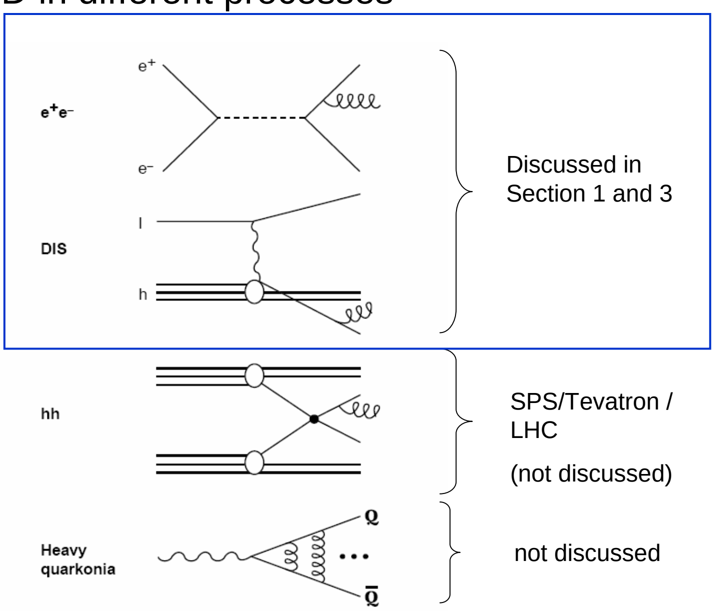
VII. Experimental Tests of QCD

1. Test of QCD in e+e- annihilation
2. Running of the strong coupling constant
3. Study of QCD in deep inelastic scattering

Disclaimer:
Due to the lack of time I have selected only a few items!

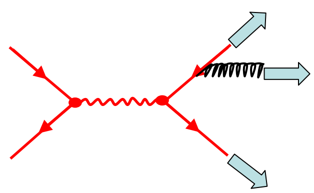
Test of QCD in different processes



1. Test of QCD in e^+e^- annihilation

1.1 Discovery of the gluon

Discovery of 3-jet events by the TASSO collaboration (PETRA) in 1977:



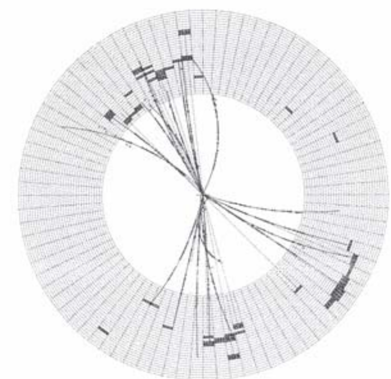


Fig. 11.12 A three-jet event observed by the JADE detector at PETRA.

3-jet events are interpreted as quark pairs with an additional hard gluon.

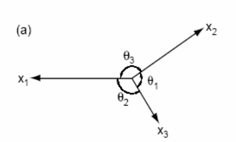
$$\frac{\#3-\text{jet events}}{\#2-\text{jet events}} \approx 0.15 \sim \alpha_s \quad \Rightarrow \quad \alpha_s \text{ is large}$$

at $\sqrt{s}$=20 GeV

1.2 Spin of the gluon

Angular distribution of jets depend on gluon spin:

Ordering of 3 jets: $E_1>E_2>E_3$



Ellis-Karlinger angle

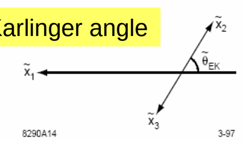


Figure 8: (a) Representation of the momentum vectors in a three-jet event, and (b) definition of the Ellis-Karliner angle.

Measure direction of jet-1 in the rest frame of jet-2 and jet-3: θ_{EK}

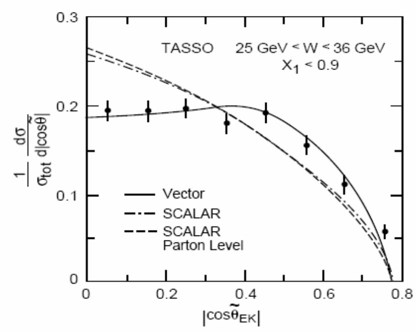


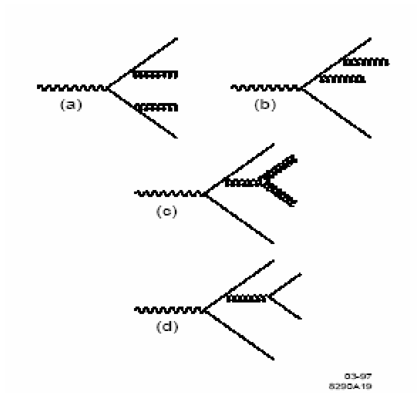
Figure 9: The Ellis-Karliner angle distribution of three-jet events recorded by TASSO at $Q \sim 30$ GeV [18]; the data favour spin-1 (vector) gluons.

Gluon spin J=1

1.3 Multi-jet events and gluon self coupling

Non-abelian gauge theory (SU(3))

4-jet events



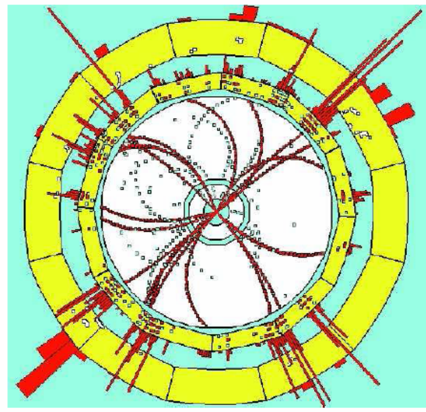


Figure 1: Hadronic event of the type $e^+e^- \to 4$ jets recorded with the ALEPH detector at LEP-I.

➡ 4 jet events allow to test the existence of gluon self coupling.

Multiple jets and jet algorithm

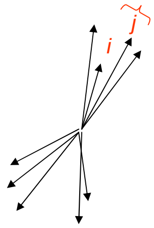
Jet Algorithm

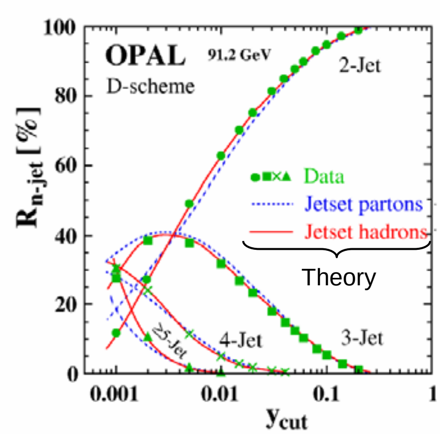
Hadronic particles i and j are grouped to a pseudo particle k as long as the invariant mass is smaller than the **jet resolution parameter**:

$$\frac{m_{ij}^2}{s} < y_{cut}$$

m_{ij} is the invariant mass of i and j.

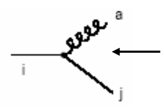
Remaining pseudo particles are **jets.**





Color factors from 4-jet events

Color factors:



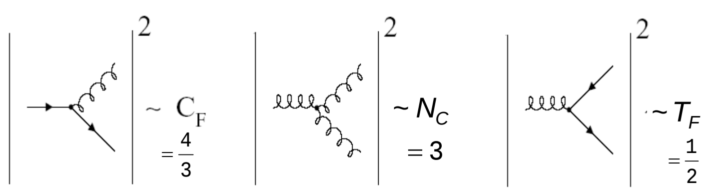
vertex factor $-i\,g_s\,(t_a)_{ij}\,\gamma^\mu$

Contribution to cross section:

$$\sim \sum_{a=1}^{8}\sum_{k=1}^{3}(t_a)_{ik}(t_a)_{kj}$$

$$\sim C_F = \frac{4}{3}$$

Different relative angular distribution



$q \to qg$: $\sim C_F = \frac{4}{3}$

$g \to gg$: $\sim N_C = 3$

$g \to q\bar{q}$: $\sim T_F = \frac{1}{2}$

Angular correlation of jets in 4-jet events

4-jet cross section:

$$\frac{1}{\sigma_0}d\sigma^4 = \left(\frac{\alpha_s C_F}{\pi}\right)^2\left[F_A + \left(1-\frac{1}{2}\frac{N_C}{C_F}\right)F_B + \frac{N_C}{C_F}F_C\right]$$
$$+ \left(\frac{\alpha_s C_F}{\pi}\right)^2\left[\frac{T_F}{C_F}N_f F_D + \left(1-\frac{1}{2}\frac{N_C}{C_F}\right)F_E\right]$$

$F_{A,B,C,D,E}$ are kinematic functions

Exploiting the angular distribution of 4-jets:

- Bengston-Zerwas angle

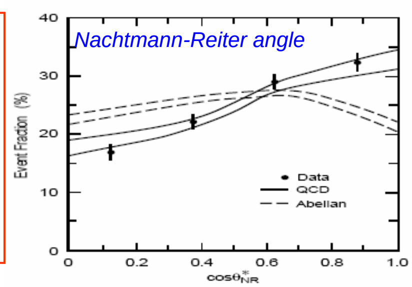
$$\cos\chi_{BZ} \propto (\vec{p}_1\times\vec{p}_2)\cdot(\vec{p}_3\times\vec{p}_4)$$

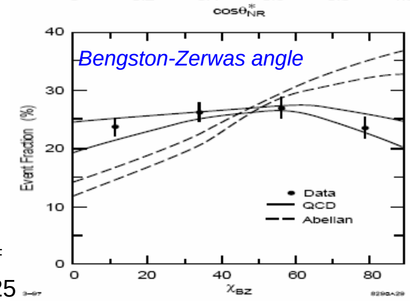
- Nachtmann-Reiter angle

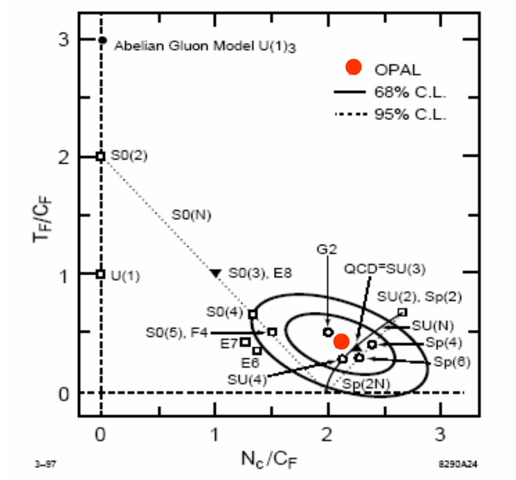
$$\cos\theta_{NR} \propto (\vec{p}_1-\vec{p}_2)\cdot(\vec{p}_3-\vec{p}_4)$$

Allows to measure the ratios T_F/C_F and N_C/C_F

SU(3) predicts: T_F/C_F = 0.375 and N_C/C_F =2.25





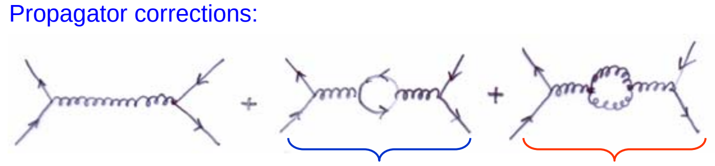


Confirms QCD prediction (SU(3)) and gluon self-coupling:

T_F/C_F = 0.375 and N_C/C_F =2.25

2. "Running" of the strong coupling α_s

Propagator corrections:



Strong coupling $\alpha_s(Q^2)$

$$\alpha_s(Q^2) = \frac{\alpha_s(\mu^2)}{1 + \alpha_s(\mu^2)\frac{1}{12\pi}(33 - 2n_f)\log\frac{Q^2}{\mu^2}}$$

$$\beta_0 = \frac{1}{12\pi}(33 - 2n_f)$$

n_f = active quark flavors

μ^2 = renormalization scale

conventionally $\mu^2 = M_Z^2$

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$

with $\Lambda_{QCD} \approx 200\,\mathrm{MeV}$

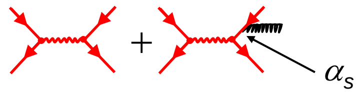
scale at which perturbation theory diverges

Measurement of Q^2 dependence of α_s

➡ α_s measurements are done at given scale Q^2: $\alpha_s(Q^2)$

a) α_s from total hadronic cross section

$e^+e^- \to q\bar{q}(g)$



Final state gluon radiation.

$$\sigma_{had}(s) = \sigma_{had}^{QED}(s)\left[1 + \frac{\alpha_s(s)}{\pi} + 1.411 \cdot \frac{\alpha_s(s)^2}{\pi^2} + \ldots\right]$$

$$R_{had} = \frac{\sigma(ee \to hadrons)}{\sigma(ee \to \mu\mu)} = 3\sum Q_q^2\left\{1 + \frac{\alpha_s}{\pi} + 1.411\frac{\alpha_s^2}{\pi^2} + \ldots\right\}$$

➡ $\alpha_s(s)$

b) α_s from hadronic event shape variables

3-jet rate: $R_3 \equiv \frac{\sigma_{3-jet}}{\sigma_{had}}$ depends on α_s

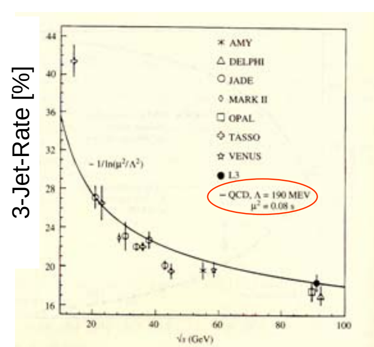
3-jet rate is measured as function of a jet resolution parameter y_{cut}

QCD calculation provides a theoretical prediction for $R_3^{theo}(\alpha_s, y_{cut})$

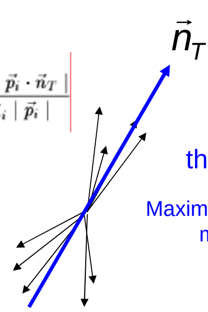
$\to$ fit $R_3^{theo}(\alpha_s, y_{cut})$ to the data to determine α_s

Similarly other event shape variables (sphericity, thrust,…) can be used to obtain a prediction for α_s

➡ $\alpha_s(s)$

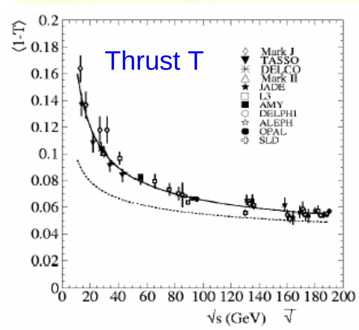


$$T = \max \frac{\sum_i |\vec{p}_i \cdot \vec{n}_T|}{\sum_i |\vec{p}_i|}$$



thrust axis

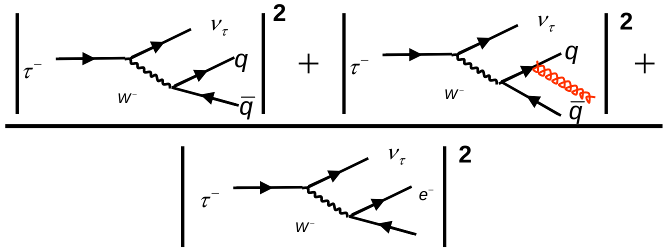
Maximizes longitudinal momentum



c) α_s from hadronic τ decays

$$R^{\tau}_{had} = \frac{\Gamma(\tau \to \nu_\tau + Hadrons)}{\Gamma(\tau \to \nu_\tau + e\bar{\upsilon}_e)} \sim f(\alpha_s)$$

$R^{\tau}_{had} =$

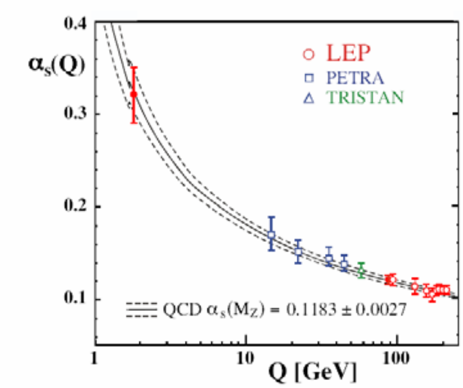


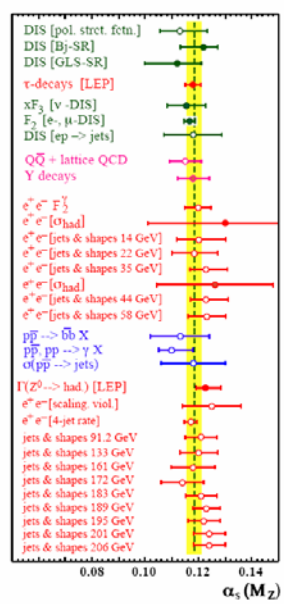
$$R^{\tau}_{had} = R^{\tau,0}_{had}(1 + \frac{\alpha_s(m_\tau^2)}{\pi} + \ldots)$$

d) α_s from DIS (deep inelastic scattering)

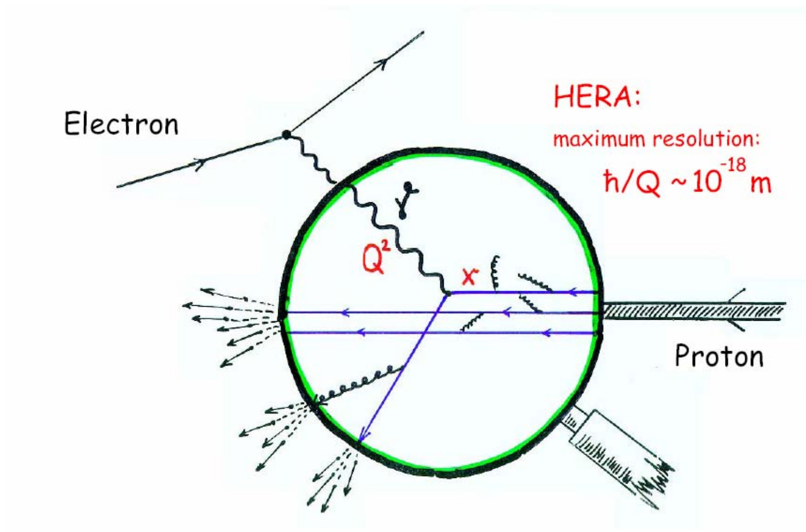
Running of α_s and asymptotic freedom

$$\alpha_s(Q^2) = \frac{1}{\beta_0 \log(Q^2/\Lambda^2_{QCD})}$$



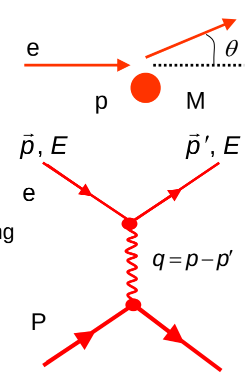


3. Study of QCD in deep inelastic scattering (DIS)



Courtesy: H.C. Schultz-Coulon

3.1 Elastic electron-proton scattering



General form of differential cross section

$$\frac{d\sigma}{d\Omega} = \underbrace{\frac{\alpha^2}{4EE'\sin^4\frac{\theta}{2}}}_{\text{Rutherford}}\{\ldots\ldots\}$$

← non pointlike scattering partners w/ spin

$$-q^2 = 4EE'\sin^4\frac{\theta}{2}$$

Spin ½ electron +

Pointlike target w/o spin
Mott scattering

$$\{\ldots\ldots\}^{elastic}_{Mott} = \left(\cos^2\frac{\theta}{2}\right)$$

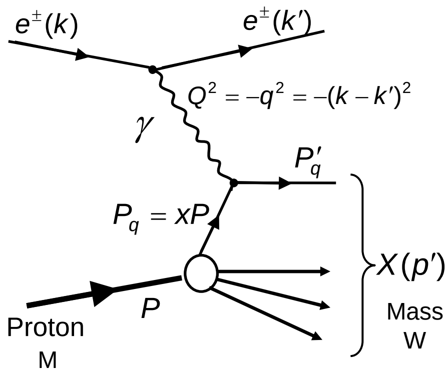
Pointlike target w/ spin
and mass M

$$\{\ldots\ldots\}^{elastic}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

Extended proton w/ spin

$$\{\ldots\ldots\}^{elastic}_{ep\to ep} = \left(\frac{G_E^2+\tau G_M^2}{1+\tau}\cos^2\frac{\theta}{2} + 2\tau G_M^2\sin^2\frac{\theta}{2}\right) \quad \text{mit } \tau = \frac{Q^2}{4M^2}$$

3.2 DIS in the quark parton model (QPM)



- Elastic scattering: W = M

  ⇒ only one free variable

$$\frac{Q^2}{2M\nu} = 1$$

- Inelastic scattering: W ≠ M

  ⇒scattering described by 2 independent variables

$$(E,\nu),\ (Q^2,x),\ (x,y),\ \ldots$$

x = fractional momentum of struck quark

y = Pq/Pk = elasticity, fractional energy transfer in proton rest frame

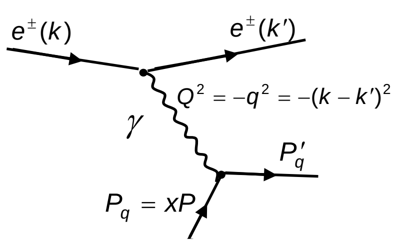
ν = E - E′ = energy transfer in lab

$$Q^2 = sxy \qquad \text{s = CMS energy}$$

$$x = \frac{Q^2}{2M\nu} \qquad \text{(Bjorken x)}$$

Cross section in quark parton model (QPM)

Elastic scattering on single quark



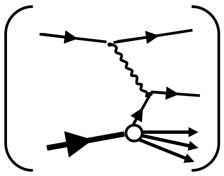
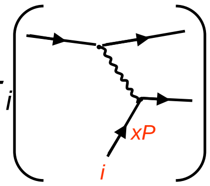
Starting point:
electron muon scattering

$$\{\ldots\ldots\}^{elastic}_{e\mu\to e\mu} = \left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2M^2}\sin^2\frac{\theta}{2}\right)$$

Electron-quark scattering:

$$\frac{d\sigma}{dQ^2} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot e_i^2\left(\cos^2\frac{\theta}{2} + \frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

↑ charge

σ  $= \sum_i q_i(x)\,\sigma_i$ 

Parton density $q_i(x)dx$: Probability to find parton i in momentum interval [x, x+dx]

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\sum_i\int_0^1 e_i^2\cdot q_i(\xi)\cdot\delta(x-\xi)d\xi\left(\cos^2\frac{\theta}{2}+\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

Structure functions

$$F_2(x) = x\sum_i\int_0^1 e_i^2 q_i(\xi)\cdot\delta(x-\xi)d\xi = x\sum_i e_i^2 q_i(x)$$

$$F_1(x) = \frac{1}{2}\sum_i e_i^2 q_i(x)$$

ignore ant-quarks!

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4}\right)\frac{E'}{E}\cdot\left(\frac{F_2(x)}{x}\cos^2\frac{\theta}{2}+2F_1(x)\frac{Q^2}{2x^2M^2}\sin^2\frac{\theta}{2}\right)$$

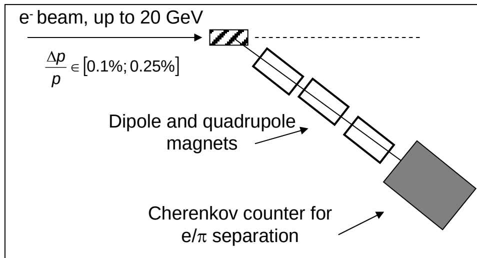
Kinematical relations

$$\frac{d^2\sigma}{dQ^2dx} = \left(\frac{4\pi\alpha^2}{Q^4x}\right)\cdot\left((1-y)F_2(x)+xy^2F_1(x)\right)$$

Deep inelastic electron-proton scattering:

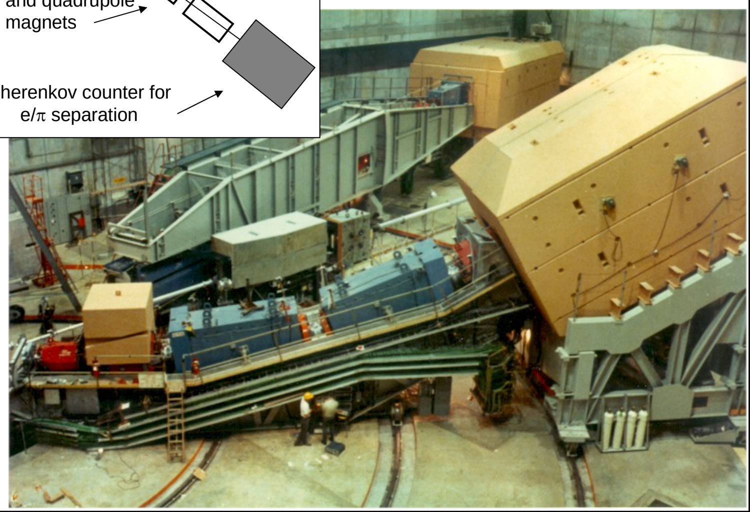
- Free partons: $F_2=F_2(x)$ ⇔ “scaling”
- Spin ½ partons: $2xF_1(x) = F_2(x)$ (Callan-Gross relation)

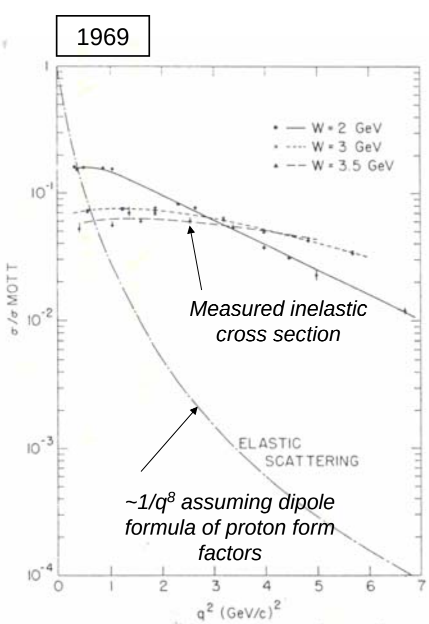
SLAC/MIT Experiment (1969)

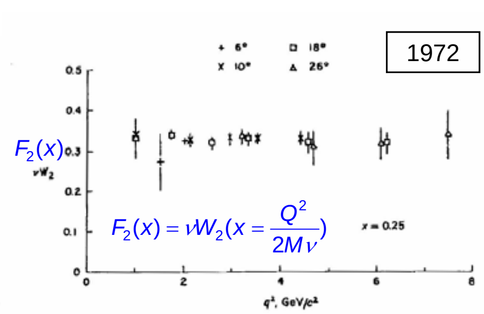


Spectrometer at fixed θ

$\frac{\Delta p}{p} \sim 0.1\% \quad \Delta\theta \sim 0.7\text{mrad}$







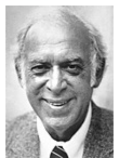
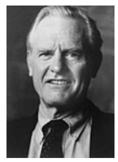
Structure function F_2 (=νW_2) depends only on the dimensionless variable x:

$$x = \frac{Q^2}{2M\nu}$$

→ **Scale invariance: "scaling"**

Indicates elastic scattering at point-like free constituents of the proton: partons

The Nobel Prize in Physics 1990

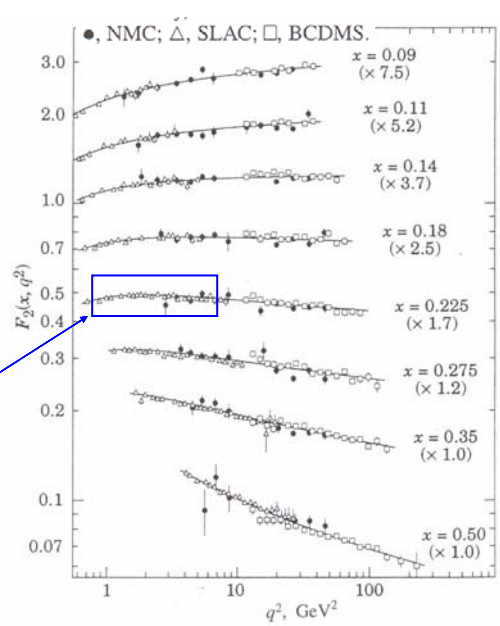
  

Jerome I. Friedman	Henry W. Kendall	Richard E. Taylor

"for their pioneering investigations concerning deep inelastic scattering of electrons on protons and bound neutrons, which have been of essential importance for the development of the quark model in particle physics"

3.5 Scaling violation

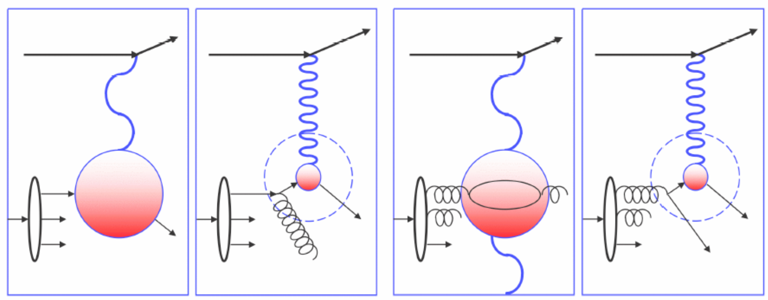
$$F_2 = F_2(x,Q^2) = x\sum_i e_i^2 q_i(x)$$



Region of 1st SLAC measurement (1972)

QCD explains observed scaling violation

Large x: valence quarks — Small x: Gluons, sea quarks

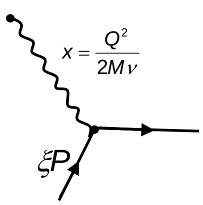


$Q^2 \uparrow \Rightarrow F_2 \downarrow$ for fixed x — $Q^2 \uparrow \Rightarrow F_2 \uparrow$ for fixed x

Scaling violation is one of the clearest manifestation of radiative effect predicted by QCD.

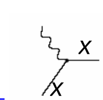
Quantitative description of scaling violation

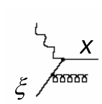
Quark Parton Model



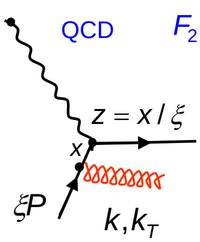
$$F_2(x) = x\sum_i e_i^2 \int_0^1 q_i(\xi)\cdot\delta(x-\xi)d\xi = x\sum_i e_i^2 q_i(x)$$

$$\delta(ax) = \frac{1}{|a|}\delta(x)$$





QCD



$$F_2(x,Q^2) = x\sum_i e_i^2 \int_0^1 \frac{d\xi}{\xi} q_i(\xi)\cdot\left[\delta\left(1-\frac{x}{\xi}\right) + \frac{\alpha_s}{2\pi}P_{qq}\left(\frac{x}{\xi}\right)\log\frac{Q^2}{\mu_0^2}\right]$$

$$\sim \frac{\alpha_s}{2\pi}P_{qq}(z)\int_{\mu_0^2}^{Q^2}\frac{dk_T^2}{k_T^2}$$

$$\sim \frac{\alpha_s}{2\pi}P_{qq}(z)\log\left(\frac{Q^2}{\mu_0^2}\right)$$

P_{qq} probability of a quark to emit gluon and becoming a quark with momentum reduced by fraction z.

μ^0 cutoff parameter

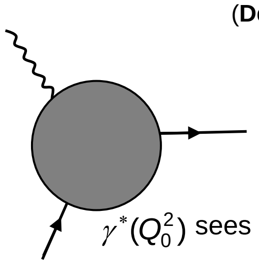
Changing to the quark densities:

$$q_i(x,Q^2) = q_i(x) + \underbrace{\frac{\alpha_s}{2\pi}\log\frac{Q^2}{\mu_0^2}\int_0^1 \frac{d\xi}{\xi} q_i(\xi) P_{qq}(\frac{x}{\xi})}_{\Delta q(x,Q^2)}$$

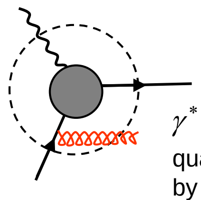
Integro-differential equation for $q(x,Q^2)$:

$$\frac{d}{d\log Q^2} q(x,Q^2) = \frac{\alpha_s}{2\pi}\int_0^1 \frac{d\xi}{\xi} q(\xi,Q^2) P_{qq}(\frac{x}{\xi})$$

DGLAP evolution equation
(**D**okshitzer, **G**ribov, **L**ipatov, **A**ltarelli, **P**arisi, 1972 – 1977)



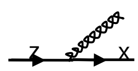
$\gamma^*(Q_0^2)$ sees q(x)

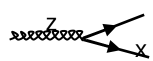


$\gamma^*(Q^2 > Q_0^2)$ sees that quarks q(x) are surrounded by softer quarks

Evolution of parton densities (quarks and gluons)

evolution of quark density with $\ln Q^2$

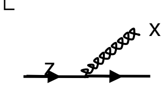


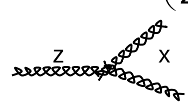


$$\frac{\partial q(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{qq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{qg}\left(\frac{x}{z}\right)\right]$$

$$\frac{\partial g(x,Q^2)}{\partial \ln Q^2} = \frac{\alpha_s}{2\pi}\int_x^1 \frac{dz}{z}\left[q(z,Q^2)P_{gq}\left(\frac{x}{z}\right) + g(z,Q^2)P_{gg}\left(\frac{x}{z}\right)\right]$$

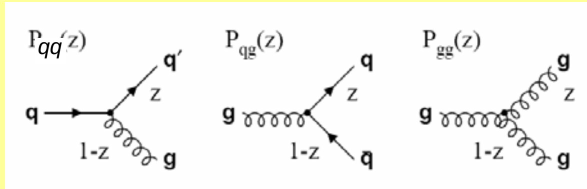
evolution of gluon density with $\ln Q^2$





Splitting functions: Probability that a parton (quark or gluon) emits a parton **(q, g)** with momentum fraction ε=x/z of the parent parton.

Splitting functions are calculated as power series in α_s up to a given order:



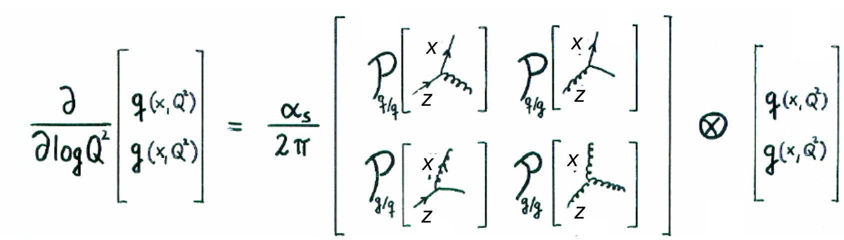
$$P_{ij}(z,\alpha_s) = P_{ij}^0(z) + \frac{\alpha_s}{2\pi} P_{ij}^1(z) + \dots$$

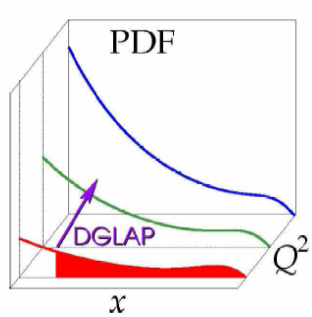
In leading order: $P_{ij}(z,\alpha_s) \equiv P_{ij}^0(z)$

$$P_{qq}(z) = \frac{4}{3}\frac{1+z^2}{1-z} \qquad P_{gq}(z) = \frac{4}{3}\frac{1+(1-z)^2}{z}$$

$$P_{qg}(z) = \frac{z^2+(1-z)^2}{2} \qquad P_{gg}(z) = 6\left(\frac{z}{1-z} + \frac{1-z}{z} + z(1-z)\right)$$

DGLAP Evolution ("symbolic"):



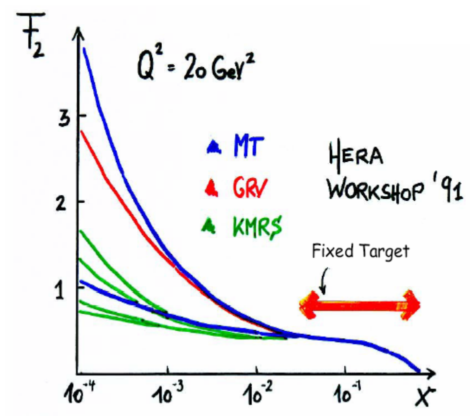


$$P \otimes f(x,Q^2) = \int_x^1 \frac{dz}{z} P\left(\frac{x}{z}\right) f(z,Q^2)$$

QCD evolution

x dependence of parton densities:

Cannot be deduced from fundamental principles.



By H.C. Schultz-Coulon

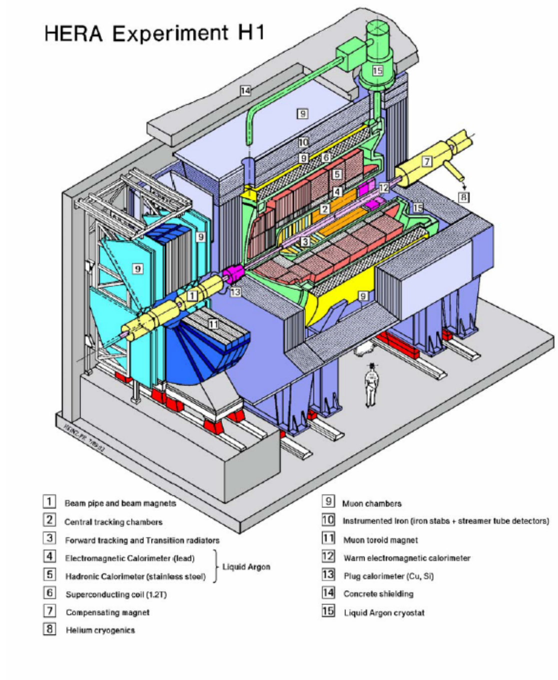
DGLAP:

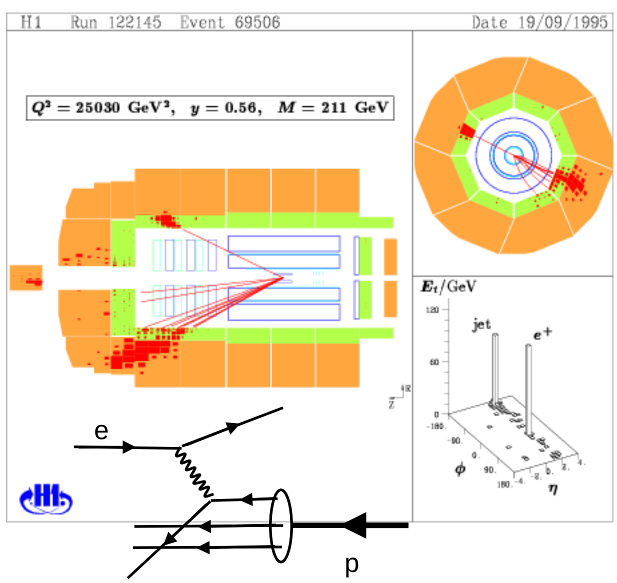
Q^2 dependence at given x but no prediction for the x dependence of the parton densities.

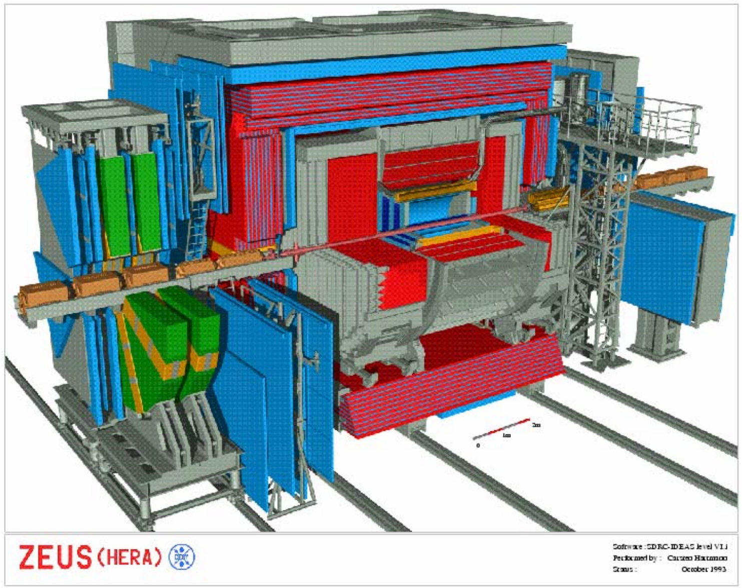
Status in 1991 (pre HERA): Data limited to a small x region. Models to extrapolate to smaller x differed significantly.

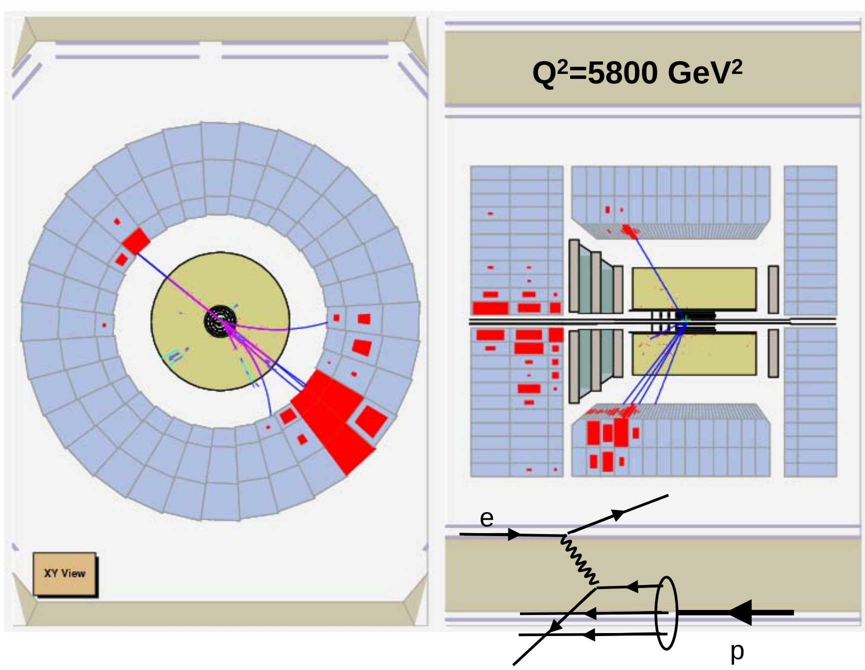
Measure structure functions (parton densities) at low x.

HERA $e^{\pm}$ 30 GeV → ← p 900 GeV $s = 4E_eE_p \approx 10^5\,\mathrm{GeV}^2$


HERA Experiment H1
1 Beam pipe and beam magnets
2 Central tracking chambers
3 Forward tracking and Transition radiators
4 Electromagnetic Calorimeter (lead)
5 Hadronic Calorimeter (stainless steel)
Liquid Argon
6 Superconducting coil (1.2T)
7 Compensating magnet
8 Helium cryogenics
9 Muon chambers
10 Instrumented Iron (iron slabs + streamer tube detectors)
11 Muon toroid magnet
12 Warm electromagnetic calorimeter
13 Plug calorimeter (Cu, Si)
14 Concrete shielding
15 Liquid Argon cryostat


H1 Run 122145 Event 69506
Date 19/09/1995
$Q^2 = 25030$ GeV^2, $y = 0.56$, $M = 211$ GeV
E_t/GeV
jet
e^+
ϕ
η
e
p


ZEUS (HERA)


Q2=5800 GeV2
XY View
e
p

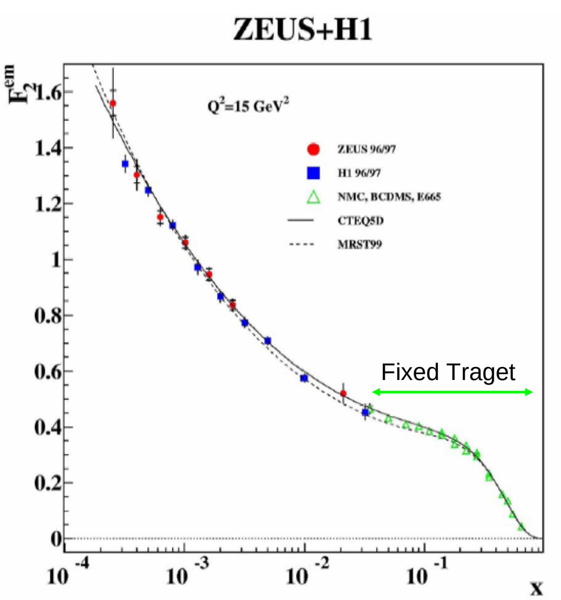
Measurement of the parton densities / F_2

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot\left(2\cdot(1-y)F_2(x,Q^2) + y^2 F_2(x,Q^2)\right)$$

e.g. for y=1 $\quad Q^2 = sxy$

$$\frac{d^2\sigma}{dx\,dQ^2} = \left(\frac{2\pi\alpha^2}{xQ^4}\right)\cdot F_2(x,Q^2)$$

$$F_2(x,Q^2) = x\sum_q e_q^2\left[q(x,Q^2) + \bar{q}(x,Q^2)\right]$$



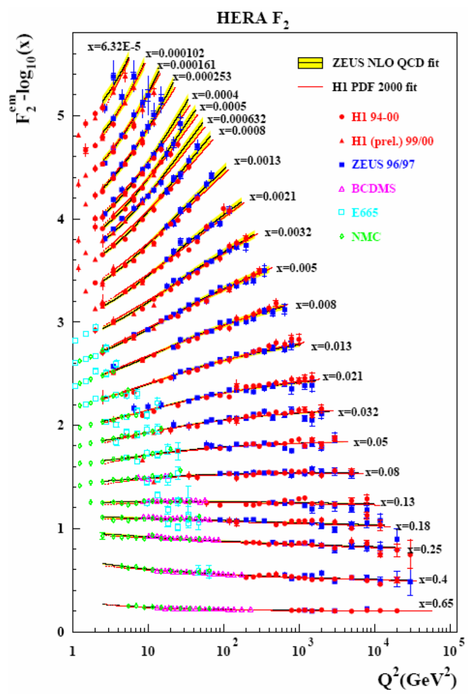
$F_2(x)$

Large increase of $F_2(x)$ for very small x - unexpected

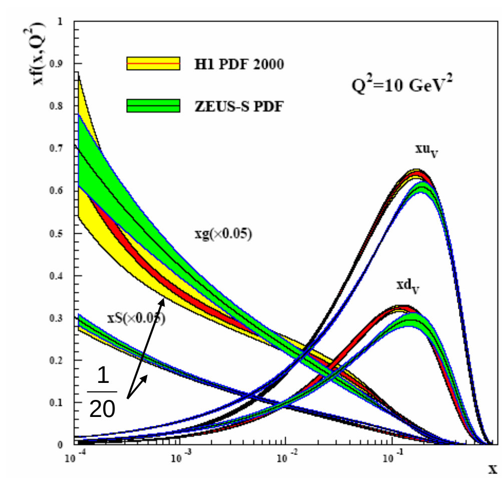
When does the rise stop ??

$F_2(x,Q^2)$

Q^2 dependence is correctly described by QCD evolution



Structure of the proton as seen by HERA



$$\#\ \text{Valenzquarks} = \int u_v(x) + d_v(x)\, dx = 3$$

$$\#\ \text{Gluonen} = \int g(x)\, dx > 30$$