$  and  $K_L \rightarrow \pi e \nu$ . This result, which is systematically less reliable, is  $|\eta_{+-}| = (2.30 \pm 0.06) \times 10^{-3}$ .

The measurement of  $\Delta m$ , undertaken with the same apparatus, is close to completion and we prefer to wait for this result before drawing a conclusion on the compatibility of the phase measurement with the superweak model.

In any case, the new value of  $|\eta_{+-}|$ , together with the more precise charge asymmetry measurements of the previous letter, can be compared with the prediction of the superweak model supplemented with unitarity:

$$|\eta_{+-}| \frac{\Gamma_S - \Gamma_L}{\sqrt{(\Gamma_S - \Gamma_L)^2 + (2\Delta m)^2}} = \text{Re } \epsilon.$$

In the foregoing letter it is found that

$$\text{Re } \epsilon = (1.67 \pm 0.08) \times 10^{-3}.$$

This has to be compared with the result of this letter:

$$\begin{aligned} |\eta_{+-}| \frac{\Gamma_S - \Gamma_L}{\sqrt{(\Gamma_S - \Gamma_L)^2 + (2\Delta m)^2}} &= (2.30 \pm 0.035) \times (0.721 \pm 0.005) \times 10^{-3} \\ &= (1.66 \pm 0.03) \times 10^{-3}. \end{aligned}$$

The agreement is very good, the precision being approximately 5%.

We wish to express our thanks to Dr. E.M. Rimmer for excellent programming assistance, to Messrs. H. Dieperink, J. Olsfors, P. Schilly and M. Vysočanský for their remarkable technical assistance throughout the experiment. We also wish to thank Dr. G. Petrucci for the design of the proton beam splitting technique which was essential to the success of this experiment, and the CERN PS staff, particularly Dr. L. Hoffmann for the design and setting-up of the beam.

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## MEASUREMENT OF THE CHARGE ASYMMETRY IN THE DECAYS

$$K_L^0 \rightarrow \pi^\pm e^\mp \nu \text{ AND } K_L^0 \rightarrow \pi^\pm \mu^\mp \nu$$

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The charge asymmetry in semi-leptonic  $K_L^0$  decays has been measured in a high statistics experiment using multi-wire proportional chambers. The asymmetry  $\delta = (N^+ - N^-)/(N^+ + N^-)$ , where  $N^+$  and  $N^-$  are the partial decay rates for  $K_L^0 \rightarrow \pi^- \ell^+ \nu$  and  $K_L^0 \rightarrow \pi^+ \ell^- \bar{\nu}$ , respectively, is found to be  $\delta_L^e = (3.41 \pm 0.18) \times 10^{-3}$  for the  $K_{e3}$  mode, and  $\delta_L^\mu = (3.13 \pm 0.29) \times 10^{-3}$  for the  $K_{\mu3}$  mode. Assuming  $CPT$  invariance and the absence of  $\Delta S = -\Delta Q$  transitions, these results lead to a value of the real part of the  $CP$ -violation parameter  $\epsilon$ ,  $\text{Re } \epsilon = (1.67 \pm 0.08) \times 10^{-3}$ .

The charge asymmetry in the semi-leptonic decay modes of the long-lived neutral kaon is, apart from the two-pion decay of the long-lived kaon, the only manifestation of  $CP$  non-invariance observed up to now. A measurement of this effect is important to the understanding of the phenomenon of  $CP$  violation, as it can be related to  $\epsilon$ , the  $CP$  mixing amplitude in the neutral K-meson state

$$\delta_L = \frac{N^+ - N^-}{N^+ + N^-} = 2 \text{Re } \epsilon \frac{1 - |x|^2}{|1 - x|^2}.$$

Here  $CPT$  invariance has been assumed.

$N^\pm$  are the number of observed  $K_{l3}$  decays with positive and negative lepton, respectively. The parameter  $x$  is defined as the ratio of  $\Delta S = -\Delta Q$  to  $\Delta S = \Delta Q$  amplitudes and vanishes if the  $\Delta S = \Delta Q$  selection rule is valid. The value of  $x$  could, in general, be different for  $K_{e3}$  and  $K_{\mu3}$  decays and may depend on the Dalitz plot variables. Recent measurements indicate, however, that  $x$  is small and not significantly different from zero [1].

The experiment was performed in a short neutral beam at the CERN Proton Synchrotron, providing neutral kaons over the momentum range 3–15 GeV/c. The main elements of the set-up [2] are shown in fig. 1. The neutral hadrons are produced by an external proton beam of 24 GeV/c hitting a 4.5 cm long platinum target,  $4 \times 4 \text{ mm}^2$  in cross-section. The secondaries are selected at an average angle of 75 mrad by a tapered uranium collimator, 2 m long, imbedded in a magnetic field of 20 kG. It is followed by a 9 m long decay volume filled with helium. The vector momenta of the charged decay products of the neutral kaons are measured in a spectrometer consisting of three multiwire proportional chambers and a bending magnet.\* All chambers are divided into a left and a right half, each equipped with a horizontal and a vertical signal plane. The wire spacing is 2 mm. A 6 m long threshold Čerenkov counter filled with hydrogen gas at atmospheric pressure is used to label electrons. Muons are identified by a coincidence signal in two counter hodoscopes behind an absorber of  $800 \text{ g/cm}^2$  of light concrete.

The use of large size multiwire proportional chambers introduces several advantageous features. Their good time resolution and zero dead-time combined with a fast selective read-out allow considerably higher

\* A fourth chamber was introduced in a later stage of the experiment, but not used in the event reconstruction.

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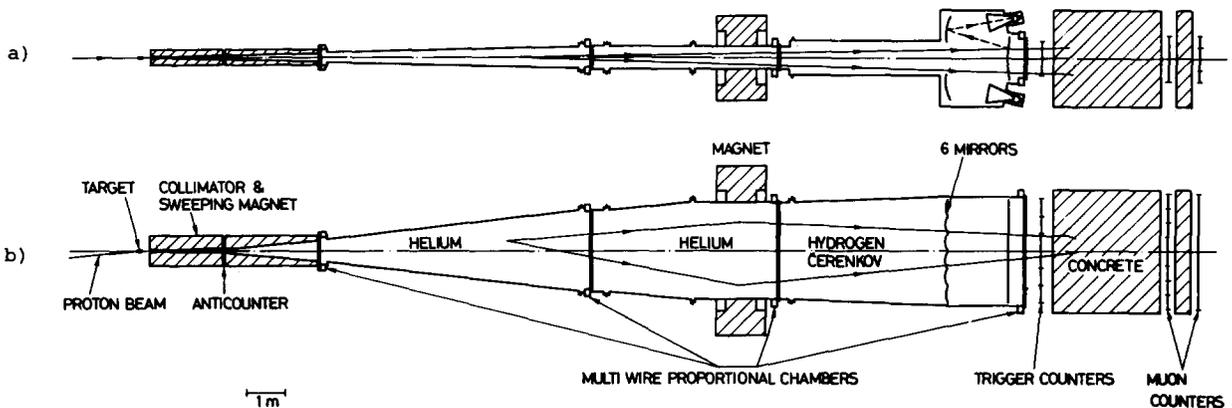


Fig. 1. Experimental lay-out. a) side view, b) top view

data acquisition rates than in earlier experiments. Furthermore, since they are d.c. operated, the proportional chambers do not require additional scintillation counters, permitting a substantial reduction of matter in the path of detected particles. The Čerenkov counter is a novel design with this in mind. The only optical element traversed by the particles is an aluminized mylar foil. The total amount of matter traversed by the decay particles in the 15 m long detector is  $0.4 \text{ g/cm}^2$ , mostly helium.

The read-out system [3] selects events with two signals in each of the wire planes so that practically all events recorded on magnetic tape can be uniquely reconstructed as a  $V^0$ . With an average rate of more than 1000 events per machine cycle of 350 m sec more than  $10^9$  events were registered. The total sample is divided into runs of equal length corresponding to about 0.5 million events each. The polarity of the momentum-analysing magnet was reversed at the end of each run.

After removing events with no signal in the lepton counters about 140 million events remained. The following selection procedure, designed to reduce possible contamination by other  $K^0$  decay modes, hyperon decays, or beam interactions in the decay volume, has been applied:

- 1) The geometrical reconstruction of the vertex is constrained by cuts on the skewness and the kinks of the tracks in the vertical projection.
- 2) The decay vertex is confined to a fiducial volume.
- 3) Electron pairs and events with poor vertex reconstruction are removed by the requirement  $\alpha_x p_z > 0.1 \text{ GeV}/c$ , where  $\alpha_x$  denotes the horizontal opening

angle of the  $V^0$ , and  $p_z$  its momentum component along the beam axis.

4) A unique and unambiguous assignment of the lepton track is required.

5) In the  $K_{e3}$  analysis an upper limit of  $8 \text{ GeV}/c$  for the reconstructed momenta of both secondaries is set to eliminate pions radiating in the Čerenkov counter. In the  $K_{\mu 3}$  data a cut is applied only on the  $\mu$  momentum,  $p_\mu > 1.6 \text{ GeV}/c$ . The threshold momentum for penetration of the absorber is  $1.45 \text{ GeV}/c$ .

6) The  $K_{3\pi}$  contamination is separated from  $K_{e3}$  decays by the requirement  $p_0'^2 < -0.003 (\text{GeV}/c)^2$ . The variable  $p_0'^2$  denotes the square of the  $\pi^0$  momentum in the frame in which the longitudinal momentum of the charged pair vanishes.

7) Events that are not compatible with the kinematics of the leptonic decays are eliminated by limits on the neutrino momentum in the  $K^0$  rest system,

$$p_\nu > 0$$

$$p_\nu'^2 = p_\nu^2 - p_L^2 > -0.003 (\text{GeV}/c)^2.$$

8) The effects of  $K_S - K_L$  interference as well as decays of the  $\Lambda$  hyperon and  $K_{2\pi}$  decays are kept to an acceptable level by a requirement on the proper time  $\tau'^*$ ,

$$K_{e3}: \tau' > 12.75 \times 10^{-10} \text{ sec}$$

$$K_{\mu 3}: \tau' > 14.75 \times 10^{-10} \text{ sec}.$$

\*  $\tau'$  is derived from the smaller of the two kinematical solutions for the kaon laboratory momentum.

Table 1  
Summary of the systematic corrections and results  
(All quantities are measured in ppm)

Corrections	$K_{e3}$	$K_{\mu 3}$
Pion absorption in hydrogen	+ 61 ± 4	+ 61 ± 4
Positron annihilation	+ 28 ± 1	-
Delta-ray production	- 100 ± 20	-
Pion decay and penetration	- 169 ± 41	- 163 ± 47
$K_{3\pi}$ contamination	0	- 167 ± 85
Wrong charge assignment	0	0 ± 19
Neutron interactions	+ 3 ± 15	+ 255 ± 87
$K_S-K_L$ interference	- 62 ± 12	+ 4 ± 4
$\Lambda$ contamination	+ 5 ± 2	- 74 ± 40
Regeneration in helium	- 17 ± 5	- 17 ± 5
Č pulse height variation	- 1 ± 2	-
Accidentals	+ 8 ± 5	+ 1 ± 2
Total correction	- 244 ± 50	- 100 ± 140
Uncorrected asymmetry	+ 3652 ± 171	+ 3232 ± 257
Corrected charge asymmetry	+ 3408 ± 178	+ 3132 ± 293

In total, 34 million  $K_{e3}$  and 15 million  $K_{\mu 3}$  events satisfied these criteria, giving a raw asymmetry of

$$\delta_L^e = (3.65 \pm 0.17) \times 10^{-3}$$

$$\delta_L^\mu = (3.23 \pm 0.26) \times 10^{-3}$$

These results have to be corrected for effects due to the presence of matter in the detector and other instrumental biases.

The geometric asymmetry of the apparatus has a negligible effect, since the essential measurement is not one of left-right asymmetry, but an asymmetry with respect to the magnet polarity. Potentially important biases are field reversal effects on the detection efficiency of the wire chambers and the phototubes. The Čerenkov counter tubes are at a distance of 6 m from the magnet. The change in pulse height was found to be negligibly small.

The differential absorption of  $\pi^+$  and  $\pi^-$  in the hydrogen filled Čerenkov counter has been evaluated from the measured  $\pi^-p$  and  $\pi^+p$  total cross-sections [4].

The corrections due to positron annihilation and production of delta electrons have been calculated according to the theoretically known cross-sections.

Pion decay (1.8%) and the penetration of the con-

crete absorber by pions (0.6%) can lead to an additional  $\mu$  signal. Such events are rejected from the  $K_{\mu 3}$  samples, because of two lepton signals. This omission of events introduces a systematic bias that was measured using  $K_{e3}$  decays with a coincidence signal in the  $\mu$  hodoscopes.

Misidentification of the muon occurs with a probability of 1.1%, whenever the muon escapes detection, mainly for geometrical reasons, and simultaneously the pion gives a  $\mu$  signal, due to decay or penetration. This is equivalent to a wrong charge assignment.

The  $K_{3\pi}$  contamination in the  $K_{e3}$  sample is negligible, because of the rejection of pions in the Čerenkov counter by a factor of the order of  $10^3$ . However,  $K_{3\pi}$  decay contributes a 4.7% background to the  $K_{\mu 3}$  data. The bulk of this is due to  $\pi \rightarrow \mu \nu$  decay, the remainder is due to pion penetration of the muon shield. The correction required has been studied for pions in  $K_{e3}$  decays.

Interactions of beam particles, chiefly neutrons, have been investigated by placing copper plates of various thicknesses at several positions in the decay volume. The number of interactions in helium and their charge asymmetry was estimated on the basis of these data. Whereas the contamination is exceed-

ingly small in the  $K_{e3}$  sample, a 10% correction is obtained for  $K_{\mu3}$ . This takes into account neutron stars as well as the production of  $\Lambda$  hyperons and neutral kaons.

Specific for a measurement in a rather short beam is a correction for  $K_S-K_L$  interference at small decay times. In addition, the contamination due to  $\Lambda$  particles produced at the target has been determined and a correction is applied.

A list of all systematic corrections is given in table 1. The corrected values for the charge asymmetry

$$K_{e3}: \delta_e = (3.41 \pm 0.18) \times 10^{-3}$$

$$K_{\mu3}: \delta_\mu = (3.13 \pm 0.29) \times 10^{-3}$$

are in good agreement with each other, and consequently there is no indication that the  $\Delta S = \Delta Q$  factor  $(1 - |x|^2)/|1 - x|^2$  is significantly different for the two semi-leptonic decay modes of the  $K_L^0$ . Furthermore these results are well consistent with earlier work [5-12] and the errors have been appreciably reduced, in particular for  $K_{\mu3}$ .

Assuming the validity of the  $\Delta S = \Delta Q$  selection rule, the combined result leads to a value for the real part of the  $CP$  mixing parameter  $\epsilon$ ,

$$\text{Re } \epsilon = (1.67 \pm 0.08) \times 10^{-3}.$$

We are grateful for the programming assistance of Dr. E.M. Rimmer and the technical help provided by Messrs. H. Dieperink, J. Olsfors, P. Schilly and Dr. M.

Vysočanský. The Heidelberg members of the Collaboration acknowledge the financial support of the Bundesministerium für Forschung und Technologie.

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