3.3 V – A Theory

Careful analysis of experimental data (parity violation, neutrino helicity spin change in nuclear β-decays, muon decay properties together w/ universality) finally led to the V-A theory of (nuclear) weak decays:

$$\boxed{M = \frac{G_F}{\sqrt{2}} \underbrace{\left(\bar{u}_p \gamma^\mu (\overset{V}{c_V} \overset{-}{-} \overset{A}{c_A \gamma^5}) u_n\right)}_{\text{nucleon}} \cdot \underbrace{\left(\bar{u}_e \gamma^\mu \overset{V-A}{(1-\gamma^5)} v_\nu\right)}_{\text{lepton / fund. fermion}}}$$

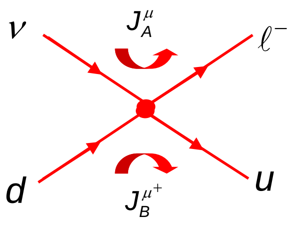
$$\underbrace{\qquad\qquad\qquad}_{\text{Composed objects}}$$

c_V, c_A vector and axial-vector couplings of nucleons:

$c_A/c_V = 1.2695 \pm 0.0029$ PDG 2004

V-A ansatz for fundamental fermions

Four-Fermion Theory



J_A and J_B are lepton and quark currents

$$J_\ell^\mu = \bar{u}_\ell \gamma^\mu (1-\gamma^5) u_\nu$$
$$J_q^\mu = \bar{u}_u \gamma^\mu (1-\gamma^5) u_d$$

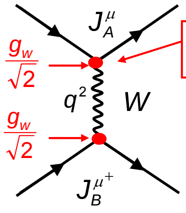
$$M = \frac{G_F}{\sqrt{2}} \cdot J_{A,\mu} \cdot J_B^{\mu +}$$

Reminder

$$u_L = \frac{1}{2}(1-\gamma^5)u$$
$$u_R = \frac{1}{2}(1+\gamma^5)u$$

Electroweak Theory

According today’s understanding the 4-fermion coupling is the $q^2 \to 0$ limit of W propagator:



g_w = coupling for weak interaction

$$M = \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_{A,\mu} \underbrace{\frac{(g_{\mu\nu} - \frac{q_\mu q_\nu}{M_W^2})}{q^2 - M_W^2}}_{\text{for } q^2 \to 0: \; = \frac{1}{M_W^2}} \cdot \frac{g_w}{\sqrt{2}} \cdot \frac{1}{2} J_B^{\nu +}$$

$$\boxed{\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}}$$

With $G_F \approx 1.16 \times 10^{-5}$ GeV^{-2} follows w $M_W \approx 80$GeV: $g_w \approx 0.65$

3.4 V-A coupling of leptons and quarks

Reminder

$$\bar u_\ell \gamma^\mu (1-\gamma^5) u_\nu = \bar u^L_\ell \gamma^\mu u^L_\nu$$

In V-A theory the weak interaction couples left-handed lepton/quark currents (right-handed anti-lepton/quark currents) with an **universal coupling strength**:

$$\frac{G_F}{\sqrt{2}} = \frac{g_w^2}{8M_W^2}$$

Charged weak transition appear only inside weak-isospin doublets:

Lepton currents:

1.	$\begin{pmatrix}\nu_e \\ e^-\end{pmatrix}_L$	$j^\mu_{e\nu} = \bar u_e \gamma^\mu (1-\gamma^5) u_\nu$
2.	$\begin{pmatrix}\nu_\mu \\ \mu^-\end{pmatrix}_L$	$j^\mu_{\mu\nu} = \bar u_\mu \gamma^\mu (1-\gamma^5) u_\nu$
3.	$\begin{pmatrix}\nu_\tau \\ \tau^-\end{pmatrix}_L$	$j^\mu_{\tau\nu} = \bar u_\tau \gamma^\mu (1-\gamma^5) u_\nu$

Quark currents:

1.	$\begin{pmatrix}u \\ d'\end{pmatrix}_L$	$j^\mu_{d'u} = \bar u_{d'} \gamma^\mu (1-\gamma^5) u_u$
2.	$\begin{pmatrix}c \\ s'\end{pmatrix}_L$	$j^\mu_{s'c} = \bar u_{s'} \gamma^\mu (1-\gamma^5) u_c$
3.	$\begin{pmatrix}t \\ b'\end{pmatrix}_L$	$j^\mu_{b't} = \bar u_{b'} \gamma^\mu (1-\gamma^5) u_t$

Problem: Not equal to the mass eigenstate

3.5 CKM matrix to describe the quark mixing

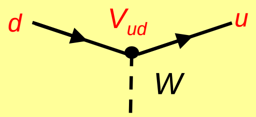
Weak eigenstates:

Mass/flavor eigenstates:

Weak transitions

One finds that the weak eigenstates of the down type quarks entering the weak isospin doublets are not equal to the their mass/flavor eigenstates:

$$\begin{pmatrix}d' \\ s' \\ b'\end{pmatrix} = \underbrace{\begin{pmatrix}V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb}\end{pmatrix}} \cdot \begin{pmatrix}d \\ s \\ b\end{pmatrix}$$



Cabibbo-**K**obayashi-**M**askawa mixing matrix

The quark mixing is the origin of the flavor number violation of the weak interaction.

Cabibbo Angle

Historical retrospect

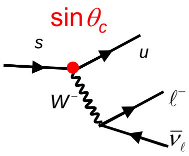
Until the early 70s, only **3 quark flavor** were known. The weak transition between quarks was described by a single quark doublet:

$$\begin{pmatrix} u \\ d' \end{pmatrix} = \begin{pmatrix} u \\ \cos\theta_c \cdot d + \sin\theta_c \cdot s \end{pmatrix}$$

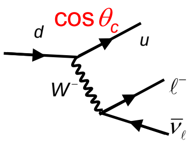
Mixing angle θ_c = Cabibbo-Angle

The mixing described automatically the suppression of $\Delta S=1$ transitions ($\sim\sin^2\theta_c$)

Historically the Cabibbo Angle was introduced to describe the suppression of s→u transitions with respect to d→u transitions (assuming a universal coupling G_F):



$M^2 \sim \sin^2\theta_c \sim 0.05$

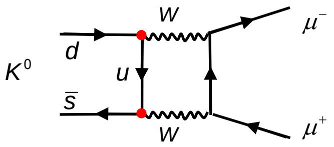


$M^2 \sim \cos^2\theta_c \sim 0.95$

Non-observed FCNC and GIM mechanism

FCNC in the 3 quark model: $K^0 \rightarrow \mu^+\mu^-$

Historical retrospect



$M \sim \sin\theta_c \cos\theta_c$

Theoretically one predicts large BR, in contradiction with experimental limits for this decay:

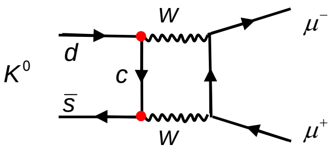
$$\frac{BR\,(K_L \rightarrow \mu^+\mu^-)}{BR\,(K_L \rightarrow all\,)} = (7.2 \pm 0.5) \cdot 10^{-9}$$

Proposal by Glashow, Iliopoulos, Maiani, 1970:

GIM

There exists a fourth quark which builds together with the s quark a second doublet:

$$\begin{pmatrix} c \\ s' \end{pmatrix} = \begin{pmatrix} c \\ -\sin\theta_c \cdot d + \cos\theta_c \cdot s \end{pmatrix}$$



$M \sim -\sin\theta_c \cos\theta_c$

Additional Feynman-Graph for $K^0 \rightarrow \mu\mu$ which compensates the first one:

Prediction of a fourth quark:
Mass prediction BR=f(m_c,…)

Weak interaction and the quark sector:

Historical retrospect

1964 Discovery of CP violation by J.H. Christenson et al.

$$BR(K_L^0 \to \pi^+\pi^-) \neq 0$$

1970 GIM mechanism predicts a forth quark: c-quark

1973 Kobayashi and Maskawa:

CP Violation can be explained through quark mixing if a complete new, third quark generation exists: in this case mixing matrix has **complex elements**.

⬇

Prediction of a 3rd quark generation

1974 Discovery of cc quark state

1977 Discovery of bb quark state

1995 Discovery of top quark

Properties of the CKM-Matrix

$$\mathbf{V_{CKM}} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

Complex,

unitary N × N Matrix:

$$V_{CKM}V_{CKM}^{+} = 1$$

	N=3	N=2
→ N^2 Parameters:	9	4
N (N - 1)/2 Euler angles (rotation angles)	$\boxed{3}$	$\boxed{1}$
Remaining parameters are phases:	6	3
2N-1 are unmeasurable phase diff	5	3
Observable phases	$\boxed{1}$	$\boxed{0}$
(N – 1)^2 observable parameters	$\boxed{4}$	$\boxed{1}$

Parameterization of CKM Matrix: 3 Angles + 1 Phase

PDG choice where $c_{ij} = \cos\theta_{ij}$, $s_{ij} = \sin\theta_{ij}$

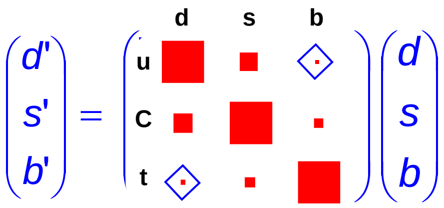
$$\begin{pmatrix} d' \\ s' \\ b' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

Wolfenstein Parameterization $\lambda,\ A,\ \rho,\ \eta$

→ hierarchy expressed by orders of λ = **sinθ_c≈ 0.22**

$$V_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & \boxed{A\lambda^3(\rho - i\eta)} \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ \boxed{A\lambda^3(1-\rho - i\eta)} & -A\lambda^2 & 1 \end{pmatrix} + O(\lambda^4)$$

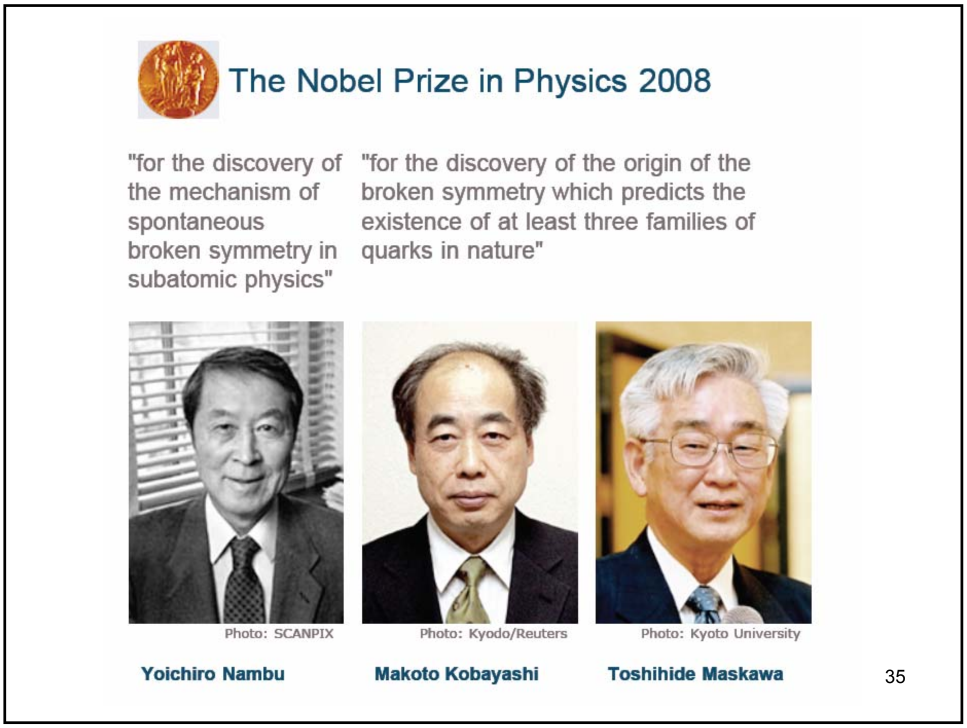
Modulus of the matrix elements: $|\mathbf{V}_{ij}|$



PDG 2006

$$|\mathbf{V}_{ij}| = \begin{pmatrix} 0.97383^{+0.00024}_{-0.00023} & 0.2272^{+0.0010}_{-0.0010} & (3.96^{+0.09}_{-0.09}) \times 10^{-3} \\ 0.2271^{+0.0010}_{-0.0010} & 0.97296^{+0.00024}_{-0.00024} & (42.21^{+0.10}_{-0.80}) \times 10^{-3} \\ (8.14^{+0.32}_{-0.64}) \times 10^{-3} & (41.61^{+0.12}_{-0.78}) \times 10^{-3} & 0.999100^{+0.000034}_{-0.000004} \end{pmatrix}$$

◇ in leading order only the elements $\mathbf{V}_{ub}$ and $\mathbf{V}_{td}$ are complex.



The Nobel Prize in Physics 2008

"for the discovery of the mechanism of spontaneous broken symmetry in subatomic physics"

"for the discovery of the origin of the broken symmetry which predicts the existence of at least three families of quarks in nature"

Photo: SCANPIX

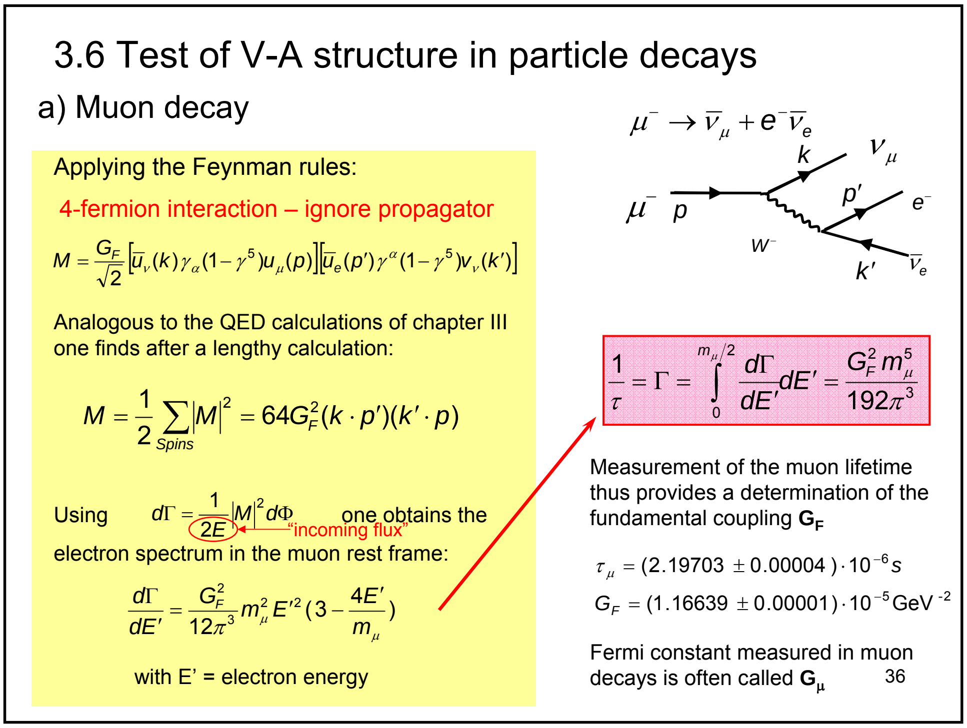
Photo: Kyodo/Reuters

Photo: Kyoto University

Yoichiro Nambu

Makoto Kobayashi

Toshihide Maskawa



3.6 Test of V-A structure in particle decays

a) Muon decay

$$\mu^- \rightarrow \overline{\nu}_\mu + e^- \overline{\nu}_e$$

Applying the Feynman rules:

4-fermion interaction – ignore propagator

$$M = \frac{G_F}{\sqrt{2}} \left[\overline{u}_\nu(k) \gamma_\alpha (1-\gamma^5) u_\mu(p) \right] \left[\overline{u}_e(p') \gamma^\alpha (1-\gamma^5) v_\nu(k') \right]$$

Analogous to the QED calculations of chapter III one finds after a lengthy calculation:

$$M = \frac{1}{2} \sum_{Spins} |M|^2 = 64 G_F^2 (k \cdot p')(k' \cdot p)$$

Using $d\Gamma = \frac{1}{\boxed{2E}} |M|^2 d\Phi$ "incoming flux" one obtains the electron spectrum in the muon rest frame:

$$\frac{d\Gamma}{dE'} = \frac{G_F^2}{12\pi^3} m_\mu^2 E'^2 (3 - \frac{4E'}{m_\mu})$$

with E' = electron energy

$$\boxed{\frac{1}{\tau} = \Gamma = \int_0^{m_\mu/2} \frac{d\Gamma}{dE'} dE' = \frac{G_F^2 m_\mu^5}{192\pi^3}}$$

Measurement of the muon lifetime thus provides a determination of the fundamental coupling $\mathbf{G_F}$

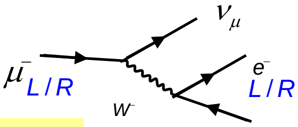
$$\tau_\mu = (2.19703 \pm 0.00004) \cdot 10^{-6} s$$

$$G_F = (1.16639 \pm 0.00001) \cdot 10^{-5} \text{GeV}^{-2}$$

Fermi constant measured in muon decays is often called $\mathbf{G_\mu}$

Test of V-A structure in the muon decay

Most general form of the matrix element for



$$M = \frac{G_F}{\sqrt{2}} \cdot \sum_{\substack{i=S,V,T \\ \lambda=L,R \\ \lambda'=\begin{cases}\pm\lambda \text{ for S,V} \\ -\lambda \text{ for T}\end{cases}}} g^i_{\lambda'\lambda} \underbrace{\left(\underset{L/R}{\bar{u}_{\lambda'}(e)}\Gamma^i v_{\lambda_i'}(\nu_e)\right)\left(\bar{u}_{\lambda_i}(\nu_\mu)\Gamma^i \underset{L/R}{u_\lambda(\mu)}\right)}_{\text{Chirality } \lambda_i, \lambda_i' \text{ determined by } \Gamma_i}$$

$$\lambda_i' = \begin{cases} \lambda' & i = S,T \\ -\lambda' & i = V \end{cases}$$

$$\lambda_i = \begin{cases} \lambda & i = V \\ -\lambda & i = S,T \end{cases}$$

Not treated during the lecture

Possible current-current couplings:

i \ λ'λ	RR	RL	LR	LL
S	x	x	x	x
V	x	x	x	x
T		x	x	

There are in general 10 complex amplitudes $g^i_{\lambda'\lambda}$

Pure V-A coupling: $g^V_{LL} = 1$

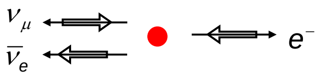
all other $g^i_{\lambda'\lambda} = 0$

Experimental determination of $g^i_{\lambda'\lambda}$ from energy spectra and spin correlation of the decay electrons from the polarized muons

Idea:

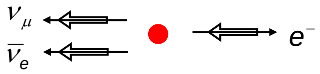
Not treated during the lecture

V-A at μ vertex ⇒ LH ν_μ



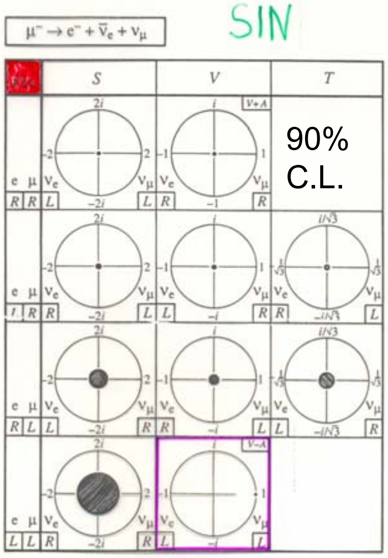
Configuration w/ max e- momentum possible

V+A at μ vertex ⇒ RH ν_μ



Due to angular momentum conservation not possible

Couplings in muon decay

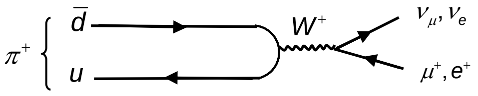


V-A theory is confirmed

b) Pion decay

$\pi^+ \to \mu^+ \nu_\mu$

$\pi^+ \to e^+ \nu_e$



Naïve expectation:

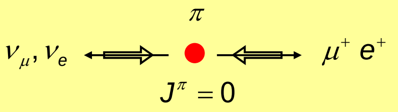
Assuming the same decay dynamics the decay rate to e⁺ should be much larger than to μ⁺ as the phase space is much bigger.

Measurement: (PDG)

$$\frac{\Gamma(\pi^+ \to e^+ \nu_e)}{\Gamma(\pi^+ \to \mu^+ \nu_\mu)} = (1.230 \pm 0.004) \cdot 10^{-4}$$

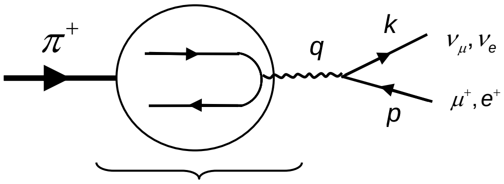
Large suppression due to a dynamic effect.

Qualitative explanation within V-A theory:



Angular momentum conservation forces the lepton into the “wrong” helicity state: suppressed ~ (1-v/c) i.e. for vanishing lepton masses the pion could not decay into leptons.

Determination of decay rates:



Quarks in pion are bound

4-fermion interaction – ignore propagator

$$M = \frac{G_F}{\sqrt{2}} \cdot (\ \pi\ )_\mu \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

$$M = \frac{G_F}{\sqrt{2}} \cdot \overbrace{(p_\mu + k_\mu) \cdot f_\pi}^{\text{scalar particles}} \cdot \left[\bar{u}_\nu \gamma^\mu (1-\gamma^5) v_\mu\right]$$

As the pion spin s_π=0, q is the only relevant 4-vector:

$$q^\mu = p^\mu + k^\mu$$

$$(\ \pi\ )_\mu = q_\mu . \underbrace{f_\pi(q^2)}$$

Pion form factor:

$$q^2 = m_\pi^2 : \quad f_\pi(q^2) = f_\pi(m_\pi^2) = f_\pi$$

Must be measured !

$$\Gamma(\pi^+ \to \mu^+\nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_\mu^2 (1 - \frac{m_\mu^2}{m_\pi^2})$$

$$\Gamma(\pi^+ \to e^+\nu_\mu) = \frac{G_F^2}{8\pi} \cdot f_\pi^2 \cdot m_\pi m_e^2 (1 - \frac{m_e^2}{m_\pi^2})$$

$$\frac{\Gamma(\pi^+ \to e^+\nu_\mu)}{\Gamma(\pi^+ \to \mu^+\nu_\mu)} = \left(\frac{m_e^2}{m_\mu^2}\right)\left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2}\right) = 1.275 \cdot 10^{-4} \qquad (1.230 \pm 0.004) \cdot 10^{-4}{}_{PDG}$$

The prediction of the V-A theory is confirmed by the experimental observation.