3.7 Neutrino scattering in V-A theory

Very small cross section for $\nu N$ scattering: $\sigma(\nu N) \approx E_\nu[\text{GeV}] \times 10^{-38} \text{ cm}^2$

$= E_\nu[\text{GeV}] \times 10 \text{ fb}$

- intense neutrino beams
- large instrumented targets

Neutrino beams

Sources of neutrino beams are 2-body decays of intense hadron beams

$\pi^\pm \rightarrow \mu^\pm \nu_\mu(\bar{\nu}_\mu) \quad K^\pm \rightarrow \mu^\pm \nu_\mu(\bar{\nu}_\mu)$

where the pions/kaons are generated in proton-nucleon interactions: $p+N \rightarrow \pi, K$

Energy spectrum

The Lorentz boost transforms the mono-energetic neutrino of the two-body decay into a flat energy spectrum:

$$0 < \frac{E_\nu}{E_{e,K}} < 1 - \frac{m_\mu^2}{m_{e,K}^2}$$

Beams contain small admixture (0.5%) of electron neutrinos!
**Generation of neutrino beams**

1. ~400 GeV proton beam on a (Be) target: secondary hadrons \( \pi, K \)
2. Momentum and charge selection of \( \pi \)'s and K's using a focusing system
3. Selected \( \pi \)'s and K's enter a decay tunnel: \( \pi^+, K^+ \rightarrow \mu^+\nu_\mu(\tau_\mu) \)
4. Remaining hadrons and decay muons are filtered by a massive absorber (~400 m iron, concrete, earth): only neutrinos after absorber

There exist 2 different focusing systems for the selection of \( \pi \)'s and K's: the two systems lead to neutrino beams with much different energy spectra and fluxes.

**Narrow-band neutrino beam:**

Deflection and focusing magnets to select and focus hadrons (one charge) of a narrow momentum range

\[
\frac{\Delta p_{K,\pi}}{p_{K,\pi}} \approx 7\% \quad \text{(at SPS } p_{K,\pi} \sim 200 \text{ GeV)}
\]

One gets a neutrino beam with a 2-component spectrum

**Narrow-band neutrino beam used if one needs to know the exact neutrino flux and wants to achieve max. neutrino energies**
Wide-band neutrino beam: Magnetic Horn

Magnetic Horn (S. van der Meer)
- Horn formed from thin aluminum skin
- Short current pulses of 100 to 180 kA
  → large short-time magnetic field perpendicular to particle direction
  → magnetic deflection similar to a paraboloid for hadrons of one charge
- Advantages: use all $\pi^+/K^+ \rightarrow$ large $\nu$ flux
- Disadvantage: large background of wrong "sign" $\nu$'s

At high energies, there are much more $\nu$ than $\bar{\nu}$?

Reason:
- High energy neutrinos are produced from K decays:
  $$pN \rightarrow \Lambda K^- \quad \text{with} \quad K^- \rightarrow \mu^+ \nu_\mu,$$
  $$|uds\rangle \langle u\bar{s}|$$

(associated strange baryon production: no anti-baryon production !!!)
To study neutrino oscillation: 
\[ \tau \] appearance for \( \nu_\mu \) beam.

\[ p + C \rightarrow (interactions) \rightarrow \pi^+, K^+ \rightarrow (decay \ in \ flight) \rightarrow \mu^+ + \nu_\mu \]
Neutrino detector: Bubble chamber BEBC (CERN)

3.7 m diameter bubble chamber, filled with liquid hydrogen, deuterium + additional muon identifier consisting of 150 m² of MWPC

Neutrino event in bubble chamber: BEBC (CERN)

Figure 3.15: Example of charged-particle production and decay in the hydrogen bubble chamber BEBC exposed to a neutrino beam at the CERN SPS. (Courtesy CERN.)
CHDS (CERN-Dortmund-Heidelberg-Saclay) Experiment

1200 t steel
**Neutrino-lepton and neutrino-quark reactions**

\[ \nu_\mu e^- \rightarrow \mu^- \nu_e \]
\[ \nu_e e^- \rightarrow \mu^- \nu_\mu \]
\[ \nu_\mu \rightarrow \mu e^- \]
\[ \nu_e \rightarrow \mu e^- \]

(Anti)neutrino-lepton interaction similar to (Anti)neutrino-quark interaction: neutrino-lepton results can be applied to deep-inelastic \( \nu N \) scattering.

a) Neutrino-electron scattering

\[ \nu_\mu e^- \rightarrow \mu^- \nu_e \]
\[ M = \frac{G_F}{\sqrt{2}} \left[ \overline{u}_e(k') \gamma_\alpha (1-\gamma^5) u_e(p) \right] \overline{u}_\mu(p') \gamma^\alpha (1-\gamma^5) u_\mu(k) \]

\[ \overline{|M|^2} = \frac{1}{2} \sum \overline{|M|^2} = \ldots = 64G^2_F (k \cdot p)(k' \cdot p') = 16G^2_F \cdot s^2 \]

Limit \( m_e = m_0 \rightarrow 0 \)

\[ s = (k + p)^2 = 2kp = 2k'p' \]

Using the phase space factor of chapter II:

\[ \frac{d\sigma}{d\Omega} (\nu_\mu e^-) = \frac{1}{64\pi^2 s} \overline{|M|^2} = \frac{G^2_F s}{4\pi^2} \]

\[ \sigma(\nu_\mu e^-) = \frac{G^2_F s}{\pi} \]

Although effective 4-fermion theory works well for low \( q^2 \) it violates unitarity bound for high \( q^2 \)!

This is a clear indication that the 4-fermion interaction is only an effective low energy approximation – not valid at high energies!!

J. Pawlowski / U. Uwer
b) Anti-Neutrino-electron scattering (V-A)

\[ \overline{\nu}_e e^- \rightarrow \overline{\nu}_\mu \mu^- \]

Crossing: \( s \leftrightarrow t \) (u)

\[ \overline{M}^2 = \frac{1}{2} \sum_{\text{Spins}} |M|^2 = 16G_F^2 \cdot t^2 = 4G_F^2 \cdot s^2 (1 - \cos \theta)^2 \]

\[ \frac{d\sigma}{d\Omega}(\overline{\nu}e^-) = \frac{G_F^2 s}{16\pi} (1 - \cos \theta)^2 \]

\[ \sigma(\overline{\nu}e^-) = \frac{G_F^2 s}{3\pi} \]

Result of V-A structure

For the charged current (CC) contribution to the (anti) neutrino electron scattering one finds

\[ \frac{\sigma_{cc}}{\sigma_{cc}} = 3 \]

Different angular distribution of (anti) neutrino scattering can be understood from a helicity discussion

\[ \frac{d\sigma}{d\Omega}(\nu_e e^- \rightarrow \nu_\mu \mu^-) = \frac{G_F^2 s}{4\pi^2} \]

\[ \frac{d\sigma}{d\Omega}(\overline{\nu}_e e^- \rightarrow \overline{\nu}_\mu \mu^-) = \frac{G_F^2 s}{16\pi^2} (1 - \cos \theta)^2 \]
c) (Anti) neutrino-quark scattering

\[ \nu_\mu d \rightarrow \mu u \]
\[ \overline{\nu}_\mu \overline{d} \rightarrow \mu \overline{u} \]
\[ \sigma(\nu_\mu d) = \frac{G_F^2 s}{4\pi} \]
\[ \sigma(\overline{\nu}_\mu \overline{d}) = \frac{G_F^2 s}{\pi} \]

Neutrinos only interact with d and anti-u quarks
Anti-neutrinos only interact with u and anti-d quarks

d) Neutrino-nucleon (iso-scalar) scattering

\[ d\sigma(\nu N \rightarrow \mu X) \]
\[ d\sigma(\nu q) \]
\[ d\sigma(\nu \overline{u}) \]
\[ d\sigma(\overline{\nu} \overline{q}) \]
\[ d\sigma(\overline{\nu} \overline{N}) \]

QPM:
\[ E' \]
\[ x = \frac{Q}{2M} \]
\[ y = \frac{\nu}{E} \]
\[ v = E - E' \]

Unter Vernachlässigung der Massen gilt:
\[ y = \frac{1 - \cos \theta}{2} \]
\[ 1 - y = \frac{1}{2} (1 + \cos \theta) \]
Total cross section after integration over \( x \) and \( y \) (0...1):

\[
\sigma(\nu N) = \frac{G_F^2 M_e^2}{2\pi} \left[ Q_i + \frac{1}{3} Q \right]
\]

\[
\sigma(\bar{\nu} N) = \frac{G_F^2 M_e^2}{2\pi} \left[ \bar{Q}_i + \frac{1}{3} Q \right]
\]

with \( Q_i = \int xQ(x)dx \)

\[
R = \frac{\sigma_{\nu N}}{\sigma_{\bar{\nu} N}} = \frac{1 + 3\bar{Q}_i/Q_i}{3 + Q_i/\bar{Q}_i}
\]

If nucleon consists only of valence quarks (\( Q=0 \)): \( R=1/3 \), because of V-A structure

Measurement: \( R = \frac{0.34}{0.67} \Rightarrow \frac{\bar{Q}_i}{Q_i} \approx 0.15 \)

\( \Rightarrow \) There are sea quarks!

### 3.8 Problems with pure V-A theory

- Cross section for \( \nu e^- \rightarrow e^- \nu_e \) in 4-fermion ansatz:
  i.e. cross section goes to infinity if \( s \rightarrow \infty \): violates unitarity

  \[
  \sigma(\nu e^-) = \frac{G_F^2 s}{\pi}
  \]

- Lee and Wu (1965) introduced a massive exchange boson. Effect of propagator:

  \[
  \frac{G_F}{\sqrt{2}} \rightarrow \frac{G_F}{\sqrt{2}} \frac{1}{1 - q^2/M_W^2}
  \]

  \( \sigma(\nu e^-) \rightarrow \text{const.} \)

  Not trivial, see e.g.:
  C. Quigg, Gauge Theory of Strong and Weak Interaction

This fix leads to a new problem, namely the violation of unitarity of the predicted W pair production!

We need a new theory: Standard Model
One out of three $\nu_e \rightarrow \nu_e$ events

Neutral current $\nu N$ events appear with a significant rate:

$$R^\nu = \frac{\sigma_{\nu N}(\nu N \rightarrow \nu X)}{\sigma_{\text{CC}}(\nu N \rightarrow \mu X)} = 0.307 \pm 0.008$$

i.e. approx. 1/3 of the $\nu N$ interactions are neutral current interactions.

---

**Structure of Neutral currents**

**Ansatz:** four-fermion interaction

$$M = \frac{8 G_{\text{NC}}}{\sqrt{2}} \cdot J_{\text{NC,1}\mu} \cdot J_{\text{NC,2} \mu}$$

as $q^2 \rightarrow 0$ approximation of:

$$J^\mu_{\mu} = \bar{u} \gamma^\mu \left( \frac{1}{2} (g_L + g_A) \gamma^5 \right) u$$

Experimental determination of the structure of the weak neutral currents:

Neutral weak interaction couples to left- and right-handed chiral fermion currents differently:

$$g_L = \frac{1}{2} (g_V + g_A) \quad g_R = \frac{1}{2} (g_V - g_A)$$

$$J^\mu_{\mu} = \bar{u} \gamma^\mu \left( g_R + \gamma^5 \frac{1}{2} \right) u \quad + \quad \bar{u} \gamma^\mu \left( g_L - \gamma^5 \frac{1}{2} \right) u$$
4.1 Vector and axial-vector couplings

<table>
<thead>
<tr>
<th></th>
<th>$g_V$</th>
<th>$g_A$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\nu$</td>
<td>$1/2$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$\ell^-$</td>
<td>$-1/2 + 2\sin^2 \theta_W$</td>
<td>$-1/2$</td>
</tr>
<tr>
<td>$u - \text{quark}$</td>
<td>$+1/2 - 4/3 \sin^2 \theta_W$</td>
<td>$1/2$</td>
</tr>
<tr>
<td>$d - \text{quark}$</td>
<td>$-1/2 + 2/3 \sin^2 \theta_W$</td>
<td>$-1/2$</td>
</tr>
</tbody>
</table>

with $\sin^2 \theta_W = 1 - \frac{M_W^2}{M_Z^2} \approx 0.223$

In case of the left-handed neutrinos: $J^\mu_\nu = \bar{\nu}_\mu \gamma^\mu \left[ \frac{1}{2} - \frac{1}{2} (1 - \gamma^5) u_\nu \right]$ pure V-A structure

(consistent, only LH neutrinos)

4.2 Effective coupling $G_{NC}$ (copy of charged current)

$J^\mu_\nu = \bar{\nu}_\mu \gamma^\mu \left( g_V - g_A \gamma^5 \right) u_
u$

$M = J_{\theta,\mu} \cdot g_Z \cdot \frac{g_{\mu\nu} - g_{\nu\mu}}{q^2 - M_Z^2} \cdot g_Z \cdot J^\nu_\nu$

$M = \frac{8 G_{NC}}{\sqrt{2}} \cdot J_{\theta,\mu} \cdot J^\mu_\nu$

As 4-fermion interaction is the $q^2 \to 0$ approximation of a massive boson exchange:

Comparison of the coupling constants in the $q^2 \to 0$ limit:

$$\frac{G_{NC}}{\sqrt{2}} = \frac{g_Z^2}{8M_Z^2} = \frac{g_W^2}{8M_W^2} \cdot \frac{g_{\nu\mu}M_W^2}{g_{\nu\mu}M_Z^2} \cdot \rho = G_\rho \sqrt{2}$$

$\rho = 1$ in the SM