i) $U(1)$ gauge theory: (Abelian Higgs model)

$$\mathcal{L}(x) = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} + \mathcal{D}_\mu \phi^* \mathcal{D}^\mu \phi - \mu^2 \phi^* \phi - \lambda (\phi^* \phi)^2$$

$$\boxed{\mu^2 < 0}$$

$$B^{\mu\nu} = \partial^\mu B^\nu - \partial^\nu B^\mu$$

$$\mathcal{D}^\mu = \partial^\mu + i g' Y B^\mu$$

Again we have $\phi_0 = v/\sqrt{2} \in \mathbb{R}$ as minimum:

Mass-term for B_μ:

$$\mathcal{D}_\mu \phi_0 \mathcal{D}^\mu \phi_0 = -(g' Y v)^2 B_\mu B^\mu$$

with mass $m_B^2 = (g' Y v)^2$.

dofs: $\phi_1, \phi_2, B_\mu^\pm$ ← Helicity $\longrightarrow$ $\sigma, \pi, B_\mu^\pm, B_\mu^L$ Longitud.

$1 + 1 + 2 \; [+\omega]$ $\qquad$ $1 + 1 + 3$

Perform gauge trafo on ϕ with $e^{i\omega}$, $\omega = -\arctan\frac{\pi}{v+\sigma}$

$$\phi \to e^{i\omega} \phi = (\cos\omega + i \sin\omega) \frac{1}{\sqrt{2}} (v + \sigma + i\pi)$$

$$= \frac{1}{\sqrt{2}} \sqrt{(v+\sigma)^2 + \pi^2} = \frac{1}{\sqrt{2}} (v + \sigma')$$

$$\text{with } \sigma' = \sqrt{(v+\sigma)^2 + \pi^2} - v = \sigma + \pi^2/2v + \dots$$

Unitary gauge

The gauge field has 'eaten up' the Goldstone Boson (Higgs (-Kibble) dinner)

iii) Electro-weak theory: $SU(2) \times U(1) \xrightarrow{SSB} U_{em}(1)$

4 gen. $\rightarrow$ 1 gen.

3 Goldstone bosons eaten up

$$\phi_0 = \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \qquad \begin{pmatrix} \phi_1 \\ \phi_2 \end{pmatrix}$$

$$e^{i\omega\underbrace{(T^3 + Y_H)}} \phi_0 = \phi_0 \quad \leftarrow \phi_2 \text{ neutral}$$

$T^3 + Y_H$ gen. of $U_{em}(1)$: $\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$

$\Rightarrow U_{em}(1)$ is unbroken symmetry

p. 13 $W^\pm, Z^0$ massive

[We have used that

$$\phi = \left(e^{i\omega^a T^a + Y_H \omega}\right) \phi_0$$

to gauge away the 3 Goldstones]

We chose SU(2) gauge transformation such that

$$\mathcal{U}\,\phi(x) = \begin{pmatrix} 0 \\ 1/\sqrt{2}\,(\rho + v) \end{pmatrix}.$$

This is left invariant under the U(1)-trafos

$$e^{i\omega(T^3 + Y_w)}$$

It follows that

(1) $$D_\nu \phi^\dagger D^\nu \phi = \frac{1}{2}\partial_\nu \rho\, \partial^\nu \rho + m_W^2 \left(1 + \rho/v\right)^2 W_\nu^+ W^{-\nu}$$
$$+ \frac{1}{2} m_Z^2 \left(1 + \rho/v\right)^2 Z_\nu^0 Z^{0\nu}$$

$\boxed{m_Z, m_W \quad \text{p. } 4-13}$

(2) $$V(\phi^+\phi) = V\left(\tfrac{1}{2}(v+\rho)^2\right)$$
$$= \frac{1}{2}\mu^2 (v+\rho)^2 + \lambda/4\, (v+\rho)^4$$
$$= \frac{1}{2} m_\rho^2\, \rho^2 \left(1 + \rho/v + \frac{1}{4}(\rho/v)^2\right)$$

with $\boxed{m_\rho^2 = 2\lambda v^2}$ Higgs mass

parameter

(3) leptons: electron $\qquad e = g/\sin\Theta_W$

$$\overline{\Psi}_e \, i\gamma^\mu \partial_\mu \Psi_e$$

p. 5-8

$$= \overline{\Psi}_e \, i\gamma^\mu \partial_\mu \Psi_e \quad - e A_\mu^0 \, j^\mu_{em}$$

$$- \frac{2e}{\sin 2\Theta_W} Z_\mu^0 \, j^\mu_{nc}$$ p. 5-9

$$\frac{1}{\sqrt{2}} \frac{e}{\sin\Theta_W} \left(W_\mu^+ j^\mu_{cc} + W_\mu^- j^{\mu\,\dagger}_{cc} \right)$$

with $\quad j^\mu_{cc} = \overline{\Psi}_e \gamma^\mu (T^1 + i T^2) \Psi_e$

$$\left[= \overline{\Psi}_e \gamma^\mu (T^1 + i T^2) (1-\gamma_5)/2 \, \Psi_e \right]$$

$$- h_4 \left(\overline{\Psi}_R \, \Phi^+ \Psi_L - \overline{\Psi}_L \, \Phi \Psi_R \right)$$

$$= m_e \overline{\Psi}_e \Psi_e (1 + \delta/v_0)$$

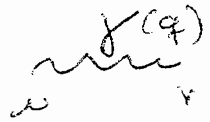
in general $\quad \Psi = \begin{pmatrix} \Psi_e \\ \Psi_\mu \\ \Psi_\tau \\ \Psi_? \end{pmatrix}$ $\quad$ allows for Mass matrix

$\Rightarrow$ mixing!

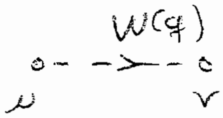
Kobayashi-Maskawa matrix $(\vartheta_1, \vartheta_2, \vartheta_3, \delta)$

Feynman rules: e.g. Waldmann 430-432

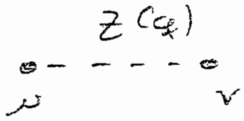
Propagators:



$$: \frac{-i\, g_{\mu\nu}}{q^2 + i\varepsilon} \leftarrow \text{Feynman gauge}$$



$$: \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_W^2}\right)}{q^2 - m_W^2 + i\varepsilon}$$

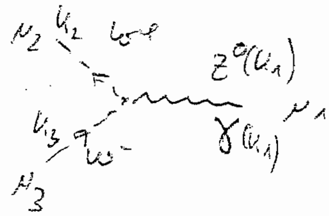


$$: \frac{-i\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{m_Z^2}\right)}{q^2 - m_Z^2 + i\varepsilon}$$

$$: \frac{i}{q^2 - m_\varphi^2 + i\varepsilon}$$

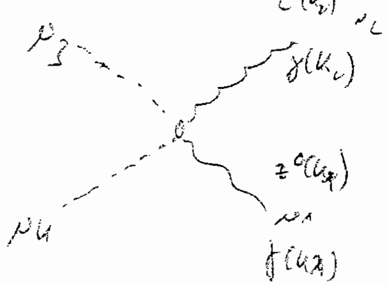
(selected vertices:)

gauge-gauge



$$\left(ie\frac{\cos\theta_W}{\sin\theta_W}\right)$$

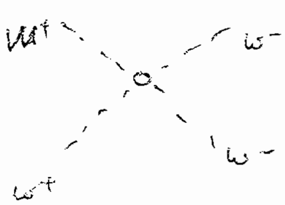
$$:(ie)\left\{(k_1-k_2)_{\mu_3}\, g_{\mu_1\mu_2} + (k_2-k_3)_{\mu_1}\, g_{\mu_2\mu_3} + (k_3-k_1)_{\mu_2}\, g_{\mu_3\mu_1}\right\}$$



$$Z^2 \quad ie^2\frac{\cos^2\theta}{\sin^2\theta}$$

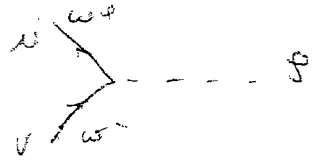
$$\gamma Z \quad ie^2\frac{\cos\theta_W}{\sin\theta_W}$$

$$:\gamma^2\left(ie^2\right)\left\{g_{\mu_1\mu_3}\, g_{\mu_2\mu_4} + g_{\mu_1\mu_4}\, g_{\mu_2\mu_3} - 2\, g_{\mu_1\mu_2}\, g_{\mu_3\mu_4}\right\}$$

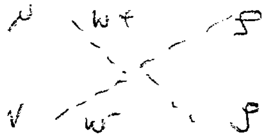


$$: -ie^2\frac{-}{\sin^2\theta_W}$$

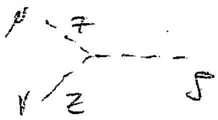
gauge - Higgs φ



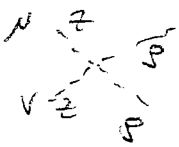
: $i g_{\mu\nu} \dfrac{2m_W^2}{v}$



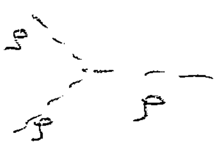
: $i g_{\mu\nu} \dfrac{2m_W^2}{v}$



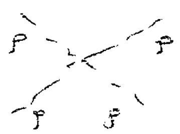
: $i g_{\mu\nu} \dfrac{2m_Z^2}{v}$



: $i g_{\mu\nu} \dfrac{2m_Z^2}{v^2}$



: $-3i \dfrac{m_\varphi^2}{v}$



: $-3i \dfrac{m_\varphi^2}{v^2}$