Anomalous magnetic moment

Standard Model script, p. 47; Nachtmann, p. 97f.

1. Magnetic moment of the electron:

Classical magnetic moment: $\vec{\mu} = \frac{1}{2}\int d^3r[\vec{r}\times\vec{j}(\vec{r})]$

Operator of the magnetic moment: $\vec{\boldsymbol{\mu}}(t) = \frac{1}{2}\int d^3x\,\left[\vec{x}\times\vec{\mathbf{j}}(\vec{x},t)\right]$

with the electron current operator $\mathbf{j}^\mu(x) = -e:\overline{\psi}(x)\gamma^\mu\psi(x):$

Calculate the expectation value of the operator in case of an one-electron system (at t=0, μ(t=0)=μ): $\left|e(\vec{k},s)\right\rangle = a_s^+(\vec{k})\left|0\right\rangle$

$$\left\langle e(\vec{k}',r)\right|\vec{\boldsymbol{\mu}}\left|e(\vec{k},s)\right\rangle = -\frac{e}{2}\int d^3x\,\vec{x}\times\left\langle e(\vec{k}',r)\right|:\overline{\psi}\,\vec{\gamma}\,\psi:\left|e(\vec{k},s)\right\rangle$$

$$= \ldots = -\frac{e}{2}\int d^3x\, e^{i(k-k')x}\,\vec{x}\times\overline{u}_r(k')\,\vec{\gamma}\,u_s(k)$$

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To interpret the result consider a wave packet, i.e. a normalized one-electron state:

$$\left|f\right\rangle = \int\frac{d^3p}{(2\pi)^3\sqrt{2p^0}}\sum_{s=\pm 1/2} f_s(p)\,a_s^+(p)\left|0\right\rangle$$

$$\left\langle f\right|\vec{\boldsymbol{\mu}}\left|f\right\rangle = \int\frac{d^3k'}{(2\pi)^3}\frac{d^3k}{(2\pi)^3 2m}\sum_{r,s} f_r^*(\vec{k}')f_s(\vec{k})\left\langle e(\vec{k}',r)\right|\vec{\boldsymbol{\mu}}\left|e(\vec{k},s)\right\rangle$$

$$= \ldots$$

$$= -\frac{e}{2m}\left\langle f\right|\vec{L}+\vec{\sigma}\left|f\right\rangle = -\frac{e}{2m}\left\langle f\right|\vec{L}+2\vec{S}\left|f\right\rangle \qquad \vec{S}=\frac{1}{2}\vec{\sigma}$$

$$= -\frac{e}{2m}\left\langle f\right|\vec{L}+g\vec{S}\left|f\right\rangle$$

$$\left\langle\vec{\mu}_e\right\rangle = -\frac{e}{2m}g\underbrace{\left(\frac{1}{2}\langle\vec{\sigma}\rangle\right)}_{\vec{S}} \qquad g=2$$

For details see Nachtmann, p. 99f.

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2. "Heuristic" motivation

Standard Model script, p. 46;

Lagrangian for QED $\mathbf{L} = \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$

Covariant derivative $D_\mu = \partial_\mu - ieA_\mu$

Hamiltonian density
$$\mathbf{H} = \bar{\psi} i\gamma^0 \dot{\psi} - \bar{\psi}(i\gamma^\mu D_\mu - m)\psi$$
$$= \psi^+ \underbrace{(-i\gamma^0\gamma^i D_i + \gamma^0 m)}_{H}\psi \qquad \gamma^i D_i = \vec{\gamma}\cdot\vec{D}$$

Evaluate non-relativistic limit of $H = \sqrt{H^2}$

$$H^2 = -\vec{D}^2 - ie\frac{1}{4}[\gamma_i, \gamma_j]F^{ij} + m^2 \qquad [\gamma^i, \gamma^j] = -2\varepsilon_{ijk}\Sigma^k \qquad \Sigma^k = \begin{pmatrix} \sigma^k & 0 \\ 0 & \sigma^k \end{pmatrix}$$

$$H^2 = -\vec{D}^2 - \frac{e}{2}\varepsilon_{ijk}\Sigma^k F^{ij} + m^2$$
$$= -\vec{D}^2 + e\vec{\Sigma}\vec{B} + m^2$$

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$$H \sim \sqrt{H^2} = m + \frac{(\vec{p} - e\vec{A})^2}{2m} + \frac{e}{2m}\vec{\Sigma}\cdot\vec{B} + O(m^{-2})$$

$\vec{\Sigma} = 2\vec{S}$

non-relativistic Schroedinger equation:

$$i\frac{d}{dt}\psi = \left(\frac{(\vec{p} - e\vec{A})^2}{2m} + \boxed{\frac{e}{2m}2\vec{S}\cdot\vec{B}} + m\right)\psi$$

Spin coupling: $\frac{e}{2m}g\vec{S}\cdot\vec{B} = \vec{\mu}\cdot\vec{B}$

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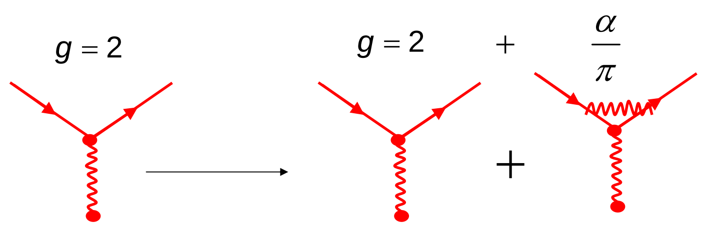
3. Effect of higher order corrections

Magnetic moment $\langle \vec{\mu}_e \rangle = -\frac{e}{2m} g \overbrace{\frac{1}{2}\langle \vec{\sigma} \rangle}^{\vec{S}}$ $\quad g = 2$



1st order: $\langle \vec{\mu}_e \rangle = -\frac{e}{2m}\left(2 + \frac{\alpha}{\pi}\right) \cdot \frac{1}{2} \cdot \langle \vec{\sigma} \rangle$

$$g = 2 + \frac{\alpha}{\pi}$$
$$a = \frac{g-2}{2} = \frac{\alpha}{2\pi}$$

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Higher order corrections to g-2

Radiative corrections g-2 are calculated to the 4-loop level:

Feynman Graphs		
O(α)		1
O(α²)		7
O(α³)	analytically	72
O(α⁴)	numerically	891
til O(α⁴)		971

Fig. 8.2 The Feynman graphs which have to be evaluated in computing the α³ corrections the lepton magnetic moments (after Lautrup et al. 1972).

Most precise QED prediction.

T. Kinoshita et al.

Kinoshita 2006 $a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.505....\left(\frac{\alpha}{\pi}\right)^4$

Kinoshita 2007 $a_e = \frac{\alpha}{2\pi} - 0.328...\left(\frac{\alpha}{\pi}\right)^2 + 1.182....\left(\frac{\alpha}{\pi}\right)^3 - 1.9144....\left(\frac{\alpha}{\pi}\right)^4$

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4. Electron g-2

Experimental method:

Storage of **single** electrons in a Penning trap (electrical quadrupole + axial B field)

$\Rightarrow$ complicated electron movement (cyclotron and magnetron precessions).

Idea: bound electron (**geonium**), transitions between excited states

$a_{e^-} = 0.001159\,652\,188\,4\,(43)$

$a_{e^+} = 0.001159\,652\,187\,9\,(43)$

H. Dehmelt et al. 1987

$a_e = 0.001159\,652\,180\,85\,(76)$

G. Gabrielse et al. 2006

Using $a_e \Rightarrow$ most precise value of α:

$\alpha^{-1}(a_e) = 137.035\,999\,084\,(51)$

For comparison α from Quanten Hall

$\alpha^{-1}(qH) = 137.036\,003\,00\,(270)$

Theory *Kinoshita 2008*

$a_e = 0.001159\,652\,182\,79\,(771)$

$a_e = 0.001159\,652\,180\,85\,(76)$

*Phys. Rev. Lett. **97**, 030801 (2006)*
*Phys. Rev. Lett. **97**, 030802 (2006)*

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5. Experimental determination of muon g-2

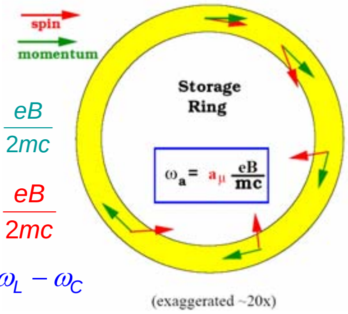
Principle:

- store polarized muons in a storage ring; revolution with cyclotron frequency ω_c
- measure spin precession around the magnetic dipole field relative to the direction of cyclotron motion

$$\omega_C = 2\frac{eB}{2mc}$$

$$\omega_L = g\frac{eB}{2mc}$$

$$\omega_a = \omega_L - \omega_C$$



Precession:

$$\vec{\omega}_a = -\frac{e}{m_\mu c}\left[a_\mu \vec{B} - (a_\mu - \frac{1}{\gamma^2 - 1})\vec{\beta}\times\vec{E}\right]$$

Difference between Lamor and cyclotron frequency (term $a_\mu \vec{B}$)

Effect of electrical focussing fields (relativistic effect).
$= 0$ for $\gamma = 29.3$
$\Leftrightarrow p_\mu = 3.094$ GeV/c

First measurements:

CERN 70s

$a_{\mu^-} = 0.001165937(12)$

$a_{\mu^+} = 0.001165911(11)$

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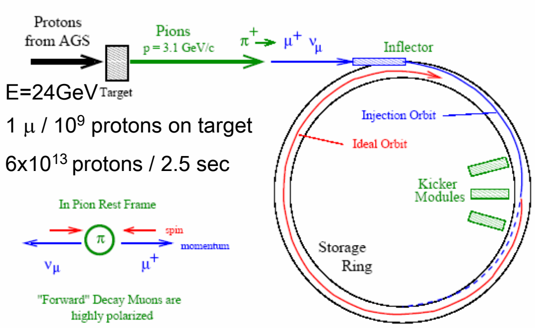
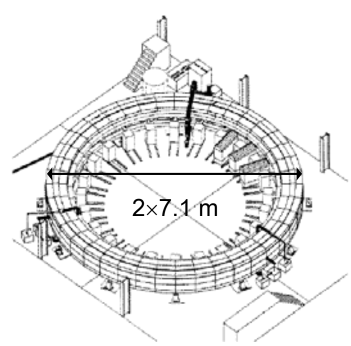
$(g\text{-}2)_\mu$ Experiment at BNL

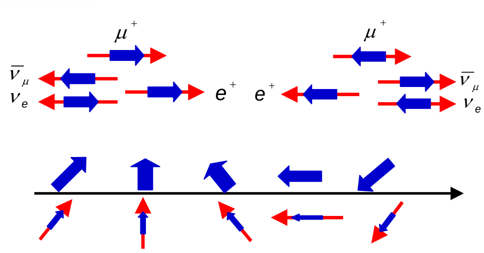




"V-A" structure of weak decay:

Use high-energy e^+ from muon decay to measure the muon polarization

Weak charged current couples to LH fermions (RH anti-fermions)



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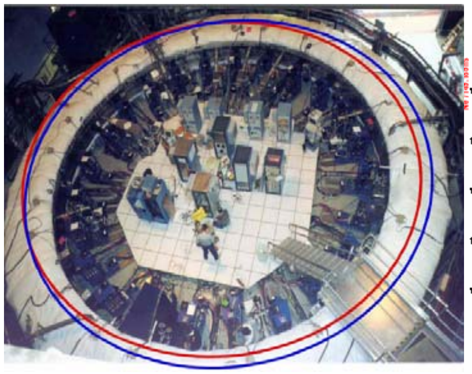
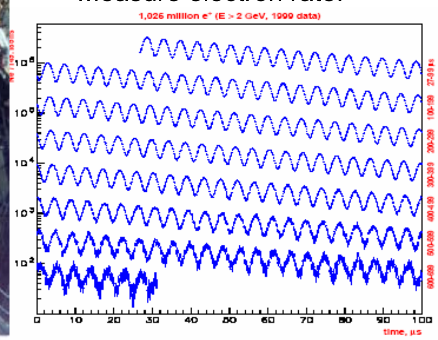
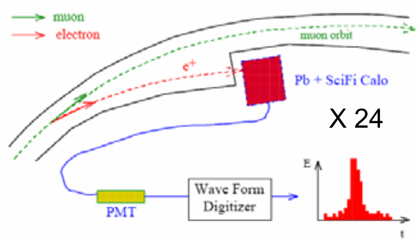


Measure electron rate:



$$N(t) = N_0\, e^{-\lambda t}\left[1 + A\cos(\omega_a t + \varphi)\right]$$



X 24

$$\frac{\omega_a}{2\pi} = 229023.59(16)\,\mathrm{Hz}$$

(0.7ppm)

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu c}\langle B\rangle}\ ?$$

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From ω_a to a_μ - How to measure the B field

$<B>$ is determined by measuring the proton nuclear magnetic resonance (NMR) frequency ω_p in the magnetic field.

$$a_\mu = \frac{\omega_a}{\frac{e}{m_\mu c}\langle B\rangle} = \frac{\omega_a}{\frac{e}{m_\mu c}\frac{\hbar\tilde{\omega}_p}{2\mu_p}} = \frac{\omega_a}{\frac{4\mu_\mu}{\hbar g_\mu}\frac{\hbar\tilde{\omega}_p}{2\mu_p}} = \frac{\omega_a/\tilde{\omega}_p}{\mu_\mu/\mu_p}(1+a_\mu)$$

$$\Downarrow$$

$$a_\mu = \frac{\omega_a/\omega_p}{\mu_\mu/\mu_p - \omega_a/\omega_p}$$

$\mu_{\mu^+}/\mu_p = 3.183\,345\,39(10)$

from hyperfine splitting in muonium

W. Liu *et al.*, Phys. Rev. Lett. **82**, 711 (1999).

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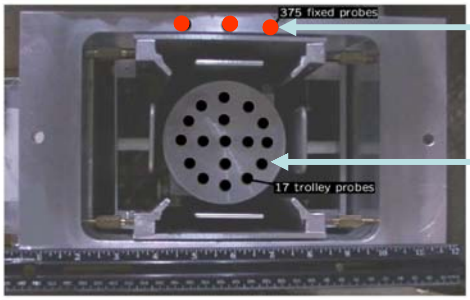
NMR trolley



375 fixed NMR probes around the ring

17 trolley NMR probes

$\tilde{\omega}_p/2\pi$ = 61 791 400(11) Hz (0.2ppm)

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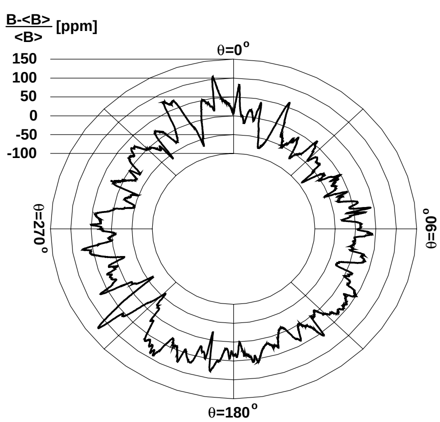
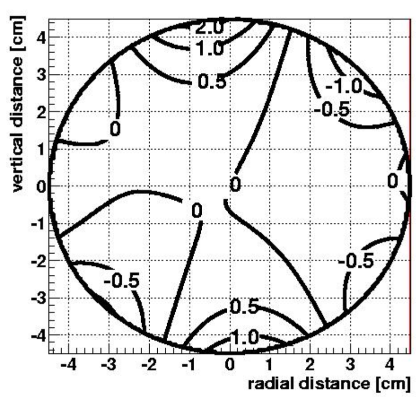
B field determination



The *B* field variation at the center of the storage region. $\langle B\rangle \approx 1.45$ T



The *B* field averaged over azimuth.

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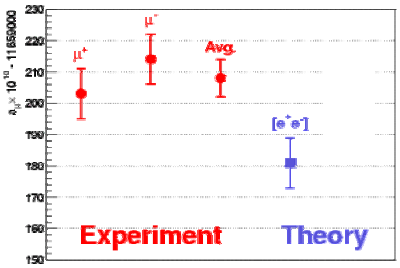
$$a_{\mu^+} = 11659\,203(8)\times 10^{-10}(0.7ppm)$$
$$a_{\mu^-} = 11659\,214(8)\times 10^{-10}(0.7ppm)$$
$$a_{\mu} = 11659\,208(6)\times 10^{-10}(0.5ppm)$$



About 2.6σ deviation:

- Often interpreted as sign of new physics: SUSY
- But careful:
  "Theory" has uncertainties … and sometimes bugs.
- Quantum loop effects (SM or new physics) are ~ m^2 and therefore more important for muons than for electrons.

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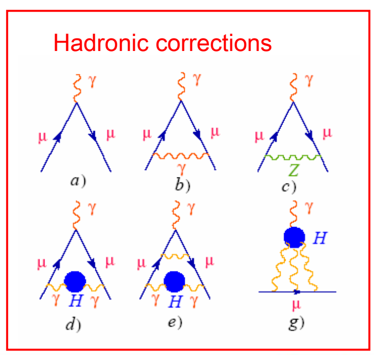
6. Theoretical prediction of a_μ

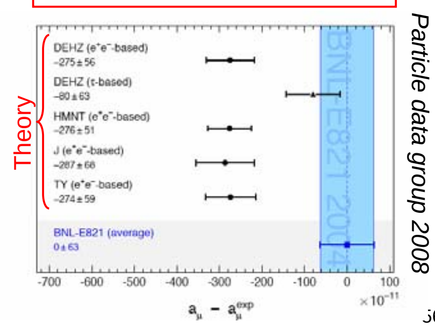
Beside pure QED corrections there are weak corrections (W, Z) exchange and „hadronic corrections“

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW}$$

(For the electron with much lower mass the hadronic and weak corrections are suppressed (~m^2), and can be neglected.)

→ Determination of hadronic corrections is difficult and is in addition based on data: hot discussion amongst theoreticians how to correctly use the data.





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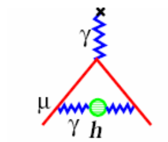
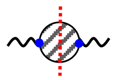
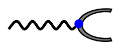
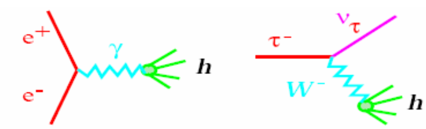
Hadronic vacuum polarization:



Hadronic corrections related to virtual intermediate hadronic states (ππ, ρ, ϕ) – cannot be calculated.

Use the "optical theorem" to relate the loop corrections to observable cross sections / branching ratios:

Im[] ∝ |  hadrons |^2



$$a_\mu(\text{had};1) = \left(\frac{\alpha m_\mu}{3\pi}\right)^2 \int_{4m_\pi^2}^{\infty} \frac{ds}{s^2} K(s)\, R(s)$$

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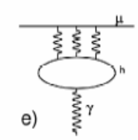
… calculations are sometimes not easy …



In 2001 Kinoshita et al. found a sign mistake in their calculation of the light-by-light scattering amplitude:

December 2001
KEK-TH-793
hep-ph/0112102

Comment on the sign of the pseudoscalar pole contribution to the muon $g-2$

Masashi Hayakawa * and Toichiro Kinoshita †

Abstract

We correct the error in the sign of the pseudoscalar pole contribution to the muon $g-2$, which dominates the $\mathcal{O}(\alpha^3)$ hadronic light-by-light scattering effect. The error originates from our oversight of a feature of the algebraic manipulation program FORM which defines the ϵ-tensor in such a way that it satisfies the relation $\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}\eta^{\mu_1\nu_1}\eta^{\mu_2\nu_2}\eta^{\mu_3\nu_3}\eta^{\mu_4\nu_4} = 24$, irrespective of space-time metric. To circumvent this problem, we replaced the product $\epsilon_{\mu_1\mu_2\mu_3\mu_4}\epsilon_{\nu_1\nu_2\nu_3\nu_4}$ by $-\eta_{\mu_1\nu_1}\eta_{\mu_2\nu_2}\eta_{\mu_3\nu_3}\eta_{\mu_4\nu_4} \pm \cdots$ in the FORM-formatted program, and obtained a positive value for the pseudoscalar pole contribution, in agreement with the recent result obtained by Knecht *et al.*

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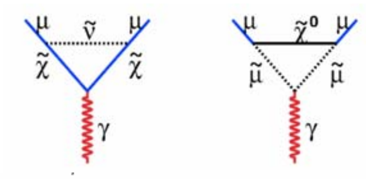
Potential SUSY contribution to muon (g-2)



$$a_\mu^{\text{SUSY}} \simeq \pm 130 \times 10^{-11} \cdot \left(\frac{100\ \text{GeV}}{m_{\text{SUSY}}}\right)^2 \tan\beta\,,$$

Potential SUSY contributions:

For muon ~40000 times larger than in case of electrons.

$$a_\mu = a_\mu^{QED} + a_\mu^{Had} + a_\mu^{EW} + a_\mu^{SUSY}$$

First sign of New Physics ??

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