Fock space

$|0\rangle$: vacuum state, normalised $\langle 0|0\rangle = 1$

with $a(\vec{k})|0\rangle = 0$

$|0\rangle$ lowest energy state !
$a(\vec{k})$ annihilates the vacuum
'Heisenberg' picture $\partial_t |0\rangle = 0$ (3.14)

All states are generated by applying a, a^+ on $|0\rangle$: (a, a^+ are annihilation and creation op respect.)

One-particle states :

$$|k\rangle = a^+(\vec{k})|0\rangle \qquad (3.15)$$

the states $|k\rangle$ are orthogonal :

$$\langle k'|k\rangle = \langle 0|a(\vec{k}')\, a^+(\vec{k})|0\rangle$$
$$= \langle 0|[a(\vec{k}'), a^+(\vec{k})]|0\rangle$$
$$= (2\pi)^3\, 2\omega\, \delta\, (\vec{k}-\vec{k}') \qquad (3.16)$$

$\Rightarrow$ general one-particle state

$$|f\rangle = \int \frac{d^3k}{(2\pi)^3 \sqrt{2k}}\, f(\vec{k})\, a^+(\vec{k})\, |0\rangle \qquad (3.17)$$

$$\Rightarrow \boxed{\langle f|f\rangle = \int d^3k/(2\pi)^3\, \frac{1}{2\omega}\, f f^*(\vec{k})}$$

N-Particle states

$$a^+(k_2)\, a^+(k_1)|0\rangle$$

$$a^+(k_3)\, a^+(k_2)\, a^+(k_1)|0\rangle$$

$$\vdots \qquad (3.18)$$

- have Bose symmetry, as

$$a^+(k_2)\, a^+(k_1) = a^+(k_1)\, a^+(k_2) \qquad (3.19)$$

- Energy-momentum is additive
  take some state $|\beta\rangle$,
  then $a^+(k)|\beta\rangle$ is a state with
  <u>one</u> additional particle with
  momentum k.

Annihilation:

$a(\vec{k})\,|\beta\rangle$ is a state,

where a particle with momentum k

is removed.

Example:

p3-5, (3.17)

$$a(\vec{k})\,|f\rangle = a(\vec{k}) \int \frac{d^3k'}{(2\pi)^3} \frac{1}{\sqrt{2\omega'}} f(k') \cdot a^+(k')\,|0\rangle$$

$$= \int \frac{d^3k'}{(2\pi)^3} \frac{1}{\sqrt{2\omega'}} f(k') \underbrace{\left[a(\vec{k}), a^+(k')\right]}_{}|0\rangle$$

p 3-4, (3.13) $\longrightarrow$ $(2\pi)^3 2\omega\, \delta(\vec{k}-\vec{k}')$

$$= \sqrt{2\omega'}\, f(k)\,|0\rangle \qquad (3.20)$$

Interpretation of $\phi(x)$

- states with defined particle number n have vanishing expectation value of ϕ, as ϕ creates and annihilates a particle, see (3.12).

This follows from $\langle 0| a^+ |0\rangle = \langle 0|a|0\rangle = 0$

$\langle k| a^+ | k'\rangle = \langle k| a |k'\rangle = 0$

$\vdots$

$$\Rightarrow \langle 0| \phi(x) |0\rangle = 0 \quad \ldots \tag{3.21}$$

- coherent states: $\langle \phi \rangle$ behave like classical wave

$$|\alpha\rangle = \frac{1}{N} \underbrace{e^{\int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega} \alpha(\vec{k}) a^+(\vec{k})} |0\rangle}_{|\alpha_0\rangle}$$

with $N = e^{\frac{1}{2} \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega} |\alpha(\vec{k})|^2} = \sqrt{\langle \alpha_0 | \alpha_0 \rangle}$ (3.22)

and $\alpha(\vec{k})$ coefficient function

$\langle \alpha | \alpha \rangle = 1$ via p. 3-8a, 3-8b

$\omega_i = \omega(\vec{k}_i) = \sqrt{\vec{k}_i^2 + m^2}\,, \quad \omega_{i'} = \omega(\vec{k}_i')$

$$\langle \alpha_0 | \alpha_0 \rangle = \sum_{n=0}^{\infty} \left(\frac{1}{n!}\right)^2 \int \prod_{i=1}^{n} \frac{d^3k_i}{(2\pi)^3} \frac{d^3k_i'}{(2\pi)^3} \frac{1}{2\omega_i} \frac{1}{2\omega_{i'}}$$

$$\left. \alpha^*(\vec{k}_i)\, \alpha(\vec{k}_i') \right] \langle 0 | a(\vec{k}_1) \circ\circ\circ a(\vec{k}_n)\, a^\dagger(\vec{k}_n') \circ\circ\circ a(\vec{k}_1') | 0 \rangle$$

—

$$\langle 0 | a(\vec{k}_1) \cdots \underbrace{a(\vec{k}_n) \cdot a^\dagger(\vec{k}_n')}_{\underbrace{[a(\vec{k}_n),\, a^\dagger(\vec{k}_n')]}_{(2\pi)^3 2\omega_n \delta(\vec{k}_n - \vec{k}_n')} + a^\dagger(\vec{k}_n')\, a(\vec{k}_n)} \cdots a(\vec{k}_1') | 0 \rangle$$

$$= (2\pi)^3 2\omega_n \delta(\vec{k}_n - \vec{k}_n') \langle 0 | a(\vec{k}_1) \circ\circ\circ a(\vec{k}_{n-1}) \circ a^\dagger(\vec{k}_{n-1}') \cdots a^\dagger(\vec{k}_1') | 0 \rangle$$

$$+ \langle 0 | a(\vec{k}_1) \circ\circ\circ a(\vec{k}_{n-1})\, a^\dagger(\vec{k}_n') \underbrace{a(\vec{k}_n)\, a^\dagger(\vec{k}_{n-1}')}_{\underbrace{[a(\vec{k}_n),\, a^\dagger(\vec{k}_{n-1}')]}_{(2\pi)^3 2\omega_n \delta(\vec{k}_n - \vec{k}_{n-1}')} + a^\dagger(\vec{k}_{n-1}') \cdot a(\vec{k}_n)} \circ\circ\circ a^\dagger(\vec{k}_1') | 0 \rangle$$

$\vdots$

$$= (2\pi)^3 2\omega_n \sum_{i=1}^{n} \langle 0 | a(\vec{k}_1) \cdots a(\vec{k}_{n-1})\, a^\dagger(\vec{k}_n') \circ\circ\circ \widehat{a^\dagger(\vec{k}_i')} \circ\circ\circ a^\dagger(\vec{k}_1') | 0 \rangle$$

$$\cdot (2\pi)^3 2\omega_n \delta(\vec{k}_n - \vec{k}_i')$$

where $\widehat{a^\dagger(\vec{k}_i')} = 1$

$$\Rightarrow \langle \alpha_0 | \alpha_0 \rangle = \sum_{n=0}^{\infty} \frac{1}{n!} \frac{1}{(n-1)!} \int \frac{d^3k_n}{(2\pi)^3} \frac{1}{2\omega_n} \alpha^*(\vec{k}_n) \alpha(\vec{k}_n)$$

$$\int \prod_{i=1}^{n-1} \left[\frac{d^3k_i}{(2\pi)^3} \frac{d^3k_i'}{(2\pi)^3} \frac{1}{2\omega_i} \frac{1}{2\omega_i'} \right] \langle 0 | a(\vec{k}_1) \cdots a(\vec{k}_{n-1}) a^+(\vec{k}'_{n-1}) \cdots a(\vec{k}'_1) | 0 \rangle$$

$$\cdot \alpha^*(\vec{k}_i) \alpha(k_i')$$

$$\vdots$$

$$= \sum_{n=0}^{\infty} \frac{1}{n!} \left[\int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega} \alpha^* \alpha(\vec{k}) \right]^n$$

$$= e^{\int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega} |\alpha(\vec{k})|^2}$$

$$\Rightarrow \boxed{N = e^{1/2 \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega} |\alpha(\vec{k})|^2}}$$

Coherent states:

$$a(\vec{k})\,|\alpha\rangle = \alpha(\vec{k})\,|\alpha\rangle$$

unchanged by annihilation (detection) of a particle with momentum k.

Scalar product:

$$\langle \alpha' | \alpha \rangle = e^{-\frac{1}{2}\int \frac{d^3k}{(2\pi)^3}\frac{1}{2\omega}\left\{ |\alpha(\vec{k})|^2 + |\alpha'(\vec{k})|^2 - 2\alpha'^{*}\alpha(\vec{k}) \right\}}$$

$$\frac{1}{N(\alpha)} \qquad \frac{1}{N(\beta)}$$

not orthonormal.

Completeness: (QM)

$$\frac{1}{\pi}\int |\alpha\rangle\langle\alpha|\, d^2\alpha = \mathbb{1}$$

Fock-space representation

$$|\alpha\rangle = \frac{1}{N}\sum_{n=0}^{\infty}\frac{1}{n!}\prod_{i=1}^{n}\left[\int \frac{d^3k_i}{(2\pi)^3}\frac{1}{2\omega_i}\,\alpha(\vec{k}_i)\right] |k_1 \ldots k_n\rangle$$

with

$$|k_1 \ldots k_n\rangle = a^{\dagger}(\vec{k}_1)\cdots a^{\dagger}(\vec{k}_n)|0\rangle$$

$\Rightarrow$

$$\langle \alpha | \Phi(x) | \alpha \rangle \tag{3.23}$$

$$= \int \frac{d^3k}{(2\pi)^3} \frac{1}{2\omega} \left\{ e^{ikx} \alpha^*(\vec{k}) + e^{-ikx} \alpha(\vec{k}) \right\}$$

using

$$\frac{1}{n!} \quad a(k) \left[\int \frac{d^3k'}{(2\pi)^3 2\omega'} \alpha(\vec{k}') \, a^+(\vec{k}') \right]^n |0\rangle$$

$$= \frac{1}{n!} \, n \, \alpha(\vec{k}') \left[\int \frac{d^3k'}{(2\pi)^3} \frac{1}{2\omega'} \alpha(\vec{k}') \, a^+(k') \right]^{n-1} |0\rangle$$

$\uparrow$
p 3-5, (3.16), see also p. 3-8a, 3-8b

$$= \alpha(\vec{k}') \, \frac{1}{(n-1)!} \left[\qquad \right]^{n-1} |0\rangle \tag{3.24}$$

and similarly

$$\frac{1}{n!} \left[\int \frac{d^3k'}{(2\pi)^3} \frac{1}{2\omega'} \alpha^*(k') \, a(\vec{k}') \right]^n a^+(k)$$

$$= \frac{1}{(n-1)!} \left[\qquad \right]^{n-1} \alpha^*(\vec{k}') \tag{3.25}$$

$\Rightarrow$ $\boxed{a(\vec{k}) |\alpha\rangle = \alpha(\vec{k}) |\alpha\rangle}$ see also p 3-8c