V. Electro-weak unification: Phenomenological approach to the Standard Model (SM)

1. Requisites
2. Weak isospin and weak hypercharge
3. Couplings to gauge fields
4. Feynman rules

I am not referring explicitly to the requirement of local gauge symmetry as was done in the theoretical part of the lecture – here a different (phenomenological) approach to the SM is presented. Starting point is the observed symmetry structure of the currents and their coupling to the bosons.

1. Requisites

a) Fundamental fermions

Leptons

$$\begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu^- \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau^- \end{pmatrix}_L \qquad \text{Left-handed doublets}$$

$$e_R^- \quad \mu_R^- \quad \tau_R^- \qquad \text{right-handed singlets}$$

Quarks

$$\begin{pmatrix} u \\ d' \end{pmatrix}_L \quad \begin{pmatrix} c \\ s' \end{pmatrix}_L \quad \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

$$u_R, d_R \quad c_R, s_R \quad t_R, b_R$$

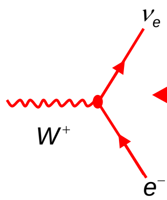
b) Fundamental interaction

Weak interaction:

- Charged current interaction: transitions inside LH doublets
- Neutral current interaction: couples to LH and RH fermions

- Electromagnetic interaction couples equally to LH and RH fermions

Charged current weak interaction

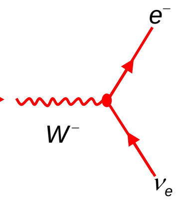


Charge raising current:

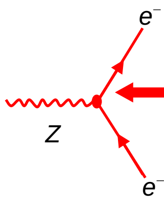
$$J_\mu^+ = \bar{u}_\nu \gamma_\mu \frac{1-\gamma^5}{2} u_e = \bar{\nu}\gamma_\mu \frac{1-\gamma^5}{2} e = \bar{\nu}_L \gamma_\mu e_L$$

Charge lowering current:

$$J_\mu^- = \bar{e}\gamma_\mu \frac{1-\gamma^5}{2}\nu = \bar{e}_L \gamma_\mu \nu_L$$



Neutral current weak interaction



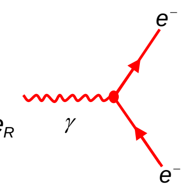
$$J_\mu^{NC} = \bar{e}\gamma_\mu \frac{1}{2}(g_V - g_A\gamma^5)e = \bar{e}\gamma_\mu \left[\underbrace{\frac{1}{2}(g_V + g_A)}_{g_L} \frac{1-\gamma^5}{2} + \underbrace{\frac{1}{2}(g_V - g_A)}_{g_R} \frac{1+\gamma^5}{2} \right] e$$

$$= g_L\, \bar{e}_L \gamma_\mu e_L + g_R\, \bar{e}_R \gamma_\mu e_R$$

Electromagnetic interaction

$$J_\mu^{em} = Q\bar{e}\gamma_\mu e = Q\bar{e}_L\gamma_\mu e_L + Q\bar{e}_R\gamma_\mu e_R$$

Charge in units of e



2. Weak isospin and weak hypercharge

Weak isospin:

In analogy to the strong isospin one can describe the particles of the LH doublets as $T_3=\pm½$ states of a particle with weak isospin T= ½.

$$\chi_L = \begin{pmatrix} \nu \\ e \end{pmatrix}_L \quad T = \frac{1}{2}, T_3 = \pm\frac{1}{2}$$

As in case of the (iso)spin one can use the raising and lowering operators defined by the Pauli matrices to express state transitions.

$$\sigma^\pm = \frac{1}{2}(\sigma^1 \pm i\cdot\sigma^2)$$

$$\sigma^i = \text{Pauli-matrices}$$

The charge current then can be written in the compact form:

$$J_\mu^\pm = \bar{\chi}_L \gamma_\mu \sigma^\pm \chi_L$$

From the SU(2) structure of the isospin formalism one expects that in addition to the currents $J^\pm$ there exists a 3rd neutral current J^3 of the form:

$$J_\mu^3 = \bar{\chi}_L\gamma_\mu \cdot T_3\sigma^3\chi_L = \frac{1}{2}\bar{\nu}_L\gamma_\mu\nu_L - \frac{1}{2}\bar{e}_L\gamma_\mu e_L$$

Weak isospin triplett of LH fermion currents

$$J_\mu^i = \bar{\chi}_L \gamma_\mu \frac{1}{2}\sigma^i \chi_L$$

Electro-weak unification

The current J^3 is not equal to the current J^{NC}:
J^{NC} contains LH and RH fermion contributions

Treat both neutral currents, J^{em} and J^{NC}, simultaneously:

As both currents contain RH contributions it should be possible to construct a linear combination which couples only to LH fermions:

Two linear combinations of J^{em} and J^{NC}

$$J_\mu^3 = \sin^2\theta_W J_\mu^{em} + J_\mu^{NC}$$
$$J_\mu^Y = \cos^2\theta_W J_\mu^{em} - J_\mu^{NC}$$

Choose θ_W such that RH fermions components in J^3 vanish.

- J^3 completes the isospin triplet J^i
- J^Y is called hypercharge current, couples via hypercharge

$$J_\mu^Y = J_\mu^{em} - J_\mu^3$$

Hypercharge

From the above definitions follows:

$$J_\mu^Y = J_\mu^{em} - J_\mu^3 = \bar\psi\gamma_\mu[Q - T_3]\psi = \bar\psi\gamma_\mu Y\psi$$

Hypercharge operator: $Y = [Q-T_3]$

(Gell-Mann Nishijima Formula)

Electro-weak quantum numbers

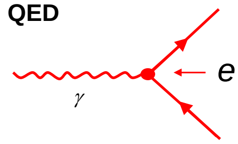
In many books (PDG) a different convention is used: $Y = 2[Q-T_3]$

Leptons	T	T_3	Q	Y
ν_e	$1/2$	$+1/2$	0	$-1/2$
e_L	$1/2$	$-1/2$	-1	$-1/2$
e_R	0	0	-1	-1

Quarks	T	T_3	Q	Y
u_L	$1/2$	$+1/2$	$2/3$	$1/6$
d'_L	$1/2$	$-1/2$	$-1/3$	$1/6$
u_R	0	0	$2/3$	$4/6$
d_R	0	0	$-1/3$	$-2/6$

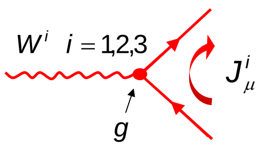
3. Current coupling to the gauge fields/bosons

QED



$$\sim -ieJ_\mu^{em} \cdot A^\mu$$

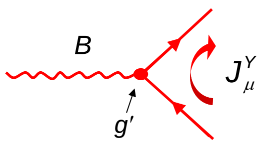
In the **electro-weak theory** the coupling between boson and fermions is defined in analogy to the coupling of the photon to the fermions currents in QED. There are in total 4 boson fields:



$$\sim -igJ_\mu^i \cdot W^{i,\mu}$$

Corresponding to $J^\pm$ and J^3 there are fields

$$W_\mu^\pm = \frac{1}{\sqrt{2}}(W_\mu^1 \mp iW_\mu^2) \quad \text{and} \quad W_\mu^3$$



$$\sim -ig'J_\mu^Y \cdot B^\mu$$

(sometimes ½ = convention: $Y=2[Q-T_3]$)

g, g′ are coupling constants.

Gauge bosons:

While the charged boson fields $W^\pm$ correspond to the observed W bosons, the neutral fields B and W^3 only correspond to linear combinations of the observed photon and Z boson:

$$A_\mu = B_\mu \cos\theta_W + W_\mu^3 \sin\theta_W \quad \leftarrow \text{massless photon}$$

$$Z_\mu = -B_\mu \sin\theta_W + W_\mu^3 \cos\theta_W \quad \leftarrow \text{massive Z boson}$$

$$B_\mu = A_\mu \cos\theta_W - Z_\mu \sin\theta_W$$

$$W_\mu^3 = A_\mu \sin\theta_W + Z_\mu \cos\theta_W$$

The weak mixing angle θ_W (Weinberg angle) is defined by the coupling strength to A^μ and Z^μ to the fermion current:

Coupling strength of A^μ to the fermion currents is “e” !!

The fermion coupling to the neutral boson fields are given by →

$$-igJ_\mu^3 \cdot W^{3,\mu} - ig'J_\mu^Y \cdot B^\mu = i\left[g\sin\theta_W J_\mu^3 + g'\cos\theta_W J_\mu^Y\right]A^\mu - i\left[g\cos\theta_W J_\mu^3 - g'\sin\theta_W J_\mu^Y\right]Z^\mu$$

Fermion coupling to the photon

$$[\quad]A^\mu \equiv eJ_\mu^{em}A^\mu = e\left[J_\mu^3 + J_\mu^Y\right]A^\mu$$

$$J_\mu^{em} = \left[J_\mu^3 + J_\mu^Y\right]$$

Comparison of the coefficients gives:

$$e = g\cdot\sin\theta_W \qquad g = \frac{e}{\sin\theta_W} \qquad g' = \frac{e}{\cos\theta_W}$$
$$e = g'\cdot\cos\theta_W$$

The couplings to the different boson types have similar strength.

Fermion coupling to the Z boson

From $-i[\quad]Z^\mu$ follows with

$$\begin{cases} J_\mu^Y = \left[J_\mu^{em} - J_\mu^3\right] \\ g\cdot\sin\theta_W = g'\cdot\cos\theta_W \end{cases}$$

$$-i[\quad]Z^\mu = -i\frac{g}{\cos\theta_W}\left[J_\mu^3 - \sin^2\theta_W J_\mu^{em}\right]Z^\mu$$
$$= -i\frac{g}{\cos\theta_W}J_\mu^{NC}Z^\mu$$

Using

$$\begin{cases} J_\mu^3 = \bar{\chi}_L\gamma_\mu \cdot T_3\tau^3\chi_L \\ J_\mu^{em} = \bar{e}\gamma_\mu Q e \end{cases}$$

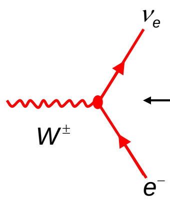
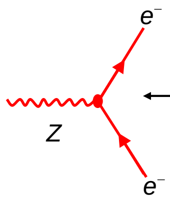
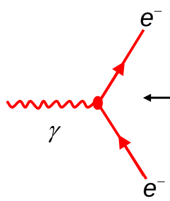
one finds

$$-i[\quad]Z^\mu = -i\frac{g}{\cos\theta_W}\left[T_3\cdot\bar{e}\gamma_\mu\frac{1-\gamma^5}{2}e - q\cdot\sin^2\theta_W\,\bar{e}\gamma_\mu e\right]Z^\mu$$

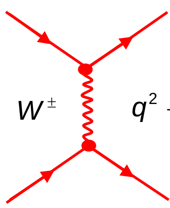
$$-i[\quad]Z^\mu = -i\frac{g}{\cos\theta_W}\left[\bar{e}\gamma_\mu\frac{1}{2}[g_V - g_A\gamma^5]e\right]Z^\mu$$

with $g_V = T_3 - 2q\sin^2\theta_W$ and $g_A = T_3$

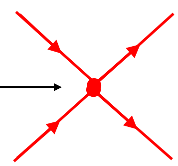
4. Feyman rules

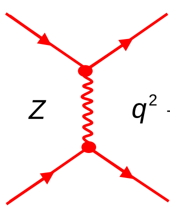
	Vertex factors	Propagator
	$-i\frac{g}{\sqrt{2}}\gamma_\mu\frac{1}{2}(1-\gamma^5)$	$\frac{g_{\mu\nu}-q_\mu q_\nu/M_W^2}{q^2-M_W^2}$
	$-i\frac{g}{\cos\theta_W}\gamma_\mu\frac{1}{2}(g_V-g_A\gamma^5)$	$\frac{g_{\mu\nu}-q_\mu q_\nu/M_Z^2}{q^2-M_Z^2}$
	$-ie\gamma_\mu$	$\frac{1}{q^2}$

Comparison of the $q^2 \to 0$ limit with the 4-femion ansatz

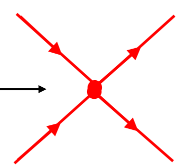


$q^2 \to 0:$ $\quad \frac{g^2}{8M_W^2} = \frac{G_F}{\sqrt{2}}$





$q^2 \to 0:$ $\quad \frac{g^2}{8\cos^2\theta_W M_Z^2} = \frac{G_{NC}}{\sqrt{2}}$



For $\frac{G_F}{\sqrt{2}} \equiv \frac{G_{NC}}{\sqrt{2}}$ follows $\cos^2\theta_W = \frac{M_W^2}{M_Z^2}$ ⬅ follows also from Higgs mechanism