IV. Weak interaction

1. Phenomenology of weak decays
2. Parity violation and neutrino helicity
3. V-A theory
4. Structure of neutral currents

The weak interaction was and is a topic with a lot of surprises:

Past: Flavor violation, P and CP violation.
Today: Weak decays used as probes for new physics

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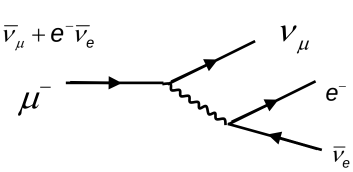
1. Phenomenology of weak decays

All particles (except photons and gluons) participate in the weak interaction. At small q^2 weak interaction is shadowed by strong and electro-magnetic effects.

- Observation of weak effects only possible if strong/electro-magnetic processes are forbidden by conservation laws:

$\mu^- \to \bar{\nu}_\mu + e^- \bar{\nu}_e$



$\tau = 2.6 \cdot 10^{-6}\,\text{s}$

Electromagnetic decay $\mu^- \to e^- \gamma$ forbidden by lepton number conservation

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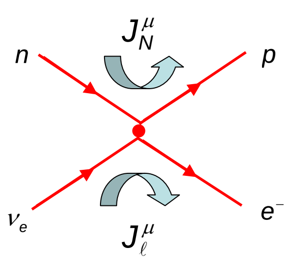
1.1 Weak interaction and nuclear β-Decay: $n \rightarrow p\, e^-\, \overline{\nu}_e$

Fermi's explanation (1933/34) of the nuclear β-decay:



Two fermionic vector currents coupled by a **weak interaction** at a single point (4-fermion interaction)

Apply "Feynman Rules"

$$M = \frac{G_F}{\sqrt{2}} \cdot J_{N,\mu} \cdot J_e^{\mu^+} = \frac{G_F}{\sqrt{2}} \cdot \left(\overline{u}_p \gamma_\mu u_n\right) \cdot \left(\overline{u}_e \gamma_\mu v_\nu\right)$$

Fermi coupling constant, dimension = $(1/M)^2$

Coupling of the currents described by coupling constant G_F – a very small number ~10^{-5} GeV^{-2}. Explains the "weakness" of the force.

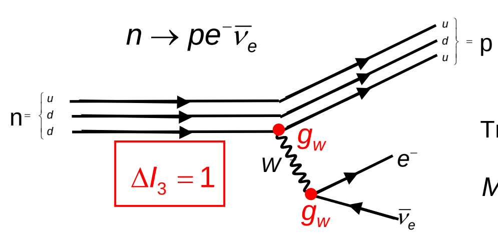
Fermi's ansatz was inspired by the structure of the electromagnetic interaction the fact that there is essentially no energy dependence observed.

Problem: Ansatz cannot explain parity violation (not a problem in the 1933) 3

Today's picture of the β-decay

- Nucleons are composed of quarks, which are the fundamental fermions. The quarks couple to the fundamental interactions.
- The weak interaction is mediated by a massive vector field (gauge boson, W).

Using the the "quark level" decay one can describe weak hadron decays (treating the quarks which are not weakly interacting as spectators)



Strong isospin

Currents: V-A structure

Transition matrix:

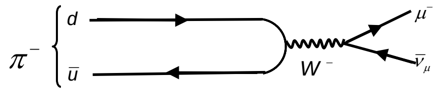
$$M = (-ig_w)^2 J^{\mu}_{quark} \left(\frac{g^{\mu\nu}}{q^2 - M_W^2} \right) J^{\nu}_{lepton}$$

"weakness" result of $(1/M_W)^2$ suppression 4

1.2 Weak hadronic decays

a) Dominant decay modes

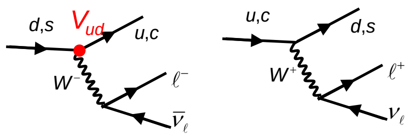
$\pi^- \to \mu^- \bar{\nu}_\mu$



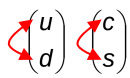
$\Delta I = 1 \quad \Delta I_3 = 1$

$d \to u\, \ell^- \bar{\nu}_\ell \qquad u \to d\, \ell^+ \nu_\ell$

$s \to c\, \ell^- \bar{\nu}_\ell \qquad c \to s\, \ell^+ \nu_\ell$



If q^2 is large enough the W can also decay to (u, $\bar{d}$) or ($\bar{u}$ d) quark pairs



Historically

$M^2 \sim \cos^2\theta_c \sim 0.95$

Cabibbo angle: $\theta_c \approx 0.22$

Today: CKM Matrix

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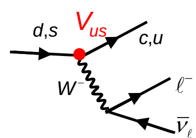
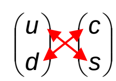
b) suppressed decay modes

$d \to c\, \ell^- \bar{\nu}_\ell \qquad u \to s\, \ell^+ \nu_\ell$

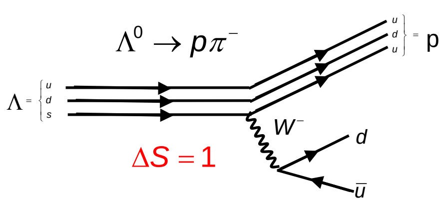
$s \to u\, \ell^- \bar{\nu}_\ell \qquad c \to d\, \ell^+ \nu_\ell$





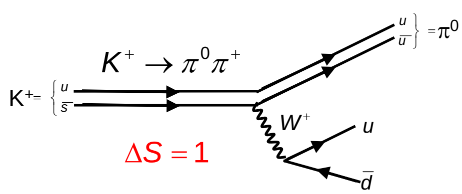
$M^2 \sim \sin^2\theta_c \sim 0.05$

$\Lambda^0 \to p\pi^-$



$\Delta S = 1$

$K^+ \to \pi^0\pi^+$



$\Delta S = 1$

Weak interaction does not conserve strong isospin, strangeness or other quark flavor numbers. Lepton number is conserved.

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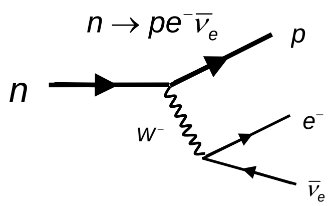
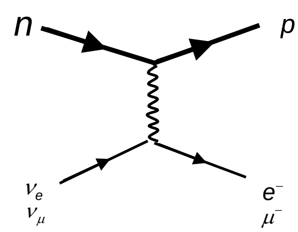
1.3 Neutrino interactions



β decay

crossing



Neutrino-nucleon scattering

Very small cross section for νN scattering: $\sigma(\nu N) \approx E_\nu[\text{GeV}]\text{x}10^{-38}\ \text{cm}^2$

$\approx E_\nu[\text{GeV}]\text{x}10\ \text{fb}$

- intense neutrino beams
- large instrumented targets

(see also DIS neutrino nucleon scattering)

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2. Parity violation

$$x'^\mu = \Lambda_P{}^\mu{}_\nu\, x^\nu$$

$$\Lambda_P = \begin{pmatrix} 1 & & & \\ & -1 & & \\ & & -1 & \\ & & & -1 \end{pmatrix}$$

Parity transformations (P) = space inversion

$$P\psi(t,\vec{x}) = \psi'(t,\vec{x}) = \psi(t,-\vec{x})$$

⇔ mirroring at plane + rotation around axis perpendicular to plane

⇒ To test P symmetry it is sufficient to study the process in the "mirrored system": physics invariant under rotation

P transformation properties: $P:$

$\vec{r} \to -\vec{r}$

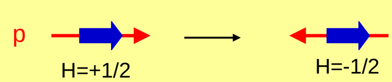
$t \to t$

$\vec{p} \to -\vec{p}$

$\vec{\ell} = \vec{r} \times \vec{p} \to \vec{\ell}$ Axial/pseudo vector

e.g.: Helicity operator

$$H = \frac{\vec{s}\cdot\vec{p}}{|\vec{p}|} \xrightarrow{P} -\frac{\vec{s}\cdot\vec{p}}{|\vec{p}|} \quad \text{(pseudo - scalar)}$$



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2.1 Historical θ/τ puzzle (1956)

P violation in pion decay: Heintze vs. Jensen

Until 1956 parity conservation as well as T and C symmetry was a “dogma”:
→ very little experimental tests done

In 1956 Lee and Yang proposed parity violation in weak processes.

Starting point: Observation of two particles θ^+ and τ^+ (Historical names) with exactly equal mass, charge and strangeness **but** with different parity:

$$\theta^+ \to \pi^+\pi^0 \quad w/ \quad P(\theta^+) = P(\pi)^2(-1)^\ell \to J^P(\theta^+) = 0^+, 1^- \qquad P(\pi)=-1$$
$$\tau^+ \to \pi^+\pi^+\pi^- \qquad P(\tau^+) = P(\pi)^3(-1)^{2\ell} \to J^P(\tau^+) = 0^-, 2^-$$

Lee + Yang: θ^+ and τ^+ same particle, but decay violates parity

⇒ particle is K^+:

$$K^+(0^-) \to \pi^+\pi^0 \qquad \text{P is violated}$$
$$K^+(0^-) \to \pi^+\pi^+\pi^- \qquad \text{P is conserved}$$

To search for possible P violation, a number of experimental tests of parity conservation in weak decays has been proposed:

1957 Observation of P violation in nuclear β decays by Chien-Shiung Wu et al.

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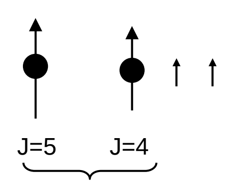
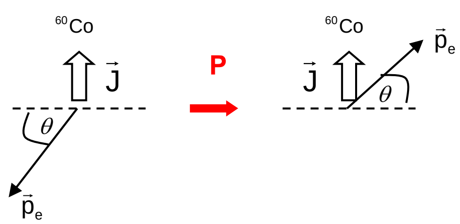
2.2 Observation of parity violation, C.S. Wu et al. 1957

Idea: Measurement of the angular distribution of the emitted e^- in the decay of polarized ^{60}Co nuclei

$$^{60}Co \to {}^{60}Ni^* + e^- + \bar{\nu}_e$$



Gamov-Teller Transition ΔJ=1



Observable: $\vec{J}\cdot\vec{p}_e$ → $-\vec{J}\cdot\vec{p}_e$

If P is conserved, the angular distribution must be symmetric in θ (symmetric to dashed line): transition rates for $\vec{J}\cdot\vec{p}_e$ and $-\vec{J}\cdot\vec{p}_e$ are identical.

Experiment: Invert Co polarization and compare the rates at the same position θ.

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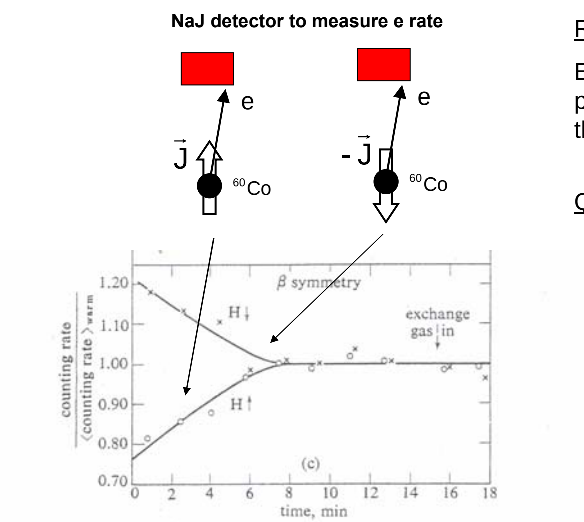
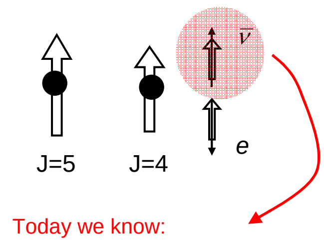


Figure 9-12 Gamma anisotropy (as determined from the two NaI counters) and beta asymmetry for the polarizing field pointing up and down as a function of time. The times for disappearance of the beta and gamma asymmetry coincide; this is the warm-up time. The warm-up time for the sample is approximately 6 min and the counting rates for the warm unpolarized sample are independent of the field direction. [From C. S. Wu, E. Ambler, R. W. Hayward, D. D. Hoppes, and R. P. Hudson, *Phys. Rev.*, **105**, 1413 (1957).]

Result:

Electron rate opposite to Co polarization is higher than along the ^{60}Co polarization:
parity violation

Qualitative explanation:



Today we know:

Consequence of existence of only left-handed (LH) neutrinos (RH anti-neutrinos)

Electron polarization in β decays

$$H_{e^-} = -\frac{1}{2}\frac{v}{c}$$

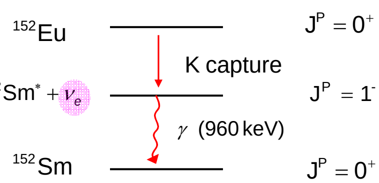
2.3 Determination of the neutrino helicity

Goldhaber et al., 1958

Indirect measurement of the neutrino helicity in a K capture reaction:

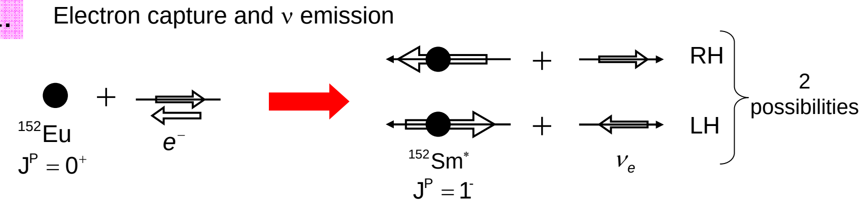
$$^{152}\mathrm{Eu} + e^- \rightarrow {}^{152}\mathrm{Sm}^* + \nu_e$$

$$E_\nu \approx 950\,\mathrm{KeV}$$



Idea of the experiment:

1. Electron capture and ν emission



Sm undergoes a small **recoil (p_{recoil} =950 KeV)**. Because of angular momentum conservation Spin J=1 of Sm* is opposite to neutrino spin.
Important: **neutrino helicity is transferred to the Sm nucleous.**

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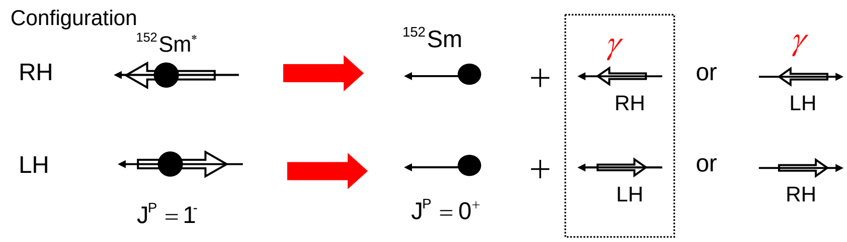
2. γ emission: ${}^{152}\mathrm{Sm}^*(J^P = 1^-) \rightarrow {}^{152}\mathrm{Sm}(J^P = 0^+) + \gamma$



Photons along the Sm recoil direction carry the polarization of the Sm* nucleus

- How to select photons along the recoil direction ? $\Rightarrow$ 3
- How to determine the polarization of these photons ? $\Rightarrow$ 4

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3. Resonant photon scattering: $\gamma + {}^{152}\mathrm{Sm} \rightarrow {}^{152}\mathrm{Sm}^* \rightarrow {}^{152}\mathrm{Sm} + \gamma$

<u>Resonant scattering:</u>

To compensate the nuclear recoil, the photon energy must be slightly larger than 960 keV.

This is the case for photons which have been emitted in the direction of the Eu$\rightarrow$Sm recoil (Doppler-effect).

Resonant scattering only possible for “forward” emitted photons, which carry the polarization of the Sm* and thus the polarization of the neutrinos.

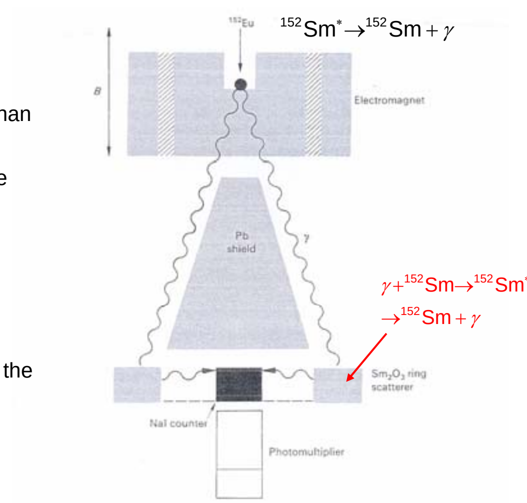
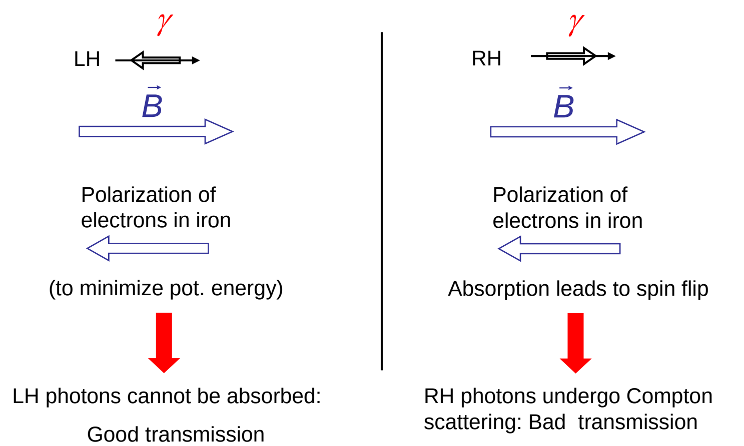


Fig. 7.8. Schematic diagram of the apparatus used by Goldhaber *et al.*, in which γ-rays from the decay of ^{152}Sm*, produced following K-capture in ^{152}Eu, undergo resonance scattering in Sm_2O_3 and are recorded by a sodium iodide scintillator and photomultiplier. The transmission of photons through the iron surrounding the source depends on their helicity and the direction of the magnetic field **B**.

4. Determination of the photon polarization

Exploit that the transmission index through magnetized iron is polarization dependent: Compton scattering in magnetized iron



Photons w/ polarization anti-parallel to magnetization undergo less absorption

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Experiment

Sm* emitted photons pass through the magnetized iron. Resonant scattering allows the photon detection by a NaJ scintillation counter. The counting rate difference for the two possible magnetizations measure the polarization of the photons and thus the helicity of the neutrinos.

Results: $P_\gamma = -0.66 \pm 0.14$

→ photons from Sm* are left-handed. The measured photon polarization is compatible with a neutrino helicity of H=–1/2.

From a calculation with 100% photon polarization one expects a measurable value P_γ~0.75. Reason is the finite angular acceptance.
→ Also not exactly forward-going γ's can lead to resonant scattering.

Summary: Lepton polarization in β decays

	e^-	e^+	ν	$\bar{\nu}$
$H=\frac{1}{2}\cdot$	-v/c	+v/c	-1	+1

3. “V-A Theory” for charged current weak interactions

3.1 Lorentz structure of the weak currents

Fermi: $M = C_V\, J_{N,\mu} \cdot J_e^{\mu +} = C_V\, (\bar{u}_p \gamma_\mu u_n) \cdot (\bar{u}_e \gamma_\mu v_\nu)$

Cannot explain the parity violation in beta decays.

More general ansatz (Gamov & Teller)

$$M = \sum_i C_i (\bar{u}_p \Gamma_i u_n) \cdot (\bar{u}_e \Gamma_i v_\nu)$$

$i = S, P, V, A, T$

$\bar{u}_p\ \Gamma_i\ u_n$

bilinear Lorentz covariants:

$\overline{\psi}\ (4 \times 4) \psi$

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$$\bar{u}_p\ \Gamma_i\ u_n = \begin{cases} S: & \bar{u}_p u_n & \text{scalar} \\ P: & \bar{u}_p \gamma^5 u_n & \text{pseudo-scalar} \\ V: & \bar{u}_p \gamma^\mu u_n & \text{vector} \\ A: & \bar{u}_p \gamma^5 \gamma^\mu u_n & \text{pseudo-vector} \\ T: & \bar{u}_p \sigma^{\mu\nu} u_n & \text{tensor} \end{cases}$$

$$\sigma^{\mu\nu} = \frac{i}{2}\left(\gamma^\mu \gamma^\nu - \gamma^\nu \gamma^\mu\right)$$

$$\gamma^5 = i\, \gamma^0 \gamma^1 \gamma^2 \gamma^3$$

$$\gamma^0 = \begin{pmatrix} I & 0 \\ 0 & -I \end{pmatrix} \quad \gamma^i = \begin{pmatrix} 0 & \sigma_i \\ -\sigma_i & 0 \end{pmatrix} \ \Rightarrow\ \gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

$$\left(\gamma^5\right)^2 = 1 \qquad \gamma^5 \gamma^\mu + \gamma^\mu \gamma^5 = 0$$

Remark: Pure P or A couplings do not lead to observable parity violation!
Mixtures like $(1 \pm \gamma^5)$ or $\gamma^\mu (1 \pm \gamma^5)$ do violate parity.

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Example: Transformation behavior

$$\psi \xrightarrow{Parity} \psi' = S_P \psi = \gamma^0 \psi \quad S_P = \gamma^0$$

See Nachtmann, p. 61: examination of Dirac eq.,
$S_P^{-1}\gamma^0 S_p = \gamma^0 \quad S_P^{-1}\gamma^i S_p = -\gamma^i$

V: $$\overline{\psi}\gamma^\mu\psi \xrightarrow{Parity} \left(\overline{\psi}\gamma^\mu\psi\right)' = \Lambda_P\left(\overline{\psi}\gamma^\mu\psi\right) = \begin{cases} \overline{\psi}\gamma^0\psi \\ -\overline{\psi}\gamma^\mu\psi \end{cases}$$

$$\overline{\psi}'\gamma^\mu\psi' = \overline{\psi}S_P^{-1}\gamma^\mu S_P\psi = \overline{\psi}\gamma^0\gamma^\mu\gamma^0\psi$$

P: $$\overline{\psi}\gamma^5\psi \xrightarrow{Parity} \overline{\psi}\gamma^0\gamma^5\gamma^0\psi = -\overline{\psi}\gamma^5\psi$$

A: $$\overline{\psi}\gamma^5\gamma^\mu\psi \xrightarrow{Parity} \overline{\psi}\gamma^0\left(\gamma^5\gamma^\mu\right)\gamma^0\psi = \overline{\psi}\gamma^5\gamma^\mu\psi$$

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3.2 Chirality operators

Operators

$$P_L \equiv \frac{1}{2}(1-\gamma^5)$$
$$P_R \equiv \frac{1}{2}(1+\gamma^5)$$

are projection operators:

$$(P_i)^2 = P_i\,, \quad P_L + P_R = 1, \quad P_L P_R = 0$$

(properties of γ^5)

working on the fermion spinors they result in the left / right handed chirality components:

$$u_L = \frac{1}{2}(1-\gamma^5)u$$
$$u_R = \frac{1}{2}(1+\gamma^5)u$$

Not observable!

In contrary to helicity, which is an observable: $\frac{1}{2}\frac{\vec{\Sigma}\cdot\vec{p}}{|\vec{p}|}$

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Reminder: Dirac spinors ➡ Eigenvectors of helicity operator

solution spin↑ i.e. helicity $\lambda = +\frac{1}{2}$

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ \frac{p_z}{E+m} \\ \frac{p_x + ip_y}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_1(p) = \sqrt{E+m} \cdot \begin{pmatrix} 1 \\ 0 \\ p/(E+m) \\ 0 \end{pmatrix}$$

solution spin↓ i.e. helicity $\lambda = -\frac{1}{2}$

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ \frac{p_x - ip_y}{E+m} \\ \frac{-p_z}{E+m} \end{pmatrix}$$

$\vec{p}$ along z

$$u_2(p) = \sqrt{E+m} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -p/(E+m) \end{pmatrix}$$

$$\frac{1}{2}\frac{\Sigma^k p^k}{|\vec{p}|} u_1 = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} u_1 = \frac{1}{2} u_1 \qquad \frac{1}{2}\frac{\Sigma^k p^k}{|\vec{p}|} u_1 = \frac{1}{2}\begin{pmatrix} \sigma^3 & 0 \\ 0 & \sigma^3 \end{pmatrix} u_1 = -\frac{1}{2} u_2$$

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Dirac spinors and chirality projection operators:

Positive helicity:

$$\frac{1-\gamma^5}{2} u_1 = \underbrace{\frac{1}{2}\sqrt{E+m} \cdot \left(1 - \frac{p}{E+m}\right)}_{\approx 0 \text{ for } E >> m} \cdot \begin{pmatrix} 1 \\ 0 \\ -1 \\ 0 \end{pmatrix} \rightarrow 0 \quad \text{for } E >> m$$

$$\gamma^5 = \begin{pmatrix} 0 & I \\ I & 0 \end{pmatrix}$$

Negative helicity:

$$\frac{1-\gamma^5}{2} u_2 = \underbrace{\frac{1}{2}\sqrt{E+m} \cdot \left(1 + \frac{p}{E+m}\right)}_{\approx \sqrt{E} \text{ for } E >> m} \cdot \begin{pmatrix} 0 \\ 1 \\ 0 \\ -1 \end{pmatrix} \rightarrow u_2 \quad \text{for } E >> m$$

In the **relativistic limit** helicity states are also eigenstates of the chirality operators.

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Polarization for particles with finite mass

Left handed spinor component of unpolarized electron:

$u_1, u_2 \rightarrow u_L, u_R$

$$u_L = \frac{1-\gamma^5}{2}\underbrace{(u_1+u_2)}_{\text{unpolarized}}$$

Not normalized

Helicity polarization of left handed chirality state u_L :

$$Pol = \frac{P(\lambda=+1/2)-P(\lambda=-1/2)}{P(\lambda=+1/2)+P(\lambda=-1/2)} = \frac{|\langle u_1|u_L\rangle|^2 - |\langle u_2|u_L\rangle|^2}{|\langle u_1|u_L\rangle|^2 + |\langle u_2|u_L\rangle|^2}$$

$$= \frac{(1-p/(E+m))^2-(1+p/(E+m))^2}{(1-p/(E+m))^2+(1+p/(E+m))^2} = -\frac{p}{E} = -\frac{v}{c}$$

i.e. the LH spinor component for a particle with finite mass is not fully in the helicity state "spin down" (λ=-½)

For massive particles there is a finite probability to measure the "wrong" helicity state!

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Neutrino-Electron Vertex

Only left-handed neutrinos are observed in beta decays: $u_\nu \rightarrow \left(\frac{1-\gamma^5}{2}\right)u_\nu$

This leads to the following electron-neutrino vertex (assuming vector coupling between LH neutrino and e): $\bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu$

If one further exploits that $P_l = 1/2(1-\gamma^5)$ is a projection operator one finds:

$$\bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu = \bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)^2u_\nu = u_e^+\left(\frac{1-\gamma^5}{2}\right)\gamma^0\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu = \overline{(u_e)}_L\gamma^\mu(u_\nu)_L$$

The left-handed neutrino thus couples only to left-handed electrons.

V-A structure: $$\bar{u}_e\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)u_\nu = \frac{1}{2}\bar{u}_e\left(\gamma^\mu - \gamma^\mu\gamma^5\right)u_\nu$$

V - A (vector – axial-vector)

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